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220. To Draw from Any Two Given Points A, B Two Straight Lines to a Point R on a Given Circle, Having Centre at C, So That They May Be Equally Inclined to CR

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to all members of our Association, and especially to mathematical teachers ; moreover, the form of publication with the additional blank pages will enable anyone to add additional notes and references about the most recent work.

It is necessary continually to remind the new members of the value of history in mathematical teaching ; the back numbers of the *Gazette* have some very happy papers on this matter. T. J. GARSTANG.

220. [K. 10. e.] To draw from any two given points  $A, B$  two straight lines to a point  $R$  on a given circle, having centre at  $C$ , so that they may be equally inclined to  $CR$ .

The following solution has the advantage that the curve made use of is independent of the data, and consequently, when once described, serves for every case that may arise.

Let  $CB=a$ ,  $CA=b$ , radius of circle  $=c$ ,  $\angle ACB=2a$ .

Take for the initial vector  $CH$ , the bisector of  $\angle ACB$ .

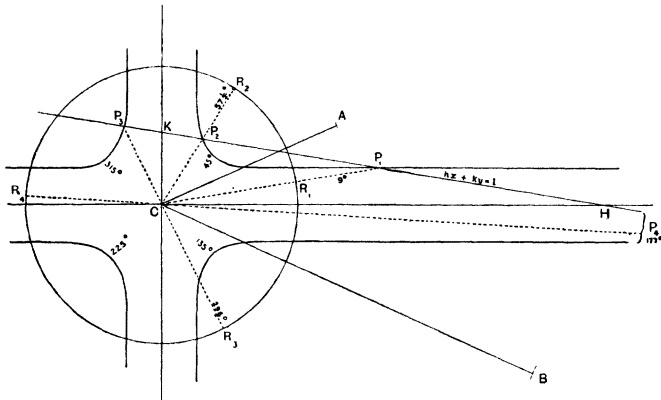
Then, if a point  $R$  move so that  $AR$  and  $BR$  are always equally inclined to  $CR$ , the polar equation of the locus is

$$\frac{r}{2ab} \{ (a-b) \sin a \operatorname{cosec} \theta + (a+b) \cos a \sec \theta \} = 1, \dots\dots\dots(1)$$

regarding  $CA$  as being on the positive side of  $CH$ .

The question becomes, what is  $\theta$  when  $r=c$ ?

Let  $c(a-b) \sin a/2ab = h$ ,  $c(a+b) \cos a/2ab = k$ ,  
then  $h \operatorname{cosec} \theta + k \sec \theta = 1. \dots\dots\dots(2)$



Now it is obvious that,  $l$  being some convenient length, the point  $(l \operatorname{cosec} \theta, l \sec \theta)$  is the intersection of the straight line

$$hx + ky = l, \dots\dots\dots(3)$$

and the curve

$$\left(\frac{l}{x}\right)^2 + \left(\frac{l}{y}\right)^2 = 1. \dots\dots\dots(4)$$

This curve is easy to plot, because its polar equation is

$$r = 2l \operatorname{cosec} 2\theta.$$

On the given figure describe the line (3); on tracing paper describe the curve and then place it on the figure. Prick through the points  $P_1, P_2, P_3, P_4$  where the curve intersects the line (3). The lines drawn through  $C$  to these 4 points cut the circle at 8 points, 4 of which,

$R_1, R_2, R_3, R_4$ , are those required. There can be no doubt as to which four they are when the corresponding values of  $\theta$  are considered. For example, although  $P_3$  in the figure falls in the 2nd quadrant, its  $\theta$  value is  $296^\circ$ , showing that  $R_3$  must be in the 4th quadrant.

By supposing the nearer of the two given points to  $C$  to be on the positive side of  $CH$ , the branch of the curve in the 3rd quadrant is of no use, for the intercepts  $CH$  and  $CK$  of the line (3) will always be positive.

H. ORFEUR.

221. [K. 10. e.] In the equation in Note, viz.

$$r\{(a-b)\sin a \operatorname{cosec} \theta + (a+b)\cos a \sec \theta\} = 2ab$$

put  $abp = c^2(a-b)\sin a$ ;  $abq = c^2(a+b)\cos a$ ,  $r = c$ .

We then have 
$$\frac{1}{2}\left(\frac{p}{\sin \theta} + \frac{q}{\cos \theta}\right) = c.$$

Let a straight line from the origin intersect the straight lines

$$r \sin \theta = p, \quad r \cos \theta = q,$$

in  $F$  and  $G$ . Bisect  $FG$  in  $P$ .

The circle  $r = c$  will cut the locus of  $P$  in the four points required.

If the four values of  $\theta$  be  $\theta_1, \theta_2, \theta_3, \theta_4$ , then  $\Sigma \theta_1$ , is a multiple of  $\pi$ .

W. H. H. HUDSON.

222. [K. 10. e.] Another solution is as follows :

Bisect  $AOE$  internally in  $CY$ , externally in  $CX$ .

Let  $H, K$  be the inverse points of  $A, B$  with respect to the circle.

Bisect  $HK$  in  $F$  and let  $G$  be the reflexion of  $F$  in  $CY$ .

Fit, mechanically, a line of length  $c$  between the rectangular axes  $CX, CY$  so that its direction passes through  $G$ .

$CR$  is parallel to the direction of this line.

There are four positions of  $R$ .

W. H. H. HUDSON.

223. [V. 1. a.] *Higher Trigonometry.*

In Mr. Hardy's most interesting article on the proof of the theorem  $e^{ix} = \cos x + i \sin x$  he recommends the proof (which he numbers (iii))

$$\begin{aligned} \text{if } y &= \cos x + i \sin x, \\ \text{then } \frac{dy}{y} &= i dx; \end{aligned}$$

$$\therefore \text{integrating, } \log y = ix + \text{constant.}$$

It would be of great use to teachers if he would point out how much knowledge of contour integration is supposed.

Is it necessary to prove

(a) For a contour  $C$  not containing an infinity of  $f(z)$

$$\int_C f(z) dz = 0 ?$$

(\beta) If  $C$  contains a simple pole at  $z = a$ , where  $f(z) = \frac{M}{z-a}$ ,

$$\int_C f(z) dz = 2\pi i M ?$$

C. O. TUCKEY.