

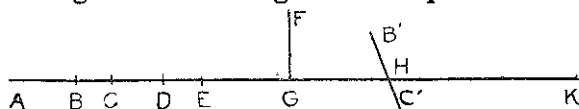
# THE FOURTH DIMENSION: AN EXPLANATION THE GEOMETRY CLASS CAN FOLLOW

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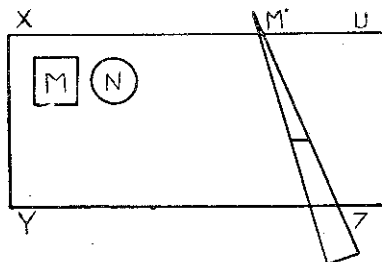
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A diversion for the geometry class, is needed now and then, and the Fourth Dimension furnishes one topic, being popular at this time. Articles on the Fourth Dimension almost invariably lead the reader on, making him think he understands, yet repeatedly waking him up to the fact that he does not understand at all. Now a presentation of the subject can be made, resembling the taking off of an aeroplane, in which the machine runs along on *terra firma* for a considerable time, and then suddenly shoots into the air to rise to any height desired. The following explanation undertakes to make the subject quite clear—certainly up to the taking off place.

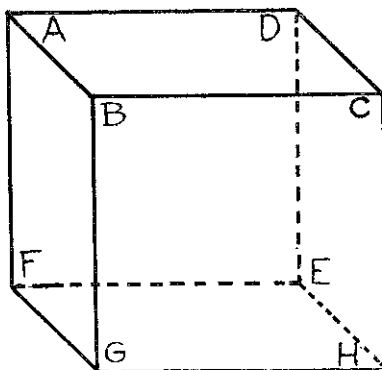
The three figures needed are given at this point.



ONE-DIMENSION SPACE.



TWO-DIMENSION SPACE.



THREE-DIMENSION SPACE.

Let AK be a space of one dimension having either no width and thickness or infinitesimal width and thickness, and let BC and DE be bodies or beings in it. Let also FG represent a perpendicular to AK. Similarly, let XYZU represent a space of two dimensions, and let M and N be bodies or beings on XYZU. Also let C (cube) represent a body or being in space of three dimensions.

We will consider: First, some of the more important positive properties of spaces of the first four dimensions; second, some negative properties of these spaces; and third, some relations of each to the next higher space.

#### I. POSITIVE PROPERTIES OF THE SEVERAL SPACES.

1. *Generation of spaces.* Space of one dimension is generated by a point in motion in a straight line; space of two dimensions by a line in motion moving parallel to itself in a plane, as  $xy$ . Space of three dimensions is generated by a plane in corresponding motion; space of four dimensions is generated by a solid in motion.

2. *Dimensions.* Space of one dimension has length only; space of two dimensions has length and breadth, perpendicular to each other; space of three dimensions has length, breadth, and height, all perpendicular to each other; and space of four dimensions has length, breadth, height, and ——— (last one not named), each perpendicular to the other three. This last dimension is out of our experience. We know nothing of it from our senses.

3. *Location of Points.* In 1-space a point is located by one ordinate; in 2-space by two coordinates,  $x$  and  $y$ , in the usual graph manner; in 3-space by three coordinates,  $x, y, z$ ; in 4-space, by four coordinates  $x, y, z$ , and  $u$ ; and, in  $n$ -space, by  $n$  coordinates.

4. *Division of Space.* A point divides 1-space into two infinite portions; a line of infinitesimal or no width and thickness divides 2-space into two infinite portions; a plane of infinitesimal or no thickness divides 3-space into two infinite portions; and, a solid of infinitesimal or no thickness in the fourth dimension direction divides 4-space into two infinite portions.

5. *Rotation.* A straight line in 1-space can not rotate about a point; a body in 2-space, as M or N, can rotate about a point; a body in 3-space can rotate about a line; a body in 4-space can rotate about a plane; and a body in 5-space can rotate about a solid.

6. *Motion of Translation.* A body in 1-space can move in only one direction (though in two "senses," positive or negative); one in 2-space in two directions, as east and north, or any combination of them; a body in 3-space in three directions, as east, north, and up, or any combination of them; and a body in 4-space can move in four directions, including the fourth dimensional direction, or any combination of them.

7. *Rigidity.* A body in 1-space is fixed in position by fixing only one point in it; a body in 2-space is fixed by fixing two points in it; a body in 3-space is held rigid by fixing three points not in the same straight line in it; a body in 4-space is held rigid by fixing four points in it not lying in the same plane.

8. *Boundaries.* Points could be fixed at any two points in 1-space, as at A and G, beyond which BC and DE could not go; in 2-space a ring could be put around M or N outside of which they could not pass; in 3-space walls could be built in the shape of a closed box out of which a being could not go, thus constituting a prison for it.

9. *Projection.* Projection in 2-space is well understood. A body in 3-space can be projected on a plane by drawing perpendiculars from every point of it which it is desired to locate to the plane. A body in 4-space can be projected on six planes, viz., XY, XZ, YZ, XU, YU, ZU, the first three of which are the reference planes of 3-space.

## II. NEGATIVE PROPERTIES OF THE SEVERAL SPACES.

1. *Non-Reversibility.* A body in 1-space can not reverse itself. Thus, AB can not turn itself around so that A is to the right of B. Similarly, a body, as M or N, can not be folded over, or reversed, so that its under surface becomes its top surface. Then, in 3-space, a closed body (as the cover of a ball) can not be turned inside out without tearing. A glove can be turned inside out, but that is because it is not a closed surface.

2. *Apparent non-existence of a higher space.* We may imagine the beings in 1-space living in a thread like hole running lengthwise through a straight iron pipe. Clearly such beings would have no conception whatever of other dimensions, because such space would be wholly without their experience. In like manner beings in flatland, like M and N, would have no knowledge of our space, and we would have no knowledge whatever of space of four dimensions. It would be wholly without our experience, and we would think it did not exist.

3. *Apparent non-existence of perpendiculars.* This is evident

from the preceding. It should be noted carefully that the perpendicular in 3-space is perpendicular to all the lines through its foot in a 2-space. It follows then that the perpendicular in the fourth dimension direction is perpendicular to all the planes and lines through its foot of 3-space.

### III. RELATION OF SPACES TO HIGHER SPACES.

1. *Disappearance.* Bodies can disappear from one space and become non-existent to other beings in that space by simply going into higher space. A body in 3-space becomes non-existent to beings in this space by merely dropping out of 3-space into 4-space.

2. *Reversibility by going into a higher space and then returning.* Thus, AB in 1-space by dropping out into 2-space and then returning can reverse itself. In 1-space if AB knew of 2-space, he could reverse himself so that DE would see AB's head where his feet had been, to DE's great astonishment. Then, in 2-space, M can move out of the plane it is in, fold itself over and go back into its old space with its under side becoming its upper side, an impossibility from the standpoint of N. Then, also, an object in 3-space can drop out into 4-space, turn itself inside out and then return into 3-space in this form!

3. *Boundaries do not hold.* In 1-space if AB knows of 2-space, he can go into it and thus get out of prison formed by points A and F, and return to 1-space outside of A and F. In 2-space M can get out of the prison ring around him by going into 3-space and then returning to 2-space outside of the ring. Similarly in 3-space, a body can get out of prison walls not by going through them, but by going into 4-space and thence returning to 3-space outside the prison walls.

Similarly knots tied in 3-space being taken into 4-space can be untied and then brought back into 3-space, since any part of the cord in 4-space can pass through any body enclosing it.

4. *Sections.* If BC in 1-space knows of 2-space he can place himself across this space in the position  $B^1C^1$ . Then the point H only can be seen in 1-space. Similarly if the being  $M^1$  places himself across his space, then only that part of him in 2-space can be seen in this space. If  $M^1$  moved across the space, he would appear to change in size and shape as he moved. In like manner a body in 4-space could show only a section of itself in 3-space, and this body would change in size and shape as it moved across the 3-space.

5. *Visibility, tangibility, and permeability of objects.* A being

out in 2-space can see all of 1-space, and can reach any point of 1-space. A being out in 3-space can see every point of 2-space, and can reach any point of 2-space. A being in 4-space can see through 3-space, can reach any point of 3-space, whether inside or outside a body, and can go into any point of a body without tearing or injuring that body in any way. Thus, a surgeon could operate for appendicitis without cutting.

6. *Birth, life, death.* Some writers have conceived the idea that living beings come from hyper-space into our space, thus being born; then, live; and, finally, passing out of our space, die. For example, AB in 1-space might come from 2-space into 1-space, live there, and, passing on out, die. Similarly, a being, as M, might come from 3-space into 2-space, thus being born, live, and then, dying, pass on out into 3-space again. A series of heavens has been suggested (as, for instance, those mentioned in the Koran), which could easily be accounted for by referring them to spaces of the fifth, sixth, and higher dimensions.

7. *Elements.* A body in 1-space consists of one line segment and two end points as B and C; a rectangular body in 2-space consists of  $2^2$  points, 2 of which lie in the original position before their line starts to generate the surface, and one line lies in this position; a cube or rectangular solid in 3-space consists of  $2^3$  points, 12 lines or sects, and 6 surfaces, of which  $2^2$  points lie in the generating surface, four lines and one plane lie in this surface.

To get a "cube" in 4-space we must imagine a cube to move in the direction of the fourth perpendicular a distance equal to one of its edges. Then, there will exist  $2^4$  points, that is, the eight original vertices and 8 more in the new position of the cube. There will be 32 edges to this figure, obtained as follows: There are four edges at each vertex, the fourth being in the fourth dimension direction. As there are 16 vertices, that will give  $16 \times 4$  edges but each is counted twice, that is, from each end. Thus there are 32 different edges, of which 8 are generated by the 8 vertices in motion, 16 are in section planes, and the other 8 are unimaginable, being in the fourth dimension.

Then, there are 24 surfaces in this fourth dimensional solid, obtained as follows: There are six planes at each vertex, resulting from taking the four edges at a vertex in pairs. Then, since there are 16 vertices, there are 96 planes in all. But each plane is counted four times, that is, once from each corner of

every square. Hence there are  $96 \div 4$  different planes. A "cube" in 4-space is bounded by eight cubes whose existence is inferred as follows: A line in 1-space is bounded by 2 points; a square in 2-space is bounded by four lines; a cube in 3-space is bounded by six squares; then a 4-space "cube" should be bounded by eight cubes. Six of these cubes are generated by the six faces of the cube moving the fourth dimension direction. The other two can not be imaged. (Observe how two points of the generator describe two sides of the square, four lines of the generator describe four faces of the cube.)

8. *Regular polyedrons.* There are five well-known regular polyedrons described in solid geometry, three bounded by equilateral triangles, one by squares, and one by regular pentagons. In 4-space it is found there are six regular polyedrons. The "cube," already described, denoted by  $C_8$ ;  $C_8$  bounded by five tetraedrons;  $C_{120}$ , bounded by 16 tetraedrons;  $C_{24}$ , bounded by 24 octaedrons;  $C_{120}$ , bounded by 120 dodecaedrons; and  $C_{600}$ , bounded by 600 tetraedrons. An explanation of how the figures for the number of bounding polyedrons were obtained is not attempted here. Suffice it to say they have all been exhaustively studied, and models of the projection of each on space of three dimensions have been made.

In conclusion, it can be said that Professor Einstein has recently announced that our universe has limits. Evidently, without any consideration of the question of higher spaces, this statement would seem wholly absurd. In the light of the preceding discussion, however, one hesitates to pronounce positively what is the fact. Thus we learn out of such discussions to suspend final judgment on all similar questions. "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

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