



# XC. The accelerated motion of a dielectric sphere

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The oscillation of the pressure did not exceed 0.12 mm.; the period of oscillation (*i. e.* from contact to contact) varied from 6 sec. downwards.

In conclusion I desire to express my thanks for the facilities afforded me by the Physical and Chemical Departments of the University of Bristol, at which the research was conducted. I am greatly indebted to Professor A. P. Chattock and Dr. James W. McBain for constant advice, and to the latter for much personal assistance.

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#### XC. *The Accelerated Motion of a Dielectric Sphere.*

By J. W. NICHOLSON, M.A., D.Sc.\*

IN a previous paper, a brief account was given of the motion of a conducting sphere whose mass is purely electrical, under the action of either a small uniform field of electric force or a small mechanical force. The solution was deduced as a limiting case from a more general problem treated by G. W. Walker †, and it was shown that there are difficulties in the results of regarding any conductor as

\* Communicated by the Author.

† *Roy. Soc. Proc. A*. vol. lxxvii. p. 260 *et seq.*; *Phil. Trans. A*. 1910, p. 145 *et seq.*

perfect when its motion is accelerated. The perfect conductor of the usual theory leads to disturbing infinities when it has no Newtonian mass. The indications that the mass of a single electron can have a Newtonian element are not very securely established; and although certain experiments can be interpreted in accordance with this view, there is always a possibility of other interpretations which do not involve it. For example, it is possible that the particles in Kaufmann's experiments are electrons not free, but attached to matter. A comprehensive examination of the conditions of motion of a small body without a Newtonian mass is therefore desirable, and this was made in the case of a conductor under the action of a small force in the previous paper. Apart from indications there obtained, it seems unlikely on general grounds that an electron can be endowed with properties analogous to those of a conductor, for there is a difficulty of attaching a physical meaning to such properties in a single electron. Moreover, the rapidity of damping of the oscillations set up when the motion of the conductor is changed, supplies a strong adverse argument.

Some interest therefore attaches to the corresponding problem of a small sphere, with a surface charge, whose interior has the properties solely of a dielectric, with no conducting element.

In the present paper, the motion of such a sphere, devoid of Newtonian mass, is investigated, and it is shown to present none of the difficulties noticed in the case of the conductor. A small field of force can produce a finite acceleration, and will give the effect of a constant acceleration after a short time, if the dielectric coefficient be not too great. If this coefficient is great, the oscillations initially set up are very permanent, and the constant acceleration is not established by a uniform field within the time during which the equations are good approximations to the motion. The problem in this case bears some resemblance to that of the perfect conductor, for the disturbance inside the sphere tends to zero as the dielectric constant increases. But the problems do not become identical, for in the case of the conductor, the charge is allowed freedom of movement on the surface, and in fact does redistribute itself in the manner previously calculated. In the dielectric sphere, it remains uniformly distributed, and the problem thus corresponds to those of accelerated motion usually treated by the quasi-stationary principle, in whose application any redistribution is ignored.

The main outlines of the necessary analysis, when both kinds of inertia are present, have been given in Walker's

second paper, although the special case is not examined. Let  $e$  be the charge on the sphere of radius  $a$ ,  $\kappa$  its dielectric constant, and  $c'$ ,  $c$  the velocities of radiation inside and out, so that

$$\kappa c'^2 = c^2. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$\xi$  is the displacement of the sphere at time  $t$ , and  $F$  the force, of a mechanical origin. As for the conductor, the field outside can be expressed in terms of a function  $\chi(ct-r)$ , small like  $F$ , in the form, valid for a certain region,

$$\begin{aligned} (X, Y, Z) = \frac{e}{r^3}(x, y, z) + \frac{e}{r^3}(-1, 0, 0) \left( r^2 \chi'' + r \chi' + \chi - \frac{e\xi}{c} \right) \\ + \frac{cx}{r^5}(x, y, z) \left( r^2 \chi'' + 3r \chi' + 3\chi - 3 \frac{e\xi}{c} \right). \quad . \quad . \quad (2) \end{aligned}$$

Inside the sphere, since there is no initial field, we may write, in terms of functions  $\psi_1(c't-r)$  and  $\psi_2(c't+r)$ , both of which are required,

$$\begin{aligned} (X, Y, Z) = \frac{c'}{r^3}(-1, 0, 0) \{ r^2(\psi_1'' + \psi_2'') + r(\psi_1' - \psi_2') + \psi_1 + \psi_2 \} \\ + \frac{c'x}{r^5}(x, y, z) \{ r^2(\psi_1'' + \psi_2'') + 3r(\psi_1' - \psi_2') + 3(\psi_1 + \psi_2) \}, \quad (3) \end{aligned}$$

the axes moving with the sphere. In order that the internal field may be finite at the centre,

$$\psi_1(c't) + \psi_2(c't) = 0. \quad . \quad . \quad . \quad . \quad (4)$$

The tangential electric and electromagnetic forces are identical to the order contemplated, and thus by the continuity of either at  $r=a$ ,

$$c \left( a^2 \chi'' + a \chi' + \chi - \frac{e\xi}{c} \right) = c' \{ a^2(\psi_1'' + \psi_2'') + a(\psi_1' - \psi_2') + \psi_1 + \psi_2 \}. \quad (5)$$

The difference of normal flux being  $4\pi\sigma$  or  $e/a^2$ , it follows that

$$c \left( a \chi' + \chi - \frac{e\xi}{c} \right) = \kappa c' (a \psi_1' - a \psi_2' + \psi_1 + \psi_2). \quad . \quad (6)$$

A determination of the mechanical force gives\* for its

\* Walker, *l. c.* p. 174.

resultant along the direction of motion the value

$$-\frac{2}{3} \frac{ec}{a} \chi''(ct-a);$$

so that

$$m\ddot{\xi} + \frac{2}{3} \frac{ec}{a} \chi''(ct-a) = F. \quad (7)$$

The initial conditions are as before,  $\xi = \dot{\xi} = 0$  at  $t=0$ ,

$$-\chi(-a) = \chi'(-a) = 0, \quad (8)$$

and the equations for determination of  $(\chi\psi_1\psi_2)$  subject to these conditions become

$$\begin{aligned} c \left\{ a^2 \chi'' + a \chi' + \left(1 + \frac{m'}{m}\right) \chi - \frac{1}{2} \frac{eF}{mc} t^2 \right\} \\ = c' \{ a^2 (\psi_1'' + \psi_2'') + a (\psi_1' - \psi_2') + \psi_1 + \psi_2 \}, \\ c \left\{ a \chi' + \left(1 + \frac{m'}{m}\right) \chi - \frac{1}{2} \frac{eF}{mc} t^2 \right\} \\ = \kappa c' \{ a (\psi_1' - \psi_2') + \psi_1 + \psi_2 \}; \quad (9) \end{aligned}$$

and Walker's particular solution for  $\chi$  involving only non-vibratory terms is

$$\chi(ct-a) = \frac{1}{2} \frac{eF}{(m+m')c^3} \left\{ c^2 t^2 - \frac{2mact}{m+m'} - 2a^2 \left( \frac{mm'}{(m+m')^2} + \frac{m}{(\kappa-1)(m+m')} \right) \right\}, \quad (10)$$

where  $m' = \frac{2}{3} \frac{e^2}{ac^2}$  and is the usual electrical inertia for slow speeds.

The vibratory terms will be of the form

$$\left. \begin{aligned} \chi(ct-r) &= A e^{-\lambda(ct-r+a)/a} \\ \psi_1(c't-r) &= B e^{-\lambda \kappa^{\frac{1}{3}}(ct-r+a)/a} \\ \psi_2(c't+r) &= -B e^{-\lambda \kappa^{\frac{1}{3}}(ct+r+a)/a} \end{aligned} \right\}, \quad (11)$$

or more strictly, summations of this form for the various values of  $\lambda$  satisfying the period equation

$$\frac{\tanh \kappa^{\frac{1}{3}} \lambda}{\kappa^{\frac{1}{3}} \lambda} = 1 + \kappa \lambda^2 \left( 1 + \frac{m'}{m} - \lambda \right) / \left\{ (\kappa-1) \left( 1 + \frac{m'}{m} - \lambda \right) - \kappa \frac{m'}{m} \lambda + \kappa \lambda^3 \right\}, \quad (12)$$

the real part of the summations being taken,  $\lambda$  being complex.

The equation may be shown to have a root zero, but no others except complex values whose real part is positive. Thus the vibratory terms ultimately decay.

We proceed to the case of a sphere without Newtonian mass. Taking the mass at first as very small, the non-vibratory part of  $\frac{1}{2}Ft^2 - \frac{2}{3}\frac{e}{ac}\chi$  becomes, on reduction,

$$\frac{mF}{2m'c^2} \left\{ c^2t^2 + 2act + 2a^2 \frac{\kappa}{\kappa-1} \right\},$$

and by (7) this is the non-vibratory part of  $m\xi$ . The vibratory portion is of the form

$$m\Sigma D e^{-\lambda ct/a} \sin\left(\frac{\mu ct}{a} + \epsilon\right), \quad . \quad . \quad . \quad (13)$$

where the root of the period equation is now written  $\lambda \pm i\mu$ , and  $D$  and  $\epsilon$  depend on  $\lambda$  and  $\mu$ .

In order that  $\xi$  may satisfy the appropriate conditions at  $t=0$ , it is necessary that

$$\begin{aligned} \Sigma D \sin \epsilon &= -\frac{m}{m'} \cdot \frac{a^2 F}{c^2} \cdot \frac{\kappa}{\kappa-1}, \\ \Sigma(\mu D \cos \epsilon - \lambda D \sin \epsilon) &= -\frac{a^2 F}{m' c^2}, \quad . \quad . \quad . \quad (14) \end{aligned}$$

the summation being for all roots  $\lambda \pm i\mu$  of the period equation

$$\frac{\tanh \kappa^{\frac{1}{2}} x}{\kappa^{\frac{1}{2}} x} = 1 + \frac{\kappa x^2}{\kappa - 1 - \kappa x}. \quad . \quad . \quad . \quad (15)$$

The acceleration is always finite, whatever the distribution of the vibrations among the possible periods. The determination of this distribution is difficult, but is not necessary for the present purpose. In addition to the decrease in amplitude which may be expected as the vibrations recede from the fundamental, there will be increased damping in the higher modes. When  $\kappa$  is not too great, the damping will not be slight even for the fundamental, which will then be the only vibration needing attention. If this is so, and if the amplitude of this vibration is sufficiently preponderant, we may write

$$\xi = \frac{F}{2m'c^2} \left( c^2t^2 + 2act + \frac{2a^2\kappa}{\kappa-1} \right) + D e^{-\lambda ct/a} \sin\left(\frac{\mu ct}{a} + \epsilon\right),$$

where 
$$D \sin \epsilon = -\frac{a^2 F}{m' c^2} \cdot \frac{\kappa}{\kappa - 1},$$

$$\mu D \cos \epsilon = -\frac{a^2 F}{m' c^2} \left(1 - \frac{\lambda \kappa}{\kappa - 1}\right),$$

and for moderately large values of  $\kappa$ ,  $\epsilon = \mu$ , and

$$\xi = \frac{F}{2m' c^2} (c^2 t^2 + 2act + 2a^2) - \frac{F a^2}{m' c^2 \mu} e^{-\lambda \sigma t/a} \sin \frac{\mu}{a} (ct + a). \quad (16)$$

The period equation is practically  $\tanh \kappa^{\frac{1}{2}} x = \kappa^{\frac{1}{2}} x$ , whose fundamental solution is  $\kappa^{\frac{1}{2}} x = \pm 4.4934i$ , so that  $\mu = 4.493/\kappa^{\frac{1}{2}}$  of the order assumed above. For period equations of the present type, the real part of the solution is much smaller. A similar case is given by Lamb\*.

We see, therefore, that for a dielectric sphere under a small mechanical force, the vanishing of the Newtonian mass causes no difficulty as regards the acceleration; and in view of the fact that the presence of this mass is doubtful, and that its absence would tend towards simplicity in the construction of the ideal electron, it seems possible that the postulation of dielectric rather than conducting property in an electron will be of service.

Such an electron, moreover, by virtue of the rigidity of its electrification, would fulfil one of the necessary conditions for the validity of the quasi-stationary principle for small accelerations. It is the possibility of redistribution of the charge which is the main difficulty of this principle, and the problems treated by Walker are sufficient to show conclusively that redistributions will ordinarily take place for conductors in accelerated motion. Now a fairly large value of  $\kappa$  for the dielectric interior of a sphere secures that the internal vectors shall be nearly zero, and this, combined with the rigidity of the charge, should be sufficient. It has been tacitly supposed throughout that the Lorentz contraction does not take place, although it is the belief of the writer that the contracted electron gives the best representation of fact, and a recent investigation by Bucherer† tends to prove this.

If the dielectric sphere with a surface charge thus fulfils the conditions of that for which the quasi-stationary principle has been used, it may be expected to yield Abraham's expression for the transverse inertia when the sphere has a

\* Camb. Phil. Trans., Stokes Commem. volume.

† *Phys. Zeit.* 1908, p. 775.

uniform motion, and an accelerating force is applied perpendicular to that motion. Now Walker has shown in the case of the conductor, that when the surface condition is the evanescence of the tangential electromagnetic force, Abraham's expression does not follow as the result of a direct calculation from the primary electromagnetic equations. This disproves the quasi-stationary principle for the initial motions of a conductor at least, although the initial condition, involving the instantaneous creation of a uniform field, is somewhat artificial.

The equations valid for a dielectric in variable motion are not yet free from doubt, and a direct calculation of the inertia in this case, as Walker points out, is not at present possible; but he concludes that the inertia of the dielectric with a large value of  $\kappa$  would be practically the same as for a conductor with equal charge, by the following argument\*.

"Since there is continuity of normal flux of disturbed electric force at the surface, the functions which determine the disturbance inside the sphere are of order  $\kappa^{-1}$  as compared with those which determine the outside field. Hence the tangential component of electric force inside, and therefore also outside, is very nearly zero. Thus since the equations for the æther are not modified by the motion of the sphere, the equation of motion and the surface forces outside differ by terms of order  $\kappa^{-1}$  from those for a perfect conductor. If this argument is valid, the assumption of perfect conduction, or of a high value of  $\kappa$  for the charged particle, would equally well explain Kaufmann's results, and give the same value for the electric inertia without limitation as to speed."

This argument appears to dispense with the necessity for complete analytical treatment. The inertia in question is that derived by Walker's analysis of the conductor with the other, and in the opinion of the writer, less likely condition in that case, that the tangential electric force is zero. Quoting the results, the initial longitudinal inertia becomes

$$\frac{e^2}{16ac^2} \left\{ \frac{4-5k^2+4k^4}{k^3(1-k^2)^{\frac{3}{2}}} \sin^{-1} k - \frac{4-13k^2+6k^4}{k^2(1-k^2)} \right\}, \quad (17)$$

and the transverse inertia is

$$\frac{e^2}{8ac^2} \left\{ \frac{4k^2-1}{k^3(1-k^2)^{\frac{3}{2}}} \sin^{-1} k + \frac{1+2k^2}{k^2} \right\}; \quad (18)$$

and these are the initial values to be regarded as true for a

\* Phil. Trans. 1910, A, p. 178.



dielectric sphere whose coefficient is large. They do not agree with the results deduced from a consideration of steady motions, without redistribution, but must apparently be regarded, with the corresponding values for a conductor, as the only values which have received a complete proof.

Meanwhile, as stated in the preface, it may well be of service, in any attempt to treat the electron as not subject to deformation, to endow it with dielectric rather than conducting properties. The analysis of this hypothesis presents no difficulty which does not appear to be shared by the other, and in a consideration of initial motions, it gives rise to great simplicity in the possible case of no Newtonian mass.

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XCI. *The Electron Theory of the Optical Properties of Metals.* By Prof. HAROLD A. WILSON, F.R.S., McGill University, Montreal\*.

THE electron theory of the optical properties of metals has been developed by Drude, J. J. Thomson, H. A. Lorentz, J. H. Jeans, and others.

Let  $N$  denote the number of free electrons per c.c. in the metal, and let  $dN$  be the number in the group with velocities between  $V$  and  $V + dV$ . The number  $dN$  remains nearly constant, although particular electrons are continually entering and leaving the group. Each such group may therefore be regarded as having a permanent existence. Since the mass of an atom is large compared with the mass of an electron, the velocity of an electron will not be much altered by collisions with atoms, and collisions with atoms must be much more numerous than collisions with electrons. Consequently the electrons in a particular group may be regarded as making many collisions, and still remaining in the same group or in a set of groups covering a small range of velocities.

When an electric force acts in the metal the electrons in each group will acquire an average velocity which will not be the same for the different groups. The motion of a group will be determined by a differential equation which will be of the same form for all the groups, but with different values of the constants for the different groups. It will therefore not be possible to represent the average velocity of all the  $N$  electrons by a single differential equation, unless we make the assumption that all the electrons have the same velocity of agitation.

\* Communicated by the Author.