



XXIX. On concave gratings for optical purposes

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the edge of the plate; in the two last-mentioned cases, the distribution corresponding to a diminution of pressure.

We see from these experiments that the nature of the electricity produced is not dependent upon the particular method by which a local heating or warming is produced, but depends essentially upon the position in the crystal of the point where these changes in temperature take place. From the result that heating the peripheral portions of the plate and heating the central portions produce opposite electrical effects, I am disposed to draw a conclusion, which indeed has not yet been experimentally demonstrated but which seems to me tolerably safe. If we suppose it possible to warm a plate so uniformly that no perceptible differences of temperature or tensions should be produced, then I believe that this heating would produce no electricity or relatively very little, although the particles of the plate suffer considerable displacement amongst themselves. If, now, we consider that even very small displacements of the particles produce very considerable quantities of electricity, if these displacements are accompanied by changes of tension in the crystal (as is the case, for example, with irregular heating of a plate), the assumption seems to be justified that change of temperature and position of the particles in itself produces no electricity, but that, on the other hand, the real cause of the evolution of electricity is to be found in changes of tension.

In what has been described I have made a first attempt to explain the electricity produced in quartz by change of temperature as due to stress produced in the crystal. I am well aware that the explanation given in the separate cases is here and there defective, and that further investigations are necessary in order to establish the exact connexion between changes of temperature and evolution of electricity.

Giessen, March 20, 1883.

XXIX. *On Concave Gratings for Optical Purposes.* By
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General Theory.

HAVING recently completed a very successful machine for ruling gratings, my attention was naturally called to the effect of irregularity in the form and position of the

* An abstract of this Paper with some other matter was given at the Physical Society of London in November last, the Paper being in my hand in its present shape at that time. As I wished to make some addi-

lines and the form of the surface on the definition of the grating. Mr. C. S. Peirce has recently shown, in the 'American Journal of Mathematics,' that a periodic error in the ruling produces what have been called "ghosts" in the spectrum. At first I attempted to calculate the effect of other irregularities by the ordinary method of integration; but the results obtained were not commensurate with the labour. I then sought for a simpler method. Guided by the fact that inverse methods in electrical distribution are simpler than direct methods, I soon found an inverse method for use in this problem.

In the use of the grating in most ordinary spectroscopes the telescopes are fixed as nearly parallel as possible, and the grating turned around a vertical axis to bring the different spectra into the field of view. The rays striking on the grating are nearly parallel; but for the sake of generality I shall assume that they radiate from a point in space, and shall investigate the proper ruling of the grating to bring the rays back to the point from which they started. The wave-fronts will be a series of spherical shells at equal distances apart: if these waves strike on a reflecting surface they will be reflected back, provided they can do so all in the same phase. A sphere around the radiant-points satisfies the condition for waves of all lengths; and this gives the case of ordinary reflection. Let any surface cut the wave-surface in any manner, and let us remove those portions of the surface which are cut by the wave-surface. The light of that particular wave-length can then be reflected back along the same path and in the same phase; and thus, by the above principle, a portion will be thus sent back. But the solution only holds for one wave-length; and so white light will be drawn out into a spectrum. Hence we have the important conclusion that a theoretically perfect grating for one position of the slit and eyepiece can be ruled on any surface, flat or otherwise. This is an extremely important practical conclusion, and explains many facts which have been observed in the use of gratings. For we see that errors of the dividing-engine can be counter-balanced by errors in the flatness of the plate; so that a bad

tions, for which I have not yet had time, I did not publish it at the time. I was much surprised soon after to see an article on this subject, which had been presented to the Physical Society and was published in the Philosophical Magazine. The article contains nothing more than an extension of my remarks at the Physical Society, and formulæ similar to those in this paper. As I have not before this published any thing except a preliminary notice of the concave grating, I expected a little time to work up the subject, seeing that the practical work of photographing the spectrum has recently absorbed all my time. But probably I have waited too long.

dividing-engine may now and then make a grating which is good in one spectrum but not in all. And so we often find that one spectrum is better than another. Furthermore Prof. Young has observed that he could often improve the definition of a grating by slightly bending the plates on which it was ruled.

From the above theorem, we see that if a plate is ruled in circles whose radius is $r \sin \mu$ and whose distance apart is $\Delta r \sin \mu$, where Δr is constant, then the ruling will be appropriate to bring the spectrum to a focus at a distance r and angle of incidence μ . Thus we should need no telescope to view the spectrum in that particular position of the grating. Had the wave-surfaces been cylindrical instead of spherical, the lines would have been straight instead of circular, but at the above distances apart. In this case the spectrum would have been brought to a focus, but would have been diffused in the direction of the lines.

In the same way we can conclude that in flat gratings any departure from a straight line has the effect of causing the dust in the slit and the spectrum to have different foci—a fact sometimes observed.

We also see that if the departure from equal spaces is small, or, in other words, the distance r is great, the lines must be ruled at distances apart represented by

$$c \left(1 - \frac{\cos^2 \mu}{r \sin \mu} x + \&c. \right)$$

in order to bring the light to a focus at the angle μ and distance r , c being a constant and x the distance from some point on the plate. If μ changes sign, the r must change in sign. Hence we see that the effect of a linear error in the spacing is to make the focus on one side shorter and the other side longer than the normal amount. Prof. Peirce has measured some of Mr. Rutherford's gratings and found that the spaces increased in passing along the grating; and he also found that the foci of symmetrical spectra were different. But this is the first attempt to connect the two. The definition of a grating may thus be very good even when the error of run of the screw is considerable, provided it is linear.

Concave Gratings.

Let us now take the special case of lines ruled on a spherical surface; and let us not confine ourselves to light coming back to the same point, but let the light return to another point. Let the coordinates of the radiant-point and focal point be $y=0$, $x=-a$, and $y=0$, $x=+a$, and let the centre of the

sphere whose radius is ρ be at x', y' . Let r be the distance from the radiant-point to the point x, y , and let R be that from the focal point to x, y . Let us then write

$$2b = R + rc,$$

where c is equal to ± 1 according as the reflected or transmitted ray is used. Should we increase b by equal quantities and draw the ellipsoids or hyperboloid so indicated, we could use the surfaces in the same way as the wave-surface above. The intersections of these surfaces with any other surface form what are known as Huyghens's zones. By actually drawing these zones on the surface we form a grating which will reflect or refract the light of a certain wave-length to the given focal point. For the particular problem in hand we need only work in the plane x, y for the present.

Let s be an element of the curve of intersection of the given surface with the plane x, y . Then our present problem is to find the width of Huyghens's zones on the surface—that is, ds in terms of db .

The equation of the circle is

$$(x - x')^2 + (y - y')^2 = \rho^2;$$

and of the ellipse or hyperbola,

$$R + rc = 2b,$$

or

$$(b^2 - a^2)x^2 + b^2y^2 = b^2(b^2 - a^2),$$

in which c has disappeared.

$$ds = \sqrt{dx^2 + dy^2}, \quad \frac{dx}{dy} = -\frac{y - y'}{x - x'},$$

$$dx \left\{ (b^2 - a^2)x - b^2y \frac{x - x'}{y - y'} \right\} = b \{ 2b^2 - (x^2 + y^2 + a^2) \} db,$$

$$dy \left\{ -(b^2 - a^2)x \frac{y - y'}{x - x'} + b^2y \right\} = b \{ 2b^2 - (x^2 + y^2 + a^2) \} db;$$

$$\therefore \frac{ds}{db} = \rho b \frac{2b^2 - (x^2 + y^2 + a^2)}{(b^2 - a^2)(y - y')x - b^2(x - x')y}.$$

This equation gives us the proper distance of the rulings on the surface; and if we could get a dividing-engine to rule according to this formula, the problem of bringing the spectrum to a focus without telescopes would be solved. But an ordinary dividing-engine rules equal spaces; and so we shall further investigate the question whether there is any part of the circle where the spaces are equal. We can then write

$$\frac{ds}{db} = C;$$

and the differential of this with regard to an arc of the circle must be zero. Differentiating and reducing by the equations

$$\frac{dx}{dy} = -\frac{y-y'}{x-x'}, \quad \frac{db}{dy} = -\frac{\rho}{C(x-x')},$$

we have

$$\begin{aligned} & \rho \left\{ 2xb(y-y') - 2yb(x-x') - \frac{\rho}{C} [6b^2 - (x^2 + y^2 + a^2)] \right\} \\ & + C \left\{ (y-y')[(b^2 - a^2)(y-y') - b^2y] - (x-x')[(b^2 - a^2)x - b^2(x-x')] \right. \\ & \quad \left. + \frac{2b\rho}{C} [x(y-y') - y(x-x')] \right\} = 0. \end{aligned}$$

It is more simple to express this result in terms of R , r , ρ and the angles between them.

Let μ be the angle between ρ and r , and ν that between ρ and R . Let us also put

$$\alpha = \frac{\mu - \nu}{2}, \quad \beta = \frac{\mu + \nu}{2}.$$

Let β , γ , δ also represent the angles made by r , R , and ρ respectively with the line joining the source of light and focus, and let

$$\eta = \frac{\beta + \gamma}{2}.$$

Then we have

$$x = \frac{R \cos \gamma + \rho \cos \beta}{2}, \quad y = \frac{R \sin \gamma + r \sin \beta}{2}, \quad a = \frac{r \cos \beta - R \cos \gamma}{2},$$

$$(b^2 - a^2)(y - y')^2 + b^2(x - x')^2 = \rho^2(b^2 - a^2 \sin^2 \delta),$$

$$b^2 - a^2 = Rr \cos^2 \alpha,$$

$$\sin \eta = \frac{R + r}{2a} \sin \alpha, \quad \cos \eta = \frac{r - R}{2a} \cos \alpha,$$

$$R = b - \frac{a}{b}x, \quad \rho = b + \frac{a}{b}x,$$

$$x = b \frac{\cos \gamma}{\cos \alpha}, \quad y = a \frac{\sin \gamma \sin \beta}{\sin \alpha \cos \alpha} = \frac{Rr}{b} \sin \eta \cos \alpha,$$

$$b^2y(y - y') + x(b^2 - a^2)(x - x') = \frac{bRr\rho}{2} (\cos \mu + \cos \nu),$$

$$2b^2 - (x^2 + y^2 + a^2) = Rr,$$

$$x(b^2 - a^2)(y - y') - b^2y(x - x') = \frac{Rr b \rho}{2} (\sin \mu + \sin \nu),$$

$$C = \frac{2}{\sin \mu + \sin \nu} = \frac{1}{\cos \alpha \sin \epsilon},$$

$$2a \cos \delta = r \cos \mu - R \cos \nu,$$

$$2a \sin \delta = r \sin \mu - R \sin \nu.$$

On substituting these values and reducing, we find

$$^*\rho = 2Rr \frac{\cos \alpha \cos \epsilon}{r \cos^2 \nu + R \cos^2 \mu},$$

whence the focal length is

$$r = \rho R \frac{\cos^2 \mu}{2R \cos \alpha \cos \epsilon - \rho \cos^2 \nu}.$$

For the transmitted beam, change the sign of R .

Supposing ρ , R , and ν to remain constant and r and μ to vary, this equation will then give the line on which all the spectra and the central image are brought to a focus.

By far the most interesting case is obtained by making

$$r = \rho \cos \mu, \quad R = \rho \cos \nu,$$

* A more simple solution is the following. $\frac{ds}{db}$ must be constant in the direction in which the dividing-engine rules. If the dividing-engine rules in the direction of the axis, the differential of this with respect to y must be zero. But we can also take the reciprocal of this quantity; and so we can write for the equation of condition,

$$\frac{d}{dy} \frac{d(R+r)}{ds} = 0.$$

Taking a circle as our curve, we can write

$$(x-x')^2 + (y-y')^2 = \rho^2$$

and

$$(x-x'')^2 + (y-y'')^2 = R^2,$$

$$(x-x''')^2 + (y-y''')^2 = r^2,$$

$$\frac{d(R+r)}{ds} = \frac{1}{\rho} \left\{ (y-y') \left(\frac{x-x''}{R} + \frac{x-x'''}{r} \right) + (x-x') \left(\frac{y-y''}{R} + \frac{y-y'''}{r} \right) \right\},$$

$$\frac{d}{dy} \frac{d(R+r)}{ds} = \frac{1}{\rho} \left\{ \frac{x-x''}{R} + \frac{x-x'''}{r} - (y-y') \left[\frac{(x-x'')(y-y'')}{R^3} + \frac{(x-x''')(y-y''')}{r^3} \right] \right. \\ \left. + (x-x') \left[\frac{(x-x'')^2}{R^3} + \frac{(x-x''')^2}{r^3} \right] \right\} = 0.$$

Making $x=0$, $y=0$, $y'=0$, $x'=\rho$, we have

$$\frac{x''}{R} + \frac{x'''}{r} - \rho \left(\frac{x''^2}{R^3} + \frac{x'''^2}{r^3} \right) = 0,$$

or

$$\rho = Rr \frac{\cos \mu + \cos \nu}{r \cos^2 \nu + R \cos^2 \mu} = 2Rr \frac{\cos \alpha \cos \epsilon}{r \cos^2 \nu + R \cos^2 \mu}.$$

tional to the line AC. Or, for any given spectrum, the wave-length is proportional to that line.

If a micrometer is fixed at C, we can consider the case as follows:

$$\frac{1}{C} = \frac{\lambda N}{2w} = \frac{1}{2}(\sin \mu + \sin \nu),$$

$$\frac{d\lambda}{du} = \frac{w}{N} \cos \mu.$$

If D is the distance the cross-hairs of the micrometer move forward for one division of the head, we can write for the point C,

$$d\mu = \frac{D}{\rho};$$

and for the same point μ is zero. Hence

$$\Delta\lambda = \frac{wD}{N\rho}.$$

But this is independent of ν ; and we thus arrive at the important fact that the value of a division of the micrometer is always the same for the same spectrum and can always be determined with sufficient accuracy from the dimensions of the apparatus and number of lines on the grating, as well as by observations of the spectrum.

Furthermore, this proves that the spectrum is normal at this point and to the same scale in the same spectrum. Hence we have only to photograph the spectrum to obtain the normal spectrum, and a centimetre for any of the photographs always represents the same increase of wave-length.

It is to be specially noted that this theorem is *rigidly true* whether the adjustments are correct or not, provided only that the micrometer is on the line drawn perpendicularly from the centre of the grating, even if it is not at the centre of curvature.

As the radius of curvature of concave gratings is usually great, the distance through which the spectrum remains practically normal is very great. In the instrument which I principally use, the radius of curvature, ρ , is about 21 ft. 4 inches, the width of the ruling being about 5.5 inches. In such an instrument the spectrum thrown on a flat plate is normal within about 1 part in 1,000,000 for six inches, and less than 1 in 35,000 for eighteen inches. In photographing the spectrum on a flat plate, the definition is excellent for twelve inches; and by use of a plate bent to 11 ft. radius, a plate of twenty inches length is in perfect focus, and the spectrum

still so nearly normal as to have its error neglected for most purposes.

It is also to be noted that this theorem of the normal spectrum applies also to the flat grating used with telescopes, and to either reflecting or transmitting gratings; but in these cases only a small portion of the spectrum can be used, as no lens can be made perfectly achromatic. And so, as the distance of the micrometer has constantly to be changed when one passes along the spectrum, its constant does not remain constant but varies in an irregular manner. But it would be possible to fix the grating, one objective, and the camera rigidly on a bar, and then focus by moving the slit or then other objective. In this case the spectrum would be normal, but would probably be in focus for only a small length only, and the adjustment of the focus would not be automatic.

Another important property of the concave grating is that all the superimposed spectra are in focus at the same point, and so by micrometric measurements the relative wave-lengths are readily determined. Hence, knowing the absolute wave-length of one line, the whole spectrum can be measured. Prof. Peirce has determined the absolute wave-length of one line with great care; and I am now measuring the coincidences. This method is greatly more accurate than any hitherto known, as by mere eye-inspection the relative wave-length can often be judged to one part in twenty thousand, and with a micrometer to 1 in 1,000,000. Again, in dealing with the invisible portion of the spectrum, the focus can be obtained by examining the superimposed spectrum. Capt. Abney, by using a concave mirror in the place of telescopes, has been enabled to use this method for obtaining the focus in photographing the ultra-red rays of the spectrum.

But nothing can exceed the beauty and simplicity of the concave grating when mounted on a movable bar such as I have described and illustrated in fig. 1. Having selected the grating which we wish to use, we mount it in its plate-holder and put the proper collimating eyepiece in place. We then carefully adjust the focus by altering the length of ρ until the cross-hairs are at the exact centre of curvature of the grating. On moving the bar the whole series of spectra are then in exact focus, and the value of a division of the micrometer is a known quantity for that particular grating. The wooden way, A C, on which the carriage moves is graduated to equal divisions representing wave-lengths, since the wave-length is proportional to the distance A C. We can thus set the instrument to any particular wave-length we may wish to study, or even determine the wave-length to

one part in at least five thousand by a simple reading. By having a variety of scales, one for each spectrum, we can immediately see what lines are superimposed on each other, and identify them accordingly when we are measuring their relative wave-length. On now replacing the eyepiece by a camera, we are in position to photograph the spectrum with the greatest ease. We put in the sensitive plate, either wet or dry, and move to the part we wish to photograph. Having exposed for that part, we move to another position and expose once more. We have no thought for the focus, for that remains perfect, but simply refer to the table giving the proper exposure for that portion of the spectrum, and so have a perfect plate. Thus we can photograph the whole spectrum on one plate in a few minutes from the F line to the extreme violet, in several strips each 20 inches long. And we may photograph to the red rays by prolonged exposure. Thus the work of days with any other apparatus becomes the work of hours with this. Furthermore each plate is to scale, an inch on any one of the strips representing *exactly* so much difference of wave-length. The scales of the different orders of spectra are *exactly* proportional to the order. Of course the superposition of the spectra gives the relative wave-lengths. To get the superposition, of course photography is the best.

Having so far obtained only the first approximation to the theory of the concave grating, let us now proceed to a second one. The dividing-engine rules equal spaces along the chord of the circular arc of the grating; the question is whether any other kind of ruling would be better. For the dividing-engine is so constructed that one might readily change it to rule slightly different from equal spaces.

The condition for theoretical perfection is that C shall remain constant for all portions of the mirror. I shall therefore investigate how nearly this is true. Let ρ be the radius of curvature, and let R and r be the true distances to any point of the grating, R_0 and r_0 being the distances to the centre. Let μ and ν be the general values of the angles, and μ_0 and ν_0 the angles referred to the centre of the mirror. The condition is that

$$\frac{2}{C} = \sin \mu + \sin \nu$$

shall be a constant for all parts of the surface of the grating. Let us then develop $\sin \mu$ and $\sin \nu$ in terms of μ_0 , ν_0 , and the angle δ between radii drawn to the centre of the grating and the point under consideration. Let δ' be the angle between R and R_0 . Then we can write immediately

$$\rho \sin \mu = \rho \sin \mu_0 \cos \delta' + R_0 \sin \delta' - \rho \cos \mu_0 \sin \delta',$$

$$\sin \mu = \sin \mu_0 \cos \delta' \left\{ 1 + \frac{R_0}{\rho \sin \mu_0} A \tan \delta' \right\},$$

where

$$A = 1 - \frac{\rho \cos \mu_0}{R_0}.$$

Developing the value of $\cos \delta'$ in terms of δ , we have

$$\cos \delta' = \cos \delta \left\{ 1 + \frac{A}{2} \left[1 + \frac{\rho \cos \mu_0}{R_0} \right] \delta^2 - \frac{\rho \sin \mu_0}{2R_0} \left[1 + A \left(1 + \frac{2\rho}{R_0} \right) \right] \delta^3 + \&c. \right\}.$$

As the cases we are to consider are those where A is small, it will be sufficient to write

$$\tan \delta' = \frac{\rho \cos \mu_0}{R_0} \delta,$$

whence we have

$$\sin \mu = \sin \mu_0 \cos \delta \left\{ 1 + \cot \mu_0 A \delta + \frac{A}{2} \left[1 + \frac{\rho \cos \mu_0}{R_0} \right] \delta^2 + \left[A^2 \cot \mu_0 \left(1 + \frac{\rho \cos \mu_0}{R_0} \right) - \frac{\rho \sin \mu_0}{2R_0} \left(1 + A \left(1 + \frac{2\rho}{R_0} \right) \right) \right] \delta^3 + \&c. \right\}$$

We can write the value of $\sin \nu$ from symmetry. But we have

$$2 \frac{db}{ds} = \sin \mu + \sin \nu.$$

In this formula db can be considered as a constant depending on the wave-length of light &c., and ds as the width apart of the lines on the grating. The dividing-engine rules lines on the curved surface according to the formula

$$2 \frac{db}{ds} = \cos \delta (\sin \mu_0 + \sin \nu_0).$$

But this is the second approximation to the true theoretical ruling. And this ruling will not only be approximately correct, but exact when all the terms of the series except the first vanish.

In the case where the slit and focus are on the circle of radius $\frac{1}{2}\rho$, as in the automatic arrangement described above, we have $A=0$, and the second and third terms of the series disappear, and we can write, since we have $\frac{R_0}{\rho} = \cos \mu_0$ and

$$\frac{r_0}{\rho} = \cos \nu_0,$$

$$2 \frac{db}{ds} \cos \delta (\sin \mu_0 + \sin \nu_0) \left\{ 1 - \frac{1}{2} \frac{\sin \mu_0 \tan \mu_0 + \sin \nu_0 \tan \nu_0}{\sin \mu_0 + \sin \nu_0} \delta^3 + \&c. \right\}$$

But in the automatic arrangement we also have $\nu_0 = 0$; and so the formula becomes

$$2 \frac{db}{ds} = \cos \delta (\sin \mu_0 + \sin \nu_0) \left\{ 1 - \frac{1}{2} \tan \mu_0 \delta^3 + \&c. \right\}.$$

To find the greatest departure from theoretical perfection, δ must refer to the edge of the grating. In the gratings which I am now making, ρ is about 260 in., and the width of the grating about 5.4 in. Hence $\delta = \frac{1}{100}$ approximately, and the series becomes

$$1 - \frac{1}{2000000} \tan \mu_0.$$

Hence the *greatest* departure from the theoretical ruling, even when $\tan \mu_0 = 2$, is 1 in 1,000,000. Now the distance apart of the components of the 1474 line is somewhat nearly one forty-thousandth of the wave-length; and I scarcely suppose that any line has been divided by the best spectro-scope in the world whose components are less than one third of this distance apart. Hence we see that the departure of the ruling from theoretical perfection is of little consequence until we are able to divide lines twenty times as fine as the 1474 line. Even in that case, since the error of ruling varies as δ^3 , the greater portion of the grating would be ruled correctly.

The question now comes up as to whether there is any limit to the resolving power of a spectro-scope. This evidently depends upon the magnifying power and the apparent width of the lines. The magnifying power can be varied at pleasure; and so we have only to consider the width of the lines of the spectrum. The width of the line evidently depends in a perfect grating upon three circumstances—the width of the slit, the number of lines in the grating, and the *true physical width of the line*. The width of the slit can be varied at pleasure; the number of lines on the grating can be made very great (160,000 in one of mine); and hence we are only limited by the true physical width of the lines. We have numerous cases of wide lines, such as the C line, the components of the D* and H lines, and numerous others which are perfectly fami-

* I have recently discovered that each component of the D line is double, probably from the partial reversal of the line as we nearly always see it in the flame-spectrum.

liar to every spectroscopist. Hence we are free to suppose that all lines have some physical width; and we are limited by that width in the resolving power of our spectroscopes. Indeed, from a theoretical standpoint we should suppose this to be true; for the molecules only vibrate freely while swinging through their free path; and in order to have the physical width $\frac{1}{100000}$ of the wave-length, the molecule must make somewhat nearly one hundred thousand vibrations in its free path; but this would require a free path of about two inches! Hence it would be only the outermost solar atmosphere that could produce such fine lines; and we can hardly expect to see much finer ones in the solar spectrum. Again*, it is found impossible to obtain interference between two rays whose paths differ by much more than 50,000 wave-lengths.

All the methods of determining the limit seem to point to about $\frac{1}{150000}$ of the wave-length as the smallest distance at which two lines can be separated in the solar spectrum by even a spectroscope of infinite power. As we can now nearly approach this limit, I am strongly of the opinion that we have nearly reached the limit of resolving-power, and that we can never hope to see very many more lines in the spectrum than can be seen at present, either by means of prisms or gratings; for the same limit holds in either case. It is not to be supposed, however, that the average wave-length of the line is not more definite than this; for we can easily point the cross-hairs to the centre of the line to perhaps 1 in 1,000,000 of the wave-length. The most exact method of detecting the coincidences of a line of a metal with one in the solar spectrum would thus be to take micrometric measurements first on one and then on the other; but I suppose it would take several readings to make the determination to 1 in 1,000,000.

Since writing the above I have greatly improved my apparatus, and can now photograph 150 lines between the H and K lines, including many whose wave-length does differ more than 1 in about 80,000. I have also photographed the 1474 and b_3 and b_4 widely double, and also one of the E lines just perceptibly double. With the eye much more can be seen; but I must say that I have not yet seen many signs of reaching a limit. The lines yet appear as fine and sharp as with a lower power. If my grating is assumed to be perfect, in the third spectrum I should be able to divide lines whose wave-length differed 1 in about 150,000, though not to photograph them.

* This method of determining the limit has been suggested to me by Prof. C. S. Hastings, of this University.

The E line has components about $\frac{1}{60000}$ of the wave-length apart. I believe I can resolve lines much closer than this, say 1 in 100,000 at least. Hence the idea of a limit has not yet been proved.

However, as some lines of the spectrum are wider than others, we should not expect any *definite* limit to resolving power, but a gradual falling-off as we increase our power. At first, in the short wave-lengths at least, the number of lines is nearly proportional to the resolving-power; but this law should fail as we approached the limit.

XXX. On Mr. Glazebrook's Paper on the Aberration of Concave Gratings. By HENRY A. ROWLAND, Professor of Physics, Johns Hopkins University, Baltimore*.

IN the June number of the Philosophical Magazine Mr. R. T. Glazebrook has considered the aberration of the concave grating, and arrives at the conclusion that the ones which I have hitherto made are too wide for their radius of curvature. As I had published nothing but a preliminary notice of the grating at that time, Mr. Glazebrook had not then seen my paper on the subject, of which I gave an abstract at the London Physical Society in November last. In this paper I arrive at the conclusion that there is practically no aberration, and that in this respect there is nothing further to be desired. The reason of this discrepancy is not far to seek. Mr. Glazebrook assumes that the spaces are equal on the arc of the circle. But I do not rule them in this manner, but the spaces are equal along the *chord* of the arc. Again, the surface is not cylindrical, but spherical.

These two errors entirely destroy the value of the paper as far as my gratings are concerned; for it only applies to a theoretical grating ruled in an entirely different manner and on a different form of surface from my own.

I am very much surprised to see the method given near the end of the paper for constructing aplanatic gratings on any surface; for this is the method by which I discovered the concave grating originally, and the figure is the same as that I put on the black board at the Meeting of the Physical Society in November last. I say I am surprised; for Mr. Glazebrook's paper was read at the Physical Society, where I had given the same method a few months before, and yet it passed without comment. Indeed I have given the same method at many of our own scientific societies. However, as Mr. Glazebrook was not present at the meeting referred to, he is entirely without blame in the matter.

* Communicated by the Author.