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361. On the Conics Passing through Four Concyelic Points

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It is a matter of satisfaction to all concerned that the question of priority should thus be placed beyond a doubt. But at the same time it is rather amusing to find that Mr. Artemas Martin also desires to place on record the fact that I have not myself discovered other identities to the authorship of which he lays claim. Needless to say I have nowhere claimed to have discovered everything. After pointing out a general method for the solution of the equation  $\sum_1^n x_i^p = y^p$ , I limited myself to the solution, the simplest, for  $p=4$ , a solution, also the simplest, for  $p=5$ , and added "and so on" (pp. 143, 146 of the monograph under review). So that the field was left free for any investigator who is undeterred by the laborious nature of research of the kind.

Of course the discovery of the two identities given above constitutes but a very small portion of my memoir, which contains a considerable amount of much more interesting material.

My monograph, completed in 1909, was not printed until 1910, and copies were sent to various learned Societies across the Atlantic. I must add that until the appearance of the note in the *Gazette* I was totally unaware that Mr. Artemas Martin had ever been engaged in researches analogous to my own. It is perhaps unfortunate that the general body of mathematical literature is assuming such proportions that only too few of the mathematicians of the Old World are able to keep themselves *au courant* with what has been or is being published in the New.

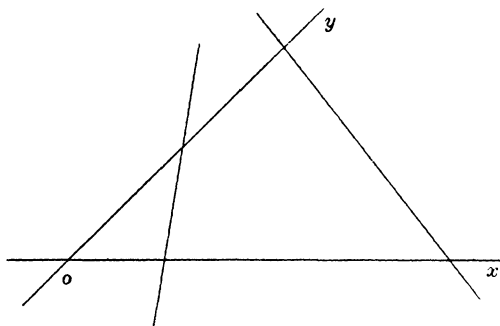
E. BARBETTE.

Liège, Oct. 1911.

**361. [L. 14. a.]** *On the conics passing through four concyclic points.*

Take two opposite sides of the quadrilateral formed by the points as axes of coordinates, and let the intercepts made by the other two opposite sides be  $a, b; a', b'$ . Then the equation of any conic through the four points is

$$\left(\frac{x}{a} + \frac{y}{b} - 1\right)\left(\frac{x}{a'} + \frac{y}{b'} - 1\right) = \lambda xy.$$



Now the points are concyclic;  $\therefore aa' = bb' \equiv k^2$ ; but also  $a + a' \equiv 2a$ ,  $b + b' \equiv 2\beta$ . Hence the various conics can be tabulated thus:

I. *Circle.*  $x^2 + 2xy \cos \omega + y^2 - 2ax - 2\beta y + k^2 = 0.$

II. *Parabolas.*  $(x \pm y)^2 - 2ax - 2\beta y + k^2 = 0.$

III. *Rect. hyp.*  $x^2 + 2xy \sec \omega + y^2 - 2ax - 2\beta y + k^2 = 0.$

I. The circle may be written  $(x - x')^2 + 2(x - x')(y - y') \cos \omega + (y - y')^2 = r^2$ , where  $(x', y')$  is the centre,  $r$  the radius.

It is easily found that

$$r^2 = (a^2 - 2a\beta \cos \omega + \beta^2) \operatorname{cosec}^2 \omega - k^2.$$

*Lemma.* The lengths of the perpendicular from the pt.  $(x', y')$  on the line  $x \pm y - p = 0$  are  $(x' \pm y' - p) \frac{\cos \frac{\omega}{2}}{\sin \frac{\omega}{2}}$ .

II. Consider the parabolas  $(x \pm y)^2 - 2ax - 2\beta y + k^2 = 0$ .

In the usual way we find that:

the equations of the axes are  $2(x \pm y) = a \pm \beta$ ,

the equations to the tangents at the vertices are

$$(a \mp \beta)(x \mp y) = k^2 - \left(\frac{a \pm \beta}{2}\right)^2,$$

and the lengths of the latera recta are

$$(a - \beta) \cos \frac{\omega}{2} \cot \frac{\omega}{2} \quad \text{and} \quad (a + \beta) \sin \frac{\omega}{2} \tan \frac{\omega}{2}.$$

The rectangular hyperbola  $x^2 + 2xy \sec \omega + y^2 - 2ax - 2\beta y + k^2 = 0$  may be written

$$\begin{aligned} & \left(x + y - \frac{a + \beta}{2} \cdot \frac{\cos \omega}{\cos^2 \frac{\omega}{2}}\right)^2 \cos^2 \frac{\omega}{2} - \left(x - y + \frac{a - \beta}{2} \cdot \frac{\cos \omega}{\cos^2 \frac{\omega}{2}}\right)^2 \sin^2 \frac{\omega}{2} \\ & = \cot^2 \omega \{2a\beta - (a^2 + \beta^2 \cos \omega)\} - k^2 \cos \omega. \end{aligned}$$

Hence the equations of the axes are

$$x + y - \frac{a + \beta}{2} \left(1 - \tan^2 \frac{\omega}{2}\right) = 0,$$

$$x - y + \frac{a - \beta}{2} \left(\cot^2 \frac{\omega}{2} - 1\right) = 0,$$

and the square of the semi-axis is

$$\cot^2 \omega \{2a\beta - (a^2 + \beta^2 \cos \omega)\} - k^2 \cos \omega. \quad \text{N. M. GIBBINS.}$$

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