SPUR-GEARING.

By DANIEL ADAMSON, *Member*, of Hyde.

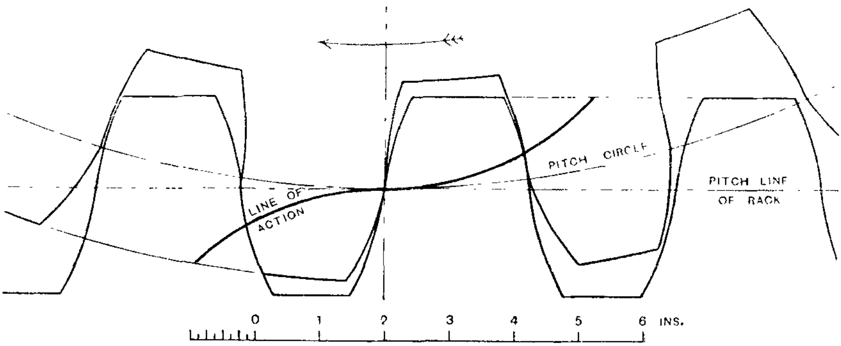
The application of toothed wheels to the transmission of power is perhaps the most interesting in the whole range of mechanical science, for the reason that nowhere else is to be found so cordial a co-operation between theory and practice. Papers upon the theory of the proper curves for the teeth of wheels have been presented to the various learned societies connected with engineering almost from the beginning of the work of each of them, while simultaneously the skill of the craftsman has kept pace with the development of the geometry of the subject. What better example of well-directed manual skill can be conceived than the work of the millwrights of, say, one hundred years ago, in dressing the teeth of iron wheels, or fitting new cogs to mortise wheels? Since that time we have had the invention of the wheel-moulding machine, a monument to the confidence of its originators, followed by gear-cutting machines, using "formers" or "formed" cutters, up to the automatic gear generating machines of the present day.

Most of the problems of a generation ago have been so well thrashed out by now that to-day a designer need not give so much consideration to the questions of outline or strength, and can devote himself to studying the materials that are likely to be most suitable for his requirements, choosing the proportions that will give the desired durability and providing for careful and accurate construction.

TOOTH SHAPES AND PROPORTIONS.

Two forms of tooth have been in general use for many years, the cycloidal curve, Fig. 1 (as described by a point on the circumference of a circle as it rolls along), and the involute curve, Fig. 2 (as made by a point in an imaginary cord unwound from a cylinder). While the involute tooth is formed of one curve, the cycloidal shape, as will be seen from the Fig., comprises two

FIG. 1.—*Cycloidal Teeth. 22-Tooth Pinion and Rack.*
Generating Circle Diameter = Half the Diameter of Smallest Wheel in Set = 12 Teeth.



curves. This double curve of the cycloidal tooth is difficult to reproduce, and if not accurately executed the gearing will not run well. Further, the exact working centres of the bearings must be strictly adhered to in order to obtain good results, and this is not always possible, whereas the involute shape is easier to originate, and there is a further advantage in that a departure from the correct distance of centres is not fatal to proper running.

The cycloidal shape for the teeth was, however, at first preferred to the involute because of a prejudice against the latter as increasing the pressure on the bearings, but more recently it has been realized that the probable loss from this cause is really very small, for the reason that whereas the thrust between bearings varies as the tangent of the angle of obliquity, the pressure on the bearings increases only as the secant (Fig. 3 and *American Machinist*, 1907, Vol. 30, page 804).

The demand for more accurately cut gearing, to give quieter

FIG. 2.—*Involute Teeth. Angle of Obliquity* $14\frac{1}{2}°$.
33-*Tooth Wheel and Rack.*

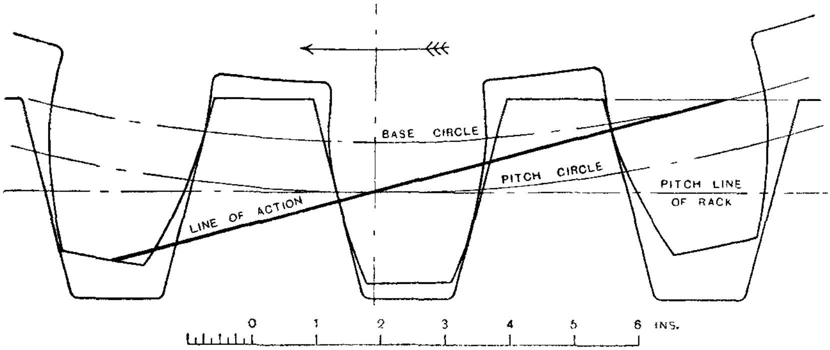
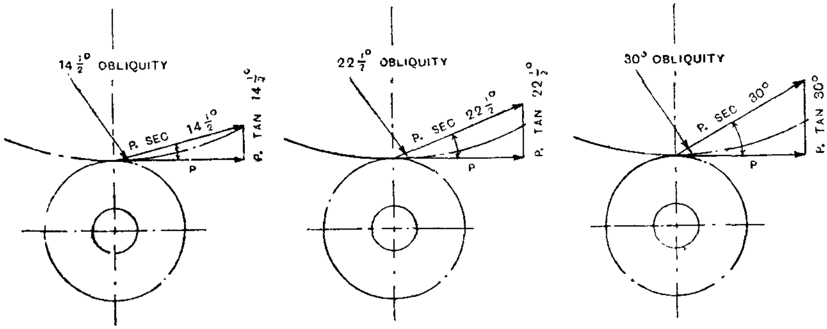


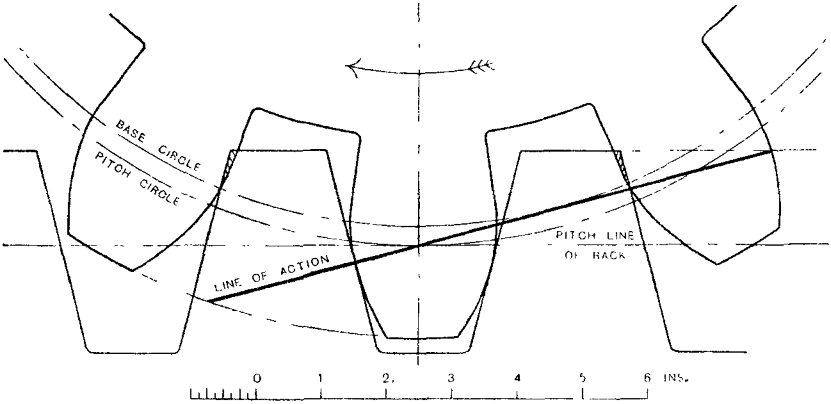
FIG. 3.—*Showing that the Bearing Pressure is not seriously increased by an increase in the Angle of Obliquity.*



running and greater freedom from shock, has encouraged the development of machinery that will generate the shapes of the teeth on correct geometrical principles, from cutters of simple shape that can be mechanically produced. This avoids the intermediate processes entailed in making "forming" cutters to outlines drawn by hand in the first instance. The involute form is suitable for such generation, because a rack-shaped cutter, or its equivalent a rotary hob-cutter, having straight-sided teeth, can be used, the only limitation being that such angles and proportions must be chosen as to avoid "interference" or "undercutting" when a pinion of small number of teeth is to be used.

In the case of a 12-tooth pinion and rack of $14\frac{1}{2}°$ obliquity, Fig. 4 (addendum = 0·3183, dedendum = 0·3683), the length of

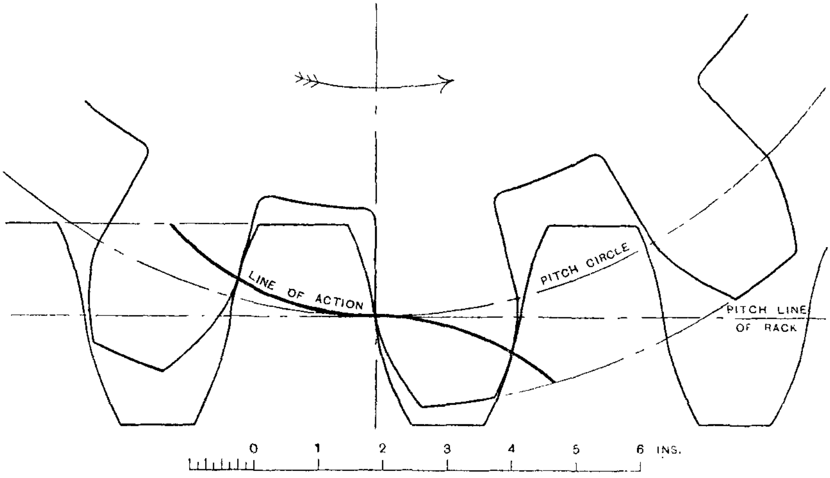
FIG. 4.—*Involute Teeth. Angle of Obliquity* $14\frac{1}{2}°$.
12-*Tooth Pinion and Rack.*
(Showing *interference* with straight-sided Rack-Teeth.)



the tooth must be reduced, or the angle of obliquity increased, or a combination of these two variables adopted to avoid "interference." Whatever system is adopted, it is now universally recognized that all wheels and pinions of a set of the same pitch must be interchangeable and work together properly as required, as distinct from the older methods where each pair of wheels was specially made. When establishing such an interchangeable system of gearing, the first factor to be taken into consideration is the minimum number of teeth in the smallest pinion of the set. This has been variously taken as 12, 13, or 14.

Professor Robert Willis (Proceedings, Inst. C.E., 1838, Vol. 2) recommended for cycloidal teeth the use of a constant describing circle of a diameter equal to the radius of the smallest pinion of the set, thus ensuring satisfactory mutual working between all wheels of the same pitch. This is the method generally adopted to-day for cycloidal teeth and gives radial flanks to the teeth of the smallest pinion (*Page's Weekly*, 29th December 1911, page 1201). Professor Willis seems to have been the first to suggest the use of circular

FIG. 5.—*Cycloidal Teeth* (Willis).
12-*Tooth Pinion and Rack.*

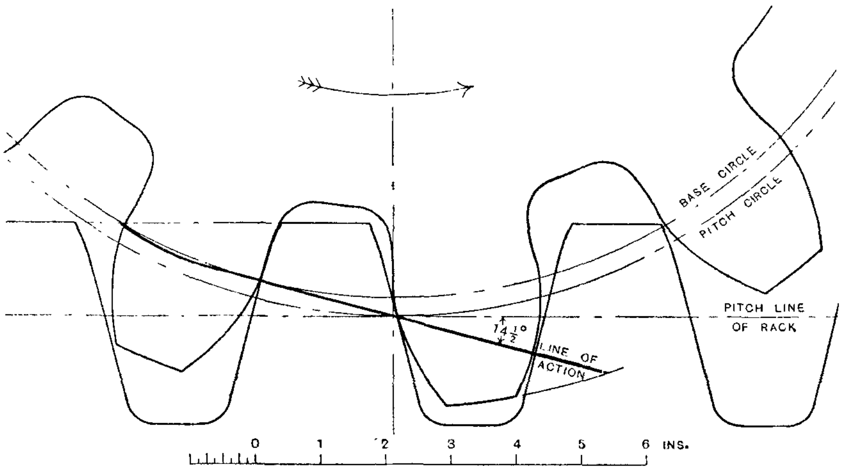


arcs to approximate the cycloidal curves (which are difficult to draw in practice), and he developed his odontagraph for this purpose. This was later superseded by Grant's odontograph (Gartside's Paper, Manchester Association of Engineers, January 1913). Professor Willis also gave a Table showing the number of rotary cutters required in a "set" for any given pitch of teeth, according to the limit of accuracy attainable in the shape of the finished cutter. A 12-tooth pinion and a rack to the proportions adopted by Willis (addendum about 0·3 of pitch, dedendum about 0·4) are shown in Fig. 5.

About fifty years ago the Brown and Sharpe Manufacturing Co. introduced their standard cutters which were afterwards adopted generally and very extensively. Their shape is a modified form of involute, the angle of obliquity being $14\frac{1}{2}°$, length of tooth above pitch line = 0·3183 of pitch, and below = 0·3683 of pitch. A 12-tooth pinion and rack to Brown and Sharpe's proportions are illustrated in Fig. 6.

About the year 1886 Mr. Wilfred Lewis advised Messrs. William Sellers and Co., of Philadelphia, to change from the

FIG. 6.—*Modified Involute* (Brown and Sharpe).
12-*Tooth Pinion and Rack.*



cycloidal system to the involute, and an obliquity of 20° was adopted by that firm (Proceedings, Engineers' Club of Philadelphia, Vol. 18, February 1901). In the Annual Report of the Engine and Boiler Insurance Company for 1887 Mr. Michael Longridge advocated shorter teeth.

In 1893 and 1895 Mr. Archibald Sharp suggested circular arcs for the outlines of wheel teeth to avoid the difficulty of forming accurate cycloidal or involute curves. He considered that his method would give a smaller departure from correct angular velocity ratio than would arise from the unavoidable errors of workmanship entailed in forming the more correct theoretical curves (Proceedings, Institution of Civil Engineers, Vol. 113, 1892–3, and Vol. 121, 1894–5). *See* Fig. 7.

Fig. 7.—*Circular Teeth* (Sharp).
12-*Tooth Pinion and Rack.*

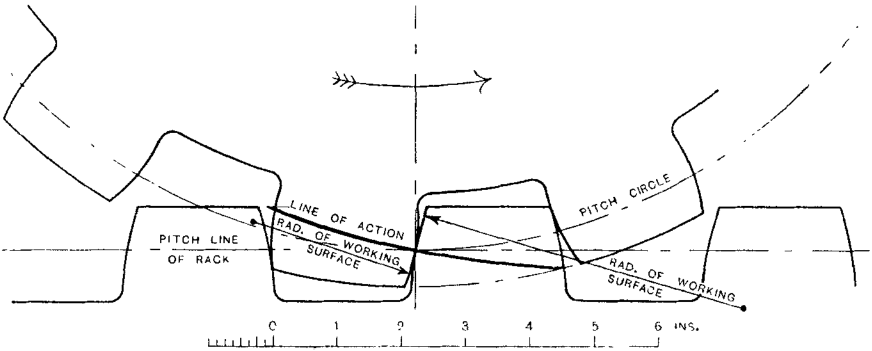
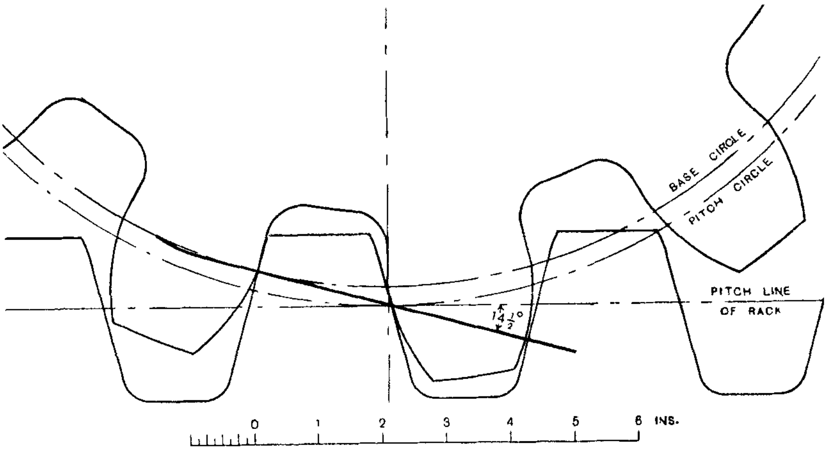


Fig. 8.—*Short Teeth* $14\frac{1}{2}°$ *Obliquity Involute* (Modified).
12-*Tooth Pinion and Rack.*

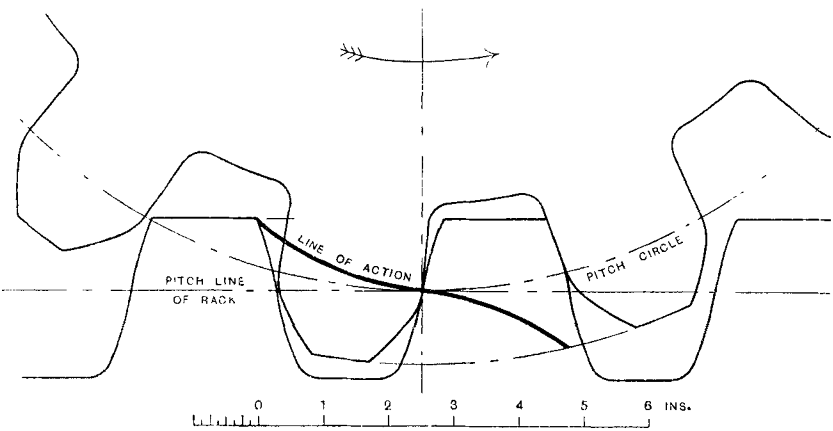


In 1899 the present Author adopted for the teeth of spur-wheels on cranes an addendum of $0 \cdot 25$ pitch with a dedendum of $0 \cdot 32$, retaining the Brown and Sharpe angle of obliquity, namely, $14\frac{1}{2}°$.

Fig. 8 shows a 12-tooth pinion and rack to these proportions, obliquity $14\frac{1}{2}°$. These proportions do not, however, prevent "interference" if a straight-sided rack-tooth is used, as is desirable for reasons of accurate reproduction. To avoid this objection, the angle of obliquity must be increased to $22\frac{1}{2}°$ if a 12-tooth pinion is to be the least of the series; but if 14 teeth can be adopted, then 20° is sufficient.

In addition to facilitating accurate construction, by permitting generation, these modifications have a further advantage in giving

FIG. 9.—*Sang's Modified Teeth* (R. H. Smith).
12-*Tooth Pinion and Rack.*



a stronger form of tooth, thus allowing a finer pitch to be used for given diameters of wheel and pinion. Reducing the pitch in this manner increases the number of teeth in contact, and thus improves the durability and reduces the noise. The rate of wear is also reduced when more teeth are in gear simultaneously, because the contact between the teeth takes place nearer the pitch line, where there is less relative sliding, and for the same reason the loss of power by friction between the teeth is reduced (Robert A. Bruce, *American Machinist*, Vol. 24, page 1290, 7th December 1901).

In 1908 Professor R. H. Smith described a method of setting out tooth outlines which was a compromise between the involute and the cycloidal, giving a contact path of "modified hour glass" shape (Society of Engineers, May 1908, "The Design and the Waste and Wear of Wheel Teeth"). The height of tooth recommended above pitch circle was equal to one quarter of the pitch. *See* Fig. 9.

Lasche's Investigations.

The most valuable publication dealing with the conditions that affect the durability of toothed gearing that has come to the notice of the Author is a series of articles by O. Lasche, of Berlin, published in 1899 (Zeitschrift des Vereines deutscher Ingenieure, Vol. 43, page 1417). As this has never been translated into English, and is not easily accessible, an extended reference thereto will perhaps be allowed. Lasche says:—

> "So long as pitch-line velocities did not exceed 600 feet per minute, the errors of workmanship and design, that were found even with gear-wheels turned out by first-class firms, did not cause any serious trouble."

To understand more clearly what occurs at the place of contact between the tooth faces, Herr Lasche investigated the variation in the amount of sliding at different parts of the tooth faces and flanks, and from this he developed a "wear characteristic." This "wear characteristic" is dependent upon the contact pressure and the amount of sliding between the contiguous teeth; and, as these quantities vary at each line of contact, it is easily understood why the rate of wear of teeth is found to vary over the different portions of the working surfaces.

When two curved surfaces are pressed together, the breadth of the line of contact is enlarged to an amount depending upon the yielding of the material and upon the rate of curvature of the two surfaces. The usual assumption is, that for a given load and a given material, with contact surfaces of varying curvatures the displaced volume remains the same for all. The depth of penetration will therefore be less, the flatter the curvature and the harder the material. Under the heavy local pressure, due to the contact of two curved surfaces of small radius, the lubricant is first squeezed out. Then the material of the teeth is deformed, and, under severe conditions or frequent repetition of loading, is quickly destroyed.

Figs. 10 and 11 are copied from diagrams in Lasche's original Paper, showing respectively the relative sliding between the working surfaces of the teeth and the "wear characteristic" developed therefrom by multiplying the amount of sliding by the estimated corresponding instantaneous load. The relative sliding shown by

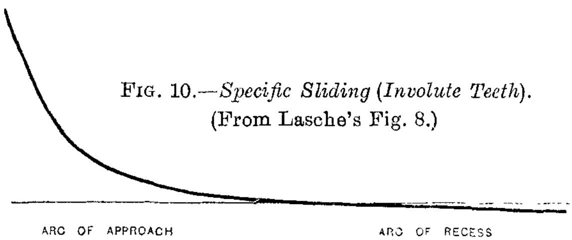


FIG. 10.—*Specific Sliding (Involute Teeth).*
(From Lasche's Fig. 8.)

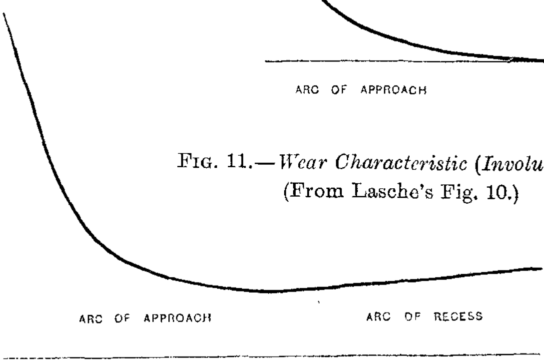


FIG. 11.—*Wear Characteristic (Involute Teeth).*
(From Lasche's Fig. 10.)

Fig. 10 may be expressed as the number of units of length of face of the driven tooth passing under one unit of length of face of the driving tooth, for a given angular movement of the gears.

These diagrams are only relative, or qualitative, as regards amount of wear to be expected. The exact quantities depend upon too many varying factors, such as elasticity of the material, lubrication, and condition of the working surfaces, to allow for reliable determination. The most noticeable feature of these diagrams is the relatively large amount of sliding that takes place at the point of the teeth, as compared with the regions near the pitch circles where the motion is more purely rolling. Also it may be pointed out that as the teeth approach the pitch point (during the arc of approach) they slide into one another, which is a less favourable condition than arises after the teeth have passed the centre line (during the arc of recess).

The next important condition affecting the working of gear-teeth is the relation between the theoretical arc of contact and the circumferential pitch, called for convenience the "duration of contact," and usually expressed in terms of the circumferential pitch. When the duration of contact is unity, each tooth in turn begins its work just as the preceding one comes out of action. If the duration of contact is greater than unity but less than two, it means that a tooth works alone until the following tooth engages, and the load is then distributed between the two teeth in gear until the first pair of teeth come out of action, leaving the second pair only in gear. Professor R. H. Smith in the Paper read before the Society of Engineers, and already alluded to, suggested a contact path of such a shape as to give a "duration of contact" of about $1\frac{1}{4}$ pitches. This ensures that each following pair of teeth is well in gear before the preceding pair has parted contact. Figs. 10 and 11 show that when the theoretical arc of contact exceeds unity, the duty will be distributed between the different teeth in contact at a time when the rate of wear would otherwise be at its highest value.

Table 1 (page 364) shows the theoretical number of teeth in contact depending upon the number of teeth in the wheel and pinion. Assuming two teeth continuously in contact, a third pair will come into action just as the first pair is leaving, and at the same time the second pair will be in contact at the pitch point. The exact division of the load, between two pairs of teeth in action under these circumstances, is uncertain, as it depends upon many factors, such as elasticity, accuracy of workmanship, etc., and recent authorities suggest that while a large arc of contact is desirable to reduce wear and tear, yet it must still be assumed that only one pair of teeth is in contact when calculating the strength of the gearing (*see* discussion on Marx's Paper, Amer. Soc. Mech. Eng.,

TABLE 1.

Duration of Contact.

For Involute Teeth 20° Obliquity. Addendum one-quarter of Circular Pitch.

No. of Teeth	12	15	20	30	36	45	60	75	100	150
12	1·17	1·19	1·21	1·25	1·26	1·28	1·29	1·30	1·32	1·33
15	1·19	1·21	1·24	1·27	1·29	1·30	1·32	1·33	1·34	1·35
20	1·21	1·24	1·27	1·30	1·32	1·33	1·35	1·36	1·37	1·38
30	1·25	1·27	1·30	1·33	1·35	1·36	1·38	1·39	1·40	1·41
36	1·26	1·29	1·32	1·35	1·37	1·38	1·40	1·41	1·42	1·43
45	1·28	1·30	1·33	1·36	1·38	1·40	1·41	1·42	1·43	1·44
60	1·29	1·32	1·35	1·38	1·40	1·41	1·43	1·44	1·45	1·46
75	1·30	1·33	1·36	1·39	1·41	1·42	1·44	1·45	1·46	1·47
100	1·32	1·34	1·37	1·40	1·42	1·43	1·45	1·46	1·47	1·48
150	1·33	1·35	1·38	1·41	1·43	1·44	1·46	1·47	1·48	1·49

1912, Vol. 34, page 1376). It is evident that the minute surfaces involved at the commencement of contact cannot carry as much as the more ample surfaces nearer the pitch line; hence the wear will quickly remove material from the points and roots of the teeth, and leave the bulk of the load to be transmitted by the teeth in contact near the pitch point.

[Büchner says that on this account involute teeth tend to approximate to cycloidal shapes when in use (Zeitschrift des Vereines deutscher Ingenieure, Vol. 46, page 279, 22nd February 1902)].

When wear takes place near the pitch circle, the resulting error of movement of the teeth points is magnified, and the points gouge out portions of the roots of the teeth of the other wheel. This accentuates the wear and gives rise to certain high stresses in a radial direction, which are not usually provided for in the design of the rims. Although new wheels may run quietly at first, yet as soon as undue wear occurs, the correct shape of the teeth is departed from, and the first requirement of toothed gearing, namely, transmission at a uniform angular velocity, is no longer fulfilled. The resulting rapid fluctuations in angular velocity give rise to chattering and hammering, which tend to destroy the structure of the material much more rapidly than the usual sliding and rolling of the surfaces.

A study of the "Wear Characteristic" diagram, Fig. 11 (page 362), will show the desirability of reducing the addendum, and by this means retaining in use only those portions of the tooth surface that endure approximately an equal amount of wear, thus retaining the correct original shape as long as possible. A reduction in the height of tooth and an increase in the angle of obliquity each improve the strength of the tooth, and the only important limitation in this direction is the reduction in duration of contact that follows from these modifications. The improved "wear characteristic" that would follow is an answer to the criticism that the increased angle of obliquity necessarily increases the amount of "back-lash."

The factors to be considered when estimating suitable proportions to resist wear are:—

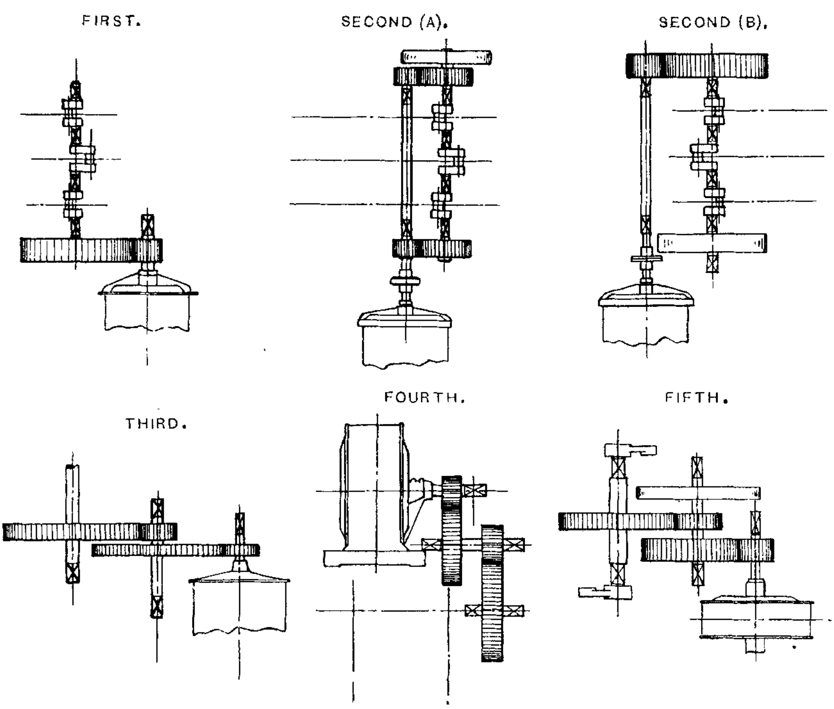
P = pressure to be transmitted at pitch line,
N = number of revolutions per minute,
e = estimated number of teeth in contact,
b = breadth of teeth,

and a Table of actual examples from practice and experiment is given in Lasche's original Paper to enable a suitable value to be placed upon $\frac{PN}{eb}$ according to the condition of the working.

The first of these examples, Fig. 12, refers to a 50 h.p. transmission at 575 revolutions per minute with a raw-hide pinion of 26 teeth, $1\frac{3}{4}$ inch pitch by 6 inches wide, gearing into an iron wheel having about 220 cast teeth. The pressure per inch of width of tooth (130 lb.) was found to be too great for rough cast teeth, and the gearing was increased in width to bring the load down to 93 lb. per inch of width; the teeth of the new wheel were machine cut, and the result was very satisfactory. The pitch-line velocity was 2,150 feet per minute.

In the second example, two raw-hide pinions of 18 teeth, $1\frac{1}{2}$ inch pitch by 6 inches wide, were arranged "in parallel" to transmit 50 h.p. between them at 575 revolutions per minute. The pressure per inch of tooth width, assuming one pinion carrying the load

FIG. 12.—*Five Examples of Transmission* (Lasche).



alone (215 lb.), was too great, the "parallel" arrangement was unsatisfactory because the pinions were not able to divide the load equally, and in addition the damp situation hastened the destruction of the raw-hide. The "parallel" arrangement was altered to a single pair of wheels and the width increased to $15\frac{3}{4}$ inches, giving a load of 80 lb. per inch of width with satisfactory results. The pitch-line velocity was 1,300 feet per minute. Fig. 12A shows the original arrangement and Fig. 12B that adopted afterwards.

The third example was designed to transmit 75 h.p. at 575 revolutions per minute, the wheel, about 136 teeth, and pinion, 35 teeth, $1\frac{1}{2}$ inch pitch, $5\frac{1}{2}$ inches wide, each being of steel with machine-cut teeth. The pinion teeth broke frequently because the first-motion pinion was rigidly connected to the large mass of the revolving armature, and the first-motion wheel was similarly very close to the second-motion pinion. The pitch-line velocity was about 2,460 feet per minute. The load transmitted works out at about 180 lb. per inch of width, which would give a working stress of only about 1,300 lb. per square inch in the teeth. Errors in workmanship amounting to 0·5 millimetre (say 0·02 inch) in pitch, however, that might have been unnoticed at lower velocities, or with more flexible attachment, gave rise to the trouble mentioned and the drive had to be entirely rebuilt.

The fourth example exhibited the same faults as the one last described. The motor was of 96 h.p. at 690 revolutions per minute driving a bore-hole pump. Pinion 23 teeth, 1·35 inch pitch, 7 inches wide, wheel about 70 teeth. The pinion was of phosphor-bronze in this case, with a load of 250 lb. per inch of width. The velocity (1,800 feet per minute) is too high for metallic wheels, unless the greatest accuracy in construction and care in installation are available.

The fifth example was a transmission of 100 h.p. with a raw-hide pinion, 26 teeth, 1·85 inch pitch, 11 inches wide, at 480 revolutions per minute, the load per inch of width being 153 lb. and the velocity 1,900 feet per minute. Although the transmitted load was not excessive, yet breakages of the teeth occurred through the same faults as in the third and fourth examples, and suggestions were made to substitute rope-driving for the first reduction, or a slow-speed motor. Each of the five examples quoted is shown diagrammatically in Fig. 12.

Four examples are given, where the experience has been quite satisfactory, of raw-hide pinions gearing with cast-iron wheels, the powers ranging from 5 to 75 h.p., the loads per inch of width from 34 to 136 lb., and the pitch-line velocities about 1,800 feet per minute. *See* Table below for particulars. The average for $\frac{PN}{eb}$ for these is about 32,000 in English units (pounds per inch).

[The average of fifty examples of raw-hide pinions "in service and giving satisfaction" given by Livingstone (*Electrician*, 19th March 1909, page 892) gives $\frac{PN}{b}$ as 112,000 and $\frac{P}{b}$ = 115 lb. per inch of face.]

Four Examples of Raw-hide Pinions with Cast-Iron Wheels.

H.P.	R.P.M.	Pitch	Breadth	Teeth in Pinion	Teeth in Wheel	Velocity	Load on Teeth in lb.	$\frac{PN}{eb}$
		inch	inches			ft. per sec.		
5	1,440	0·49	2·95	28	112	27·56	99	25,400
15	960	0·99	4·72	25	100	32·8	246	25,000
40	720	1·236	7·87	25	100	30·84	705	35,900
75	575	1·496	10·24	25	100	29·53	1,380	43,000

Three examples given by Lasche of delta-metal pinions working satisfactorily with steel wheels on crane motors varying from 15 to 75 h.p. have loads per inch of width varying from 175 to 350 lb., the pitch-line velocity being from 1,000 to 1,200 feet per minute; average for $\frac{PN}{eb}$ is about 100,000, *see* Table below.

[As confirmation of the above, the Author may be allowed to say that he adopted, more than twelve years ago, a maximum

Three Examples of Delta Metal with Steel Wheels on Cranes.

H.P.	R.P.M.	Pitch	Breadth	Teeth in Pinion	Teeth in Wheel	Velocity	Load on Teeth in lb.	$\frac{PN}{eb}$
		inch	inches			ft. per sec.		
15	960	0·618	2·36	24		19·68	415	93,600
40	720	0·618	4·72	24		17·72	1,235	104,500
75	575	0·866	7·08	24		16·40	2,470	111,300

of 130,000 for $\frac{PN}{eb}$ for intermittent crane work for steel pinions gearing with steel wheels, and 53,300 for raw-hide pinions under similar conditions, assuming a duration of contact equal to $1\frac{1}{3}$ pitch, and the results have been quite satisfactory.]

Referring to the variation in stresses due to irregularities in pitch, Lasche gives calculations of a typical example from practice showing that while at moderate speeds this is not serious, yet as the excess load due to irregularities will increase as the square of the velocity, it is evident why trouble is experienced, at higher peripheral speeds, that cannot be accounted for by calculations based upon the transmitted load alone. Hence the greater care that is essential for success at the high speeds now in use. (An abstract of Lasche's calculations on this point is given in Appendix II, page 390.)

Professor R. H. Smith also refers to the increased stress caused by irregularities in the pitch or shape of wheel teeth (Society of Engineers, 1908).

Another important detail elaborated by Lasche is the advisability of separating revolving masses, as, for example, armatures of motors, from adjoining masses such as the large gear-wheels of the second motion, by some elastic medium such as a length of shaft or a flexible coupling, so as to allow slight variations in angular velocity, due to unavoidable errors in gear-cutting, to take place without giving rise to destructive loads between the teeth. Satisfactory running had been obtained at the date of Lasche's Paper (1899) (by careful workmanship, installation and lubrication) with milled cast-iron wheels and raw-hide pinions, at 960 revolutions per minute, 2,400 feet per minute, and 112 lb. per inch of face, and with cast-steel wheels and delta-metal pinions at 720 revolutions per minute and 1,800 feet per minute. Wear was hardly noticeable on the delta-metal and cast-steel combination after millions of revolutions at pressures up to 700 lb. per inch of width. For lubrication of raw-hide, a mixture of talc graphite and resin proved satisfactory.

[From these references to Lasche's investigations, it is very evident that durability depends upon accuracy of construction as much as upon correctness of design and suitability of the material used.]

MATERIALS.

It is upon the question of the most suitable materials that the Author hopes to learn most, and with a view to eliciting some discussion, he proposes to review briefly the requirements of the conditions involved. For many years cast-iron was universally adopted for spur-gearing, and then gun-metal was used to some extent, but the elastic limit of these materials is too low for use under modern conditions as to speed and load (Christie, Proceedings, Engineers' Club of Philadelphia, Vol. 18, page 43, February 1901).

Cast-steel has been largely used in recent years in place of cast-iron, with satisfactory results, but the demand for greater reliability and economy in space has called for forged steel of varying qualities. The milder qualities of steel first adopted, say up to 0·3 per cent. carbon, gave trouble by abrasion (Christie, *loc. cit.*), and also by change of tooth form under the percussive action they experience when in use under heavy loads, and case-hardening the teeth has been adopted to some extent to overcome this weakness.

It is now generally agreed that for important gearing, where space is limited, the elastic limit of the steel must be raised from, say, 20 tons to 40 tons per square inch (Litchfield, Trans. Amer. Soc. Mech. Eng., 1908, page 972), and the only difference of opinion is as to whether this should be done by merely increasing the carbon contents to, say, 0·6 per cent. or more, or by using other alloys such as nickel, manganese, chromium, vanadium, etc., with or without suitable heat-treatment to toughen and harden the steel. One objection raised to high-carbon steel is the risk of brittleness ensuing from careless treatment (Logue, *American Machinist*, 1908, Vol. 31, Part 1, page 95). Revillon recommends nickel steel, because all requirements can be met without a complicated heat treatment (Iron and Steel Institute, Carnegie Memoirs, Vol. 1, page 218).

It should not be overlooked that while heat treatment may be quite feasible with large masses of steel, such as armour-plates and gun tubes, where the value of each piece makes it possible to maintain an expensive laboratory and staff to check the processes and the results, this is not so convenient in ordinary machine-shops. Whatever material is adopted, it is advisable to make the pinion of harder material than the wheel (Logue, *American Machinist*, 1908, Vol. 31, Part 2, page 115) in order to divide the wear equally between the two, and thus prolong their useful life by enabling the teeth of both wheel and pinion to retain their original accuracy and shape as long as possible. If undue wear affects the pinion, this in turn reacts upon the teeth of the wheel and both suffer. More attention will have to be given to the constituents of steel used by engineers as requirements become more onerous. Some engineers connected with large steel works are already including in their specifications of machinery to be purchased a stipulation that phosphorus and sulphur in steel forgings and castings must not exceed 0·04 per cent.

ALLOWABLE WORKING STRESSES.

As to allowable working stresses in the teeth of wheels, there is a great diversity of opinion due no doubt to the varying standards of workmanship in the examples used by the various authorities for their bases.

E. R. Walker in 1868 published some data, based upon a breaking load of 2,000 lb., for a tooth 1 inch wide by 1 inch pitch (corresponding to a maximum fibre stress in the teeth of about 15,000 to 33,000 lb. per square inch, depending upon whether a pinion tooth or a rack tooth is taken, or, say, 24,000 as an average), with a Table giving factors of safety recommended, varying from 3 at very slow speeds to 14 at 40 feet per second (2,400 feet per minute) (J. H. Cooper's Paper in the Journal of the Franklin Institute, 1879, Vol. 108, pages 15–16). *See* Fig. 13 (page 372).

F. Reuleaux's "The Constructor," originally published over 50 year ago, gives permissible stresses as follows:—

Wood	2,544 lb. per square inch.
Cast-iron	4,240 ,, ,, ,, ,,
Steel	14,112 ,, ,, ,, ,,

These are for speeds not exceeding 100 feet per minute, and are to be reduced for higher velocities down to about half their value for a velocity of 2,500 feet per minute.

In 1892 Wilfred Lewis published a Table (Proceedings, Engineers' Club of Philadelphia, 1893, Vol. 10), suggesting working stresses as follows (page 374):—

FIG. 13.—*Ratio of Allowable Load to Velocity.* *E. R. Walker*, 1868.

Velocity of rim of wheel in ft. per min.	Very slow without shock.	180	300	600	900	1,200	1,800	2,400
Factor of safety.	3	4	5	6	8	10	12	14
Corresponding Max. working stress in lb. per sq. in. (estimated).	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700

NOTE.—The actual values vary from 60 per cent. to 150 per cent. of these values, depending upon whether the teeth of 12-tooth pinions or racks are taken.

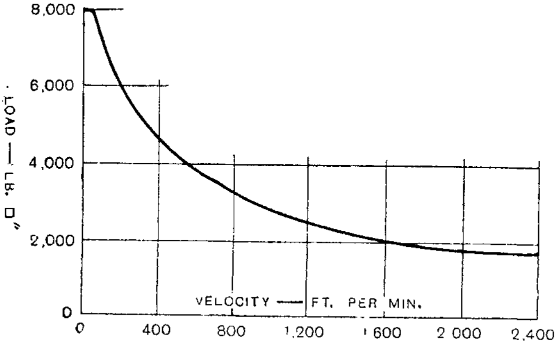


FIG. 14.—*Ratio of Allowable Load to Velocity.* *Wilfred Lewis*, 1893.

Velocity of teeth in ft. per min.	100 or less	200	300	600	900	1,200	1,800	2,400
Safe working stress for C.I. in lb. per sq. in.	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700

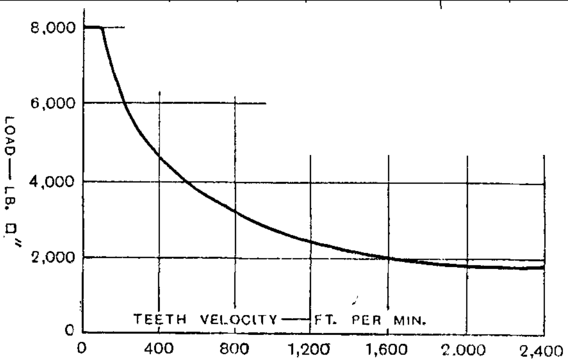


FIG. 15.—*Ratio of Allowable Load to Velocity. Marx and Cutter from Tests,* 1915. *Brown and Sharpe* $14\frac{1}{2}°$ *Involute Cast-iron.*

Pitch velocity in ft. per min.	0000	100	200	300	600	900	1,200	1,800	2,000
Coefficient . .	1·00	0·795	0·730	0·675	0·565	0·500	0·455	0·410	0·400
Corresponding working stress with a factor of safety of, say, 4.	9,000	7,200	6,600	6,100	5,100	4,500	4,100	3,700	3,600

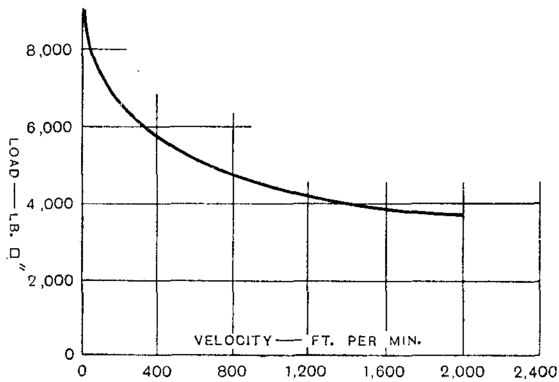
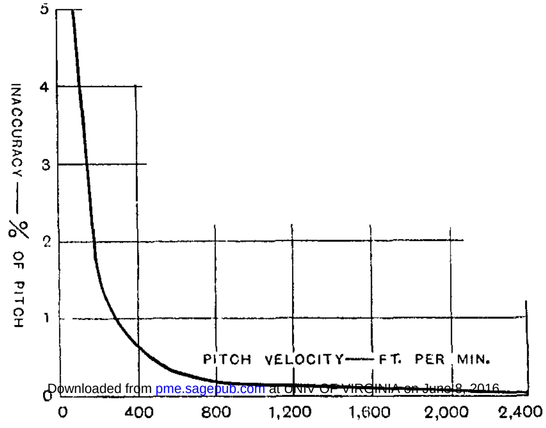


FIG. 16.—*Ratio of Allowable Inaccuracy to Velocity. Author, Suggested,* 1916.

Velocity of pitch circle in ft. per min.	100 or less	200	300	600	900	1,200	1,500	1,800	2,100	2,400
Inaccuracy in per cent. of pitch.	5 per cent.	1·76	0·96	0·34	0·185	0·12	0·09	0·065	0·052	0·042



Cast-iron . . .	8,000 lb. per square inch	In outer fibres due to bending.
Steel . . .	20,000 ,, ,, ,, ,,	

for velocities of 100 feet per minute or less, and these are to be reduced to about one-fifth for a velocity of 2,400 feet per minute. *See* Fig. 14.

These values correspond with E. R. Walker's recommendations, and it is a remarkable fact that Lewis's Paper, which is still acknowledged (Marx's Paper, Trans. Amer. Soc. Mech. Eng., 1912, Vol. 34) to be the standard work of reference on this subject, should be based upon empirical figures, that so long ago as 1868 had been in use for several years (J. H. Cooper's Paper, Journal Franklin Institute, 1879, Vol. 108, pages 15–16). *See* also Appendix I (page 381).

Maximum Speeds.

The maximum speeds allowed by different authorities also vary considerably owing, no doubt, to the different experience of each individual.

C. W. Drake (*Electric Journal*, 1912, page 554) states that noise begins at 600 feet per minute, but does not become disagreeable under about double this speed.

G. J. Leire (*Machinery*, July 1905, page 565) says that to avoid noise the pitch-line speed ought not to exceed 1,000 feet per minute.

W. H. Thornbery, M.I.Mech.E. (Staffordshire Iron and Steel Institute, 12th April 1902), gives 1,800 feet per minute as the greatest speed at which ordinary cast-iron wheels may be safely run, and up to 3,000 feet per minute for machine-cut wheels. The same authority quotes examples up to 4,000 and 5,000 feet per minute, but recommends caution before adopting such speeds.

Emile Geyelin (Proceedings, Engineers' Club of Philadelphia, 1894, Vol. 11, page 142) gives particulars of bevel-wheels at Niagara transmitting 1,100 h.p. at $\frac{260}{200}$ revolutions per minute where mortise-wheels were adopted because of the expected rapid variations in the load. The wheels have 43 and 33 teeth respectively, $5\frac{1}{2}$ inches pitch and 20 inches wide, with a pitch-line velocity of 3,930 feet per minute. E. Graves 6½ years later (Proceedings, Engineers' Club of Philadelphia, 1901, Vol. 18, page 57) refers to the same wheels as running satisfactorily, except that the wooden teeth only last from 6 to 8 weeks.

E. Graves gives in the same Journal (Vol. 18, page 56) particulars of some cast-steel bevel-wheels running in the same room as the above, and which had been put to work about 2 years later. These transmit 1,300 h.p. at $\frac{260}{200}$ revolutions per minute. The teeth are 5 inches pitch, 20 inches wide, and the velocity is 3,900 feet per minute. Their wearing power as regards abrasion was quite satisfactory, but troubles arose from time to time through the teeth breaking out. The normal stress due to the transmitted load alone works out at 2,100 lb. per square inch, or only one-tenth of the allowable stress under a static load, or, say, perhaps one-fifteenth to one-twentieth of the elastic limit of the material, or one-thirtieth of the ultimate breaking stress.

That a velocity of 2,500 feet per minute is quite feasible is proved by an example quoted by Christie (Engineers' Club of Philadelphia, 1901, Vol. 18, page 44), where a pair of spur-wheels transmit 3,300 h.p. at 260 revolutions per minute with a pitch-line velocity of 2,500 feet per minute, a load on teeth of nearly 2,100 lb. per inch of face, and a maximum fibre stress due to transmitted load of 6,300 lb. per square inch (assuming one pair of teeth in action). The pinion was made from a fluid-pressed steel forging having the following analysis:—Carbon, 0·86 per cent.; manganese, 0·51 per cent.; silicon, 0·27 per cent.; phosphorus and sulphur, each below 0·03 per cent. The wheel was an annealed steel casting having the analysis of carbon, 0·47 per cent.; manganese, 0·66 per cent.; phosphorus and sulphur, each below 0·5 per cent.

On the New York Subway (Litchfield's Paper, Trans. Amer. Soc. Mech. Eng., 1908, page 967) teeth failed by breakage before they were very far worn, the maximum stress running to 13,370 lb. per square inch (calculated on the original size of tooth, or about 16,500 lb. on the worn tooth) with material having an elastic limit of 45,000 lb., the pinion running at about 550 revolutions per minute, or 1,168 feet per minute at pitch circle.

An example that has come to the Author's notice in private correspondence (E. and G., Bolton, November 1910) is a pair of machine-cut bevel-wheels, ratios 3 to 1, transmitting 1,500 h.p., pitch-line velocity 3,000 feet per minute, teeth $10\frac{1}{2}$ inches wide; load 1,570 lb. per inch of width; the first wheels were made $3\frac{1}{2}$ inches pitch and transmitted 600 h.p. satisfactorily, but failed under occasional peak loads of 1,500 h.p., and were replaced by others of $2\frac{1}{2}$ inches pitch which proved quite satisfactory, the other conditions remaining unaltered.

Another example that has given trouble is a set of spur-gearing to transmit 120 h.p. at $\frac{488}{110}$ revolutions per minute, $\frac{156}{35}$ teeth 2 inches pitch, 8 inches broad, velocity 2,860 feet per minute, load 172 lb. per inch of face. Paper pinions and also a cast-iron one (cut teeth) failed in use and a steel pinion was found to be too noisy. The paper pinions lasted about 7,000 working hours.

It may be pointed out that, while it is the irregularity in the teeth that gives rise to noise in the first instance, yet, if the construction of the wheel is favourable to resonance, the noise will be multiplied to a disagreeable extent.

DURABILITY.

Consideration, as yet, has only been given to permissible loads and speeds having regard to risk of failure from too high stresses, but some guidance is also necessary for the designer in respect of durability.

Dr. Otto Schaefer (*Dingler's Polytechnisches Journal*, 1910, Vol. 325) gives a Table showing the variation in loading, according to the number of revolutions estimated to be made by a wheel before wear has so weakened the teeth as to bring the stress due to bending up to a predetermined maximum. The article refers to hoisting machinery in particular. Table 2 (page 377) gives Dr. Schaefer's values for K (a variable dependent upon the material used and the circumstances of the drive) in the simplified formula for strength of teeth:—

TABLE 2.—*For K.*

(*Converted into British Units*). *Maximum Stress Allowed.*

Total life in millons of revolutions	17,000 lb. per sq. inch	14,000 lb. per sq. inch	11,400 lb. per sq. inch	8,500 lb. per sq. inch	5,700 lb. per sq. inch	2,850 lb. per sq. inch
0	1,350	1,120	898	685	455	228
1	1,292	1,080	870	655	442	228
2	1,238	1,040	840	640	442	214
3	1,180	1,010	812	626	428	214
4	1,138	968	798	612	428	214
5	1,094	940	785	598	413	214
6	1,051	910	770	598	413	214
7	1,023	883	742	584	413	214
8	997	868	726	570	399	214
9	968	840	713	555	399	214
10	940	826	698	555	384	214
12	883	782	670	526	384	200
14	840	740	640	512	370	200
16	797	712	612	498	356	200
18	769	685	598	484	356	200
20	740	655	570	470	342	200
30	612	555	485	442	314	185
40	526	484	428	370	285	172
50	413	384	356	299	242	157
60	342	327	299	270	214	142
100	297	284	256	226	200	134
120	257	242	226	212	171	123
150	228	212	200	185	157	111
200	185	171	157	142	128	97
250	142	142	137	127	111	86
300	128	123	120	109	100	77
400	101	98	94	88	80	64
500	84	81	81	74	69	56

$$W = P \times F \times K,$$

where W = safe working load in pounds,
P = circular pitch in inches,
F = width of face in inches,
K = as in Table 2 (page 377).

In explanation of the Table, it may be observed that the number of revolutions made by a pinion running at, say, 500 revolutions per minute for 10 years of 300 days of 10 hours, working for one-third of the time (as, for example, on a crane), amounts to $500 \times 0 \cdot 6 \times 10^6$, and this figure (multiplied by, say, $0 \cdot 5$ on the assumption that the gearing is only loaded to one-half its full capacity on an average) gives 150×10^6. On referring to the Table it will be seen that for a life of 150×10^6 revolutions the value of K in the above formula should be taken at 228 for steel at 17,000 lb. per square inch maximum working stress in the worn tooth and 157 for cast-iron at 5,700 lb. per square inch.

Dr. Schaefer suggests maximum stresses of about 17,000 and 14,000 lb. per square inch for forged steel and steel castings respectively, and 5,700 lb. per square inch for cast-iron, and in the original article shows by comparison with actual examples, and also with an accepted formula by Bach, that his deductions are reasonably correct.

[These values seem very low for steel.]

CONCLUSION.

Reference has been made to Reuleaux and to Papers by Lewis, Lasche, Schaefer, and Marx, all of whom, in one way or another, directly or indirectly, make provision for reducing the estimated working stresses in the teeth when these are to run at higher speeds, and each Author is able to point to more or less satisfactory proof of the reliability of his data, so that it may be reasonably assumed that all contain a considerable amount of truth, although the question has been approached from different directions.

Lasche makes the most helpful suggestion by indicating how the stresses induced by variations in the angular velocity of wheels, due to unavoidable errors in pitch, may be many times the transmitted load alone, and also that these excesses will vary as the square of the velocity. Examples have been quoted of wheels that have failed by breakage of steel teeth after a few months' work, although these failures could not be accounted for by the peripheral load due to the power transmitted, and in fact an example is given where steel teeth (when gearing with wooden mortise teeth) have transmitted equivalent loading satisfactorily for years, the mortise teeth absorbing the shocks but having very short lives in consequence.

Assuming that Lasche is correct, and that the varying angular velocity resulting from inaccuracies in the cutting of the teeth of wheels causes excess loads which are large compared with the normal loading, the resulting stresses must be taken as practically equal in magnitude and of opposite sign. Under these conditions Dr. Unwin shows ("Elements of Machine Design," 1909, page 41) that the breaking stress (for one million changes of load) is only about one-half the primitive yield-stress of the material because of the Wöhler effect of the repeated stresses. (*See* also *Engineering*, Vol. XI, 1871, for the most complete record in English of Wöhler's work.)

Referring now to Graves' examples on page 375, if the transmitted load (equal to one-twentieth of the elastic limit of the material) is raised by these alternating stresses to one-half the elastic limit or anything of that order, it seems that the excess load (in this particular instance) at 3,900 feet per minute is ten times the transmitted load. Arguing largely from theoretical grounds, Lasche showed in 1899 that the possible excess load induced by an actual measured error of pitch of 0·02 inch in a wheel of about $1\frac{1}{2}$ inch pitch running at 2,460 feet per minute amounted to twenty-eight times the peripheral transmitted load. *See* Appendix II.

It seems to the Author a very reasonable assumption to make that the additional stresses thus induced in a wheel running at a high velocity are very large, and accordingly it is not sufficient to consider the stress due to the transmitted load, and to reduce this merely for the higher velocities. In the examples just quoted, where the excess loads are suggested as being ten times, or twenty-eight times, the transmitted load, no reduction in the stress originally allowed would meet the case. However small the transmitted load, the wheels would break, due to the alternating stresses alone. What is wanted is a standard of accuracy for different velocities. If 0·02 inch in a pitch of $1\frac{1}{2}$ inch (say $1\frac{1}{3}$ per cent.), in Lasche's third example, can cause trouble at 2,460 feet per minute, then to secure a factor of safety of, say, three, the inaccuracy must be reduced to less than $\frac{1}{2}$ per cent. for this velocity, and if a velocity of, say, 3,900 feet is desired (as in Christie's example) the error must be reduced to less than 0·2 per cent., the excess load increasing directly as the amount of inaccuracy, and as the square of the velocity.

Figs. 13, 14, and 15 (pages 372–3) show the allowable working stress according to Walker, Lewis, and Marx respectively, while Fig. 16 embodies the conclusion arrived at in this Paper.

H. F. Moore and F. B. Seeley have recently shown (American Society for Testing Materials, June 1915) that it is reasonable to assume that the allowable working stress should vary inversely as the one-eighth power of the expected number of repetitions of stress during the working life of the piece concerned. This means that, in addition to improving the standard of accuracy of *high-speed* gearing so as to reduce the excess loads resulting from variations in angular velocity, there must be further improvement in order to reduce the total stress to such a figure as will allow the required number of rotations (repetitions of stress) to be made without endangering the safety of the mechanism concerned, the number of rotations being generally greater per unit of time with *high-speed* gearing than with low-speed.

The Author considers that these somewhat detached notes are hardly worthy of being called a Paper, being in fact merely a collection made during the past twenty years of information and references for the Author's own guidance in the design of electric overhead travelling-cranes of varying capacities. He has endeavoured to show that the present state of knowledge on this important mechanical and metallurgical subject is far from complete, and hopes that he has given some indication of the direction in which members can afford valuable help, by giving examples from their experience of successful and unsuccessful practice in the use of spur-gearing of different materials under varying conditions.

The thanks of the Author are due to Mr. Thomas Bevan, M.Sc. (Tech.), for assistance in reading the proof, preparing the illustrations, and revising the Appendixes.

The Paper is illustrated by 16 Figs. in the letterpress, and is accompanied by two Appendixes with 2 Figs.

APPENDIX I.

LIST OF REFERENCES (*Arranged Chronologically*).

[*The square brackets enclose references to pages of the present Paper.*]

Euler's "New Commentaries," Vol. XI, St. Petersburg, about 1760. According to Willis the involute form was first suggested by Euler in his second Paper on "The Teeth of Wheels."

Camus on "The Teeth of Wheels," translated by G. I. Hawkins. Second edition, published by J. S. Hodson, London, 1837.

Professor Robert Willis: Proc. Inst C.E., 1838, Vol. 2. [Page 357.] Also, "Principles of Mechanism." Published by Parker, London, 1841.

Edward Sang: "New General Theory of the Teeth of Wheels." Published by A. and C. Black, London, 1852.

J. H. Cooper: Journal, Franklin Institute, 1879, Vol. 108, pages 15–16. *Paper* on "Power Transmitting Mechanism." The following is taken from the Paper:—

From "Practical Rules for the Proportions of Cog-Wheels and Shafting," by E. R. Walker (of Haigh Foundry, Wigan), Newcastle-under-Lyme, 1868 [page 371], we select the following observations and rules, changing the notations:—

"The results will be found to give a good average margin of strength with ordinary materials. The

rules are based upon and have been tested by a large number of examples in actual operation, and they have been used in a considerable practice for some years.

"To find the extreme strain which the teeth of any wheel are capable of transmitting.

Calling X = breaking load of tooth in lb.

S = working ,, ,, ,, ,, ,,

p = pitch of teeth in inches.

f = face ,, ,, ,, ,,

m = 3 for very slow speed without shock.

,, = 4 when rim of wheel runs 3 F. ps.

,, = 5 ,, ,, ,, ,, ,, 5 ,, ,,

,, = 6 ,, ,, ,, ,, ,, 10 ,, ,,

,, = 8 ,, ,, ,, ,, ,, 15 ,, ,,

,, = 10 ,, ,, ,, ,, ,, 20 ,, ,,

,, = 12 ,, ,, ,, ,, ,, 30 ,, ,,

,, = 14 ,, ,, ,, ,, ,, 40 ,, ,,

we have $$X = 2{,}000\, p.f \qquad . \quad . \quad . \quad (1)$$

"The extreme or breaking load being found, and the speed given to find the working load the wheel should bear,

$$S = \frac{x}{m}."$$

(Mr. E. R. Walker was Manager of Haigh Foundry, Wigan, about 1868, and is believed to have afterwards become Manager of Talk-o'-th'-Hill Colliery in Staffordshire.)

John Walker and others (Trans. Amer. Soc. Mech. Eng., 1885, Vol. 6, pages 862–3). Short notes as to maximum speeds and the relation thereto of pitch and workmanship, 6,000 feet per minute given as a maximum.

G. B. Grant (*American Machinist*, 31st October 1885, Vol. 8, pages 2–3) considers the curves for working surfaces of gear teeth and shows the involute to be superior to the epicycloid for efficiency. C. A. Smith (*American Machinist*, 1886, Vol. 9, page 6) criticizes Grant's article.

Wilfred Lewis: Amer. Soc. Mech. Eng., 1886, Vol. 7, pages 273–310, and *American Machinist*, 1886, Vol. 9, page 4. Experiments by Wm. Sellers and Co. on power transmission by gearing to ascertain the speeds and pressures liable to cause "cutting." Also calculations for efficiency.

Brown and Sharpe Mfg. Co.: Practical Treatise of Gearing: First Edition, 1886.

W. Harkness: Proc. Amer. Assoc. Advance. of Science, 1886, page 183. Formulæ for strength, etc., of brass, wood, and iron teeth.

F. Reuleaux: Trans. Amer. Soc. Mech. Eng., 1887, Vol. 8, pages 45–85. Friction in toothed gearing and wear on teeth.

A. B. Couch and others: Trans. Amer. Soc. Mech. Eng., 1887, Vol. 8, pages 699–704. Discussion on data for working pressure on gear-teeth.

Michael Longridge: Annual Reports of Engine and Boiler Insurance Co., 1887–8–9. [Page 358.]

Archibald Sharp: "Wheel Teeth," Proc. Inst. C.E., Vol. 113, 1892–3, and Vol. 121, 1894–5. [Page 358.]

Wilfred Lewis: Proc. Engineers' Club of Philadelphia, Investigation of the Strength of Gear-Teeth, January, 1893, Vol. 10. Also *American Machinist*, 1893, Vol. 16, pages 3–4. [Page 371.]

Reuleaux, "The Constructor," 1893 Edition, page 146, quotes Tredgold as recommending 400 lb. per inch of width, but adds that pressures as high as 1,400 have been successfully used. He also shows that $\frac{P \times N}{b}$ ought not to exceed 28,000, and may be reduced to 12,000 or even 6,000. For wooden teeth he mentions 15/20,000, with an example given of 7,000 to 8,000 having run 26½ years (page 147). [Page 371.]

Emile Geyelin: Proc. Engineers' Club of Philadelphia, April 1894 Vol. 11, page 142. [Page 374.]

F. R. Jones: Trans. Amer. Soc. Mech. Eng., 1897, Vol. 18, pages 766–794. Diagrams for relative strength of gear-teeth.

J. B. Mayo: Trans. Amer. Soc. Mech. Eng., 1898, Vol. 19, pages 109–118. A strength of gear chart.

L. Lecornu: *Revue de Mécanique*, 1898, Vol. 2, pages 24–42.

Toothed gearing in England and America; line of action, form of teeth, efficiency, strength, etc.

G. B. Grant: A Treatise on Gear Wheels. Eighth Edition, 1899. Gives a bibliography and, after considering involute and cycloidal shapes as well as others, strongly advocates the universal adoption of the involute.

O. Lasche: Zeit. des Ver. deut. Ing., 1899, Vol. 43, page 1417. [Pages 361–9; and also Appendix II, page 390.]

Wilfred Lewis: "Interchangeable Gearing," Proc. Engineers' Club of Philadelphia, Vol. 18, February 1901, page 51; and in *American Machinist*, 1901, Vol. 24, pp. 218–9. [Page 358.]

Christie (Proc. Engineers' Club of Philadelphia, Feb. 1901, Vol. 18, page 44, and in *Iron Age*, Vol. 67, 28th February 1901, page 19) gives an example 2,100 lb. per inch of face at 2,500 feet per minute. $\frac{PN}{b} = 286{,}000$. Carbon 0·86 per cent. Highest recorded velocity 3,900 feet per minute for mortise-wheels—680 lb. per inch of face, life short. He suggests that the product of speed by pressure divided by circular pitch—that is, $\frac{\text{Velocity} \times \text{Load}}{\text{Pitch}}$, should not exceed 1,000,000. [Page 370.]

E. Graves: Proc. Engineers' Club of Philadelphia, 1901, Vol. 18, page 57. [Page 375.]

American Machinist, 1901, Vol. 24, page 689. Table of Strength of Gear-Teeth in terms of diametral pitch.

American Machinist: Vol. 24, page 1079, 12th October 1901. List of proportions and strength of spur-gear teeth adopted by Joseph Adamson and Co.

Robert A. Bruce, Lieut. R.N.V.R. (*American Machinist*, 1901, Vol. 24, pages 1140, 1269, and 1288) furnished an article advocating increased obliquity and shorter teeth. [Page 360.]

American Machinist: Vol. 25, 15th February 1902, page 145, "The Strength of Shrouded Teeth," by Wilfred Lewis. A single shroud may add 10 per cent. and double shroud 30 per cent. to the strength of teeth, but the space occupied would be better employed in increasing the width of the face of the teeth.

W. H. Thornbery (Staffordshire Iron and Steel Inst., 1902) gives several different values varying from about 400 lb. to 800 lb. per inch of width on cast-iron teeth of 4-inch circular pitch. These give the very low stresses of 1,000 lb. to 2,000 lb. per square inch of material. [Page 374.]

Büchner: Zeit. des Ver. deut. Ing., 1902, Vol. 46, pages 159 and 278. [Page 364.]

H. D. Williams (*American Machinist*, 1903, Vol. 26, page 257) wrote a Paper on Measurements of Contacts, comparing the resistance to crushing of curved surfaces of different radii in contact.

William Kent: Mechanical Engineers' Pocket Book (Seventh Edition, 1904, pages 900–5). Several formulæ for calculating the strength of gear-teeth are compared and examples quoted of gearing running at high speeds up to 3,000 feet per minute.

Cheddie (*American Machinist*, 1905, Vol. 28, page 220) gives his ideas as to relative loads on cast-iron, gun-metal, cast-steel, and forged steel, in the ratios of 1, 1½ and 2½ respectively, with modifications for speed and shock as in "lifting machinery."

G. J. Leire: *Machinery*, 1905, July, page 565. [Page 374.]

C. H. Logue: *American Machinist*, 1907, Vol. 30, Part I, page 804. [Page 355 and Fig. 3.]

Frank Burgess: *American Machinist*, 1907, Vol. 30, Part I, pages 935–6. A long tooth gives a better movement than a short one.

Engineering: 21st February 1908, Vol. 85, page 241. Recommendation of the Wuest system for all cases where high reduction, high powers, or high speeds are encountered.

Thomas Humpage: "Manufacture of Spur-Gearing," Inst. Mech. E., July 1908.

In Nicolson and Smith's "Lathe Design," published by Longmans, Green, London (1908), will be found, on pages 158–162, an extended reference to Stribeck, Bach, and Lasche's investigations.

Robert H. Smith: "Design and Waste and Wear of Wheel Teeth," Society of Engineers, May 1908. [Page 361.]

R. E. Flanders (Trans. Amer. Soc. Mech. Eng., 1908, Vol. 30, page 921, and *American Machinist*, 1909, Vol. 32, page 307) advocates shorter teeth and increased pressure angle.

N. Litchfield: Trans. Amer. Soc. Mech. Eng., 1908, page 967. [Pages 370 and 375.]

C. H. Logue: *American Machinist*, 1908, Vol. 31, Parts I and II, pages 95 and 115. [Page 370.] Considers the effect of wear as well as strength when calculating the size of gear-teeth.

L. P. M. Revillon: Iron and Steel Inst., Carnegie Memoirs, 1908–9, Vol. 1, page 160. [Page 370.]

Iron Age: 30th April 1908, Vol. 81, page 1382. Manganese steel gear-wheels and pinions that lasted five times as long as ordinary steel, on travelling cranes.

C. H. Logue (*American Machinist*, 1909, Vol. 32, Part 1, page 917, and Vol. 32, Part 2, page 571) suggests varying the working stresses for the teeth of wheels according to the method of construction and presumed accuracy of workmanship, but the figures given seem to be only personal opinions unsupported by evidence either of a practical or a theoretical nature.

R. Livingstone: *Electrician*, 19th March 1909, page 892. [Page 368.]

W. C. Unwin: "Elements of Machine Design," 1909, Part 1, page 41 [page 379] and page 399; when the whole load is assumed to come on one corner of the tooth, the safe load P is shown to increase as the square of the pitch.

R. Plessing (Zeit. des Ver. deut. Ing., 1910, Part 2, Vol. 54, page 1682), advocates increasing the pressure angle, and points out how this can be done with existing generating machines, and shows, by calculation, the required difference in diameter of base circle and side accordingly.

J. D. Steven: "Tooth Gearing," Inst. Mech. E., July 1910.

H. I. Brackenbury: Trans. Amer. Soc. Mech. Eng., 1910, Vol. 32, pages 739 *et seq.* References to strength of gearing, etc., and also in discussion.

Wilfred Lewis: Trans. Amer. Soc. Mech. Eng., 1910, Vol. 32, page 803. Refers to investigation by a committee, and gives a summary of experiments.

Sir C. A. Parsons: Trans. Inst. Naval Arch., 1910, Vol. 52, pages 169–83. The use of helical spur-gearing in the application of the marine steam-turbine. Again in the same publication, 1911, Vol. 53, Part I, pages 29–36, and Part II, pages 79–95.

Otto Schaefer: *Dingler's Polytechnisches Journal*, 1910. [Page 376.]

Wilfred Lewis: Amer. Soc. Mech. Eng. and Inst. Mech. E. Joint Meeting, July 1910. Suggested interchangeable system of involute gearing with a pressure angle of $22\frac{1}{2}°$, and an addendum of 0·275 circular pitch.

C. H. Logue: *American Machinist*, 1910, Vol. 33, Part 2, pages 67, 139, and 527. Comments on gear efficiency. References to N.Y. Subway Gears. [Page 375.] A suggested modification in the Lewis formula, giving allowable increase in load for generated teeth.

Eden, Rose, and Cunningham: "The Endurance of Metals," Inst. Mech. E., October 1911.

P. C. Day: Trans. Amer. Soc. Mech. Eng., 1911, Vol. 33, pages 681–715. Herring-bone gears on the Wuest system. See also *Page's Weekly*, February 1912, Vol. 20, pages 237–9, and pages 341–4.

Rikli (Zeit. des Ver. deut. Ing., 1911, Vol. 55, page 1435): Tests of the efficiency of transmission by spur-gearing show that the loss is only about 2 per cent.

R. E. Flanders: Stevens Inst. Indicator, April 1911, Vol. 28, pages 141–59. Practical considerations in the design and construction of spur-gearing; involute versus cycloidal; cast versus cut teeth; standard versus special shapes; and some remarks on strength.

Wallwork: *Engineering*, 20th January 1911, Vol. 91, page 86. Testing appliances for reduction gears.

Page's Weekly: 29th December 1911, pages 1201 *et seq.* [Page 357.]

W. C. Betz: *American Machinist*, 1912, Vol. 37, page 322. Heat treatment of gears.

H. T. Millar: *Machinery*, 1912, Vol. 19, page 99. A logarithmic chart for finding the strength of gear-teeth.

G. H. Marx: Trans. Amer. Soc. Mech. Eng., 1912, Vol. 34, page 1376. [Pages 364 and 374.]

C. W. Drake (*Electric Journal*, June 1912, page 554) states that raw-hide may be run up to 2,000 or 3,000 feet per minute. [Page 374.]

G. T. White: Toothed Gearing. Published by Scott Greenwood, London, 1912.

American Society of Mechanical Engineers: "Standard Involute Gearing," 1913, September Journal, page 1405. Report of a Committee. A majority of this Committee recommended a standard system ranging from a 12-tooth pinion to a rack, the angle of obliquity to be $22\frac{1}{2}°$ with addendum $\frac{7}{8}$ module (0·28 pitch) and dedendum 1 module (0·318 pitch).

Burgess (*American Machinist*, 1913, Vol. 36, page 113) shows the increased number of blows under a hammer test that alloy steels will sustain as compared with 0·3 per cent. carbon steel and cold rolled steel.

Electric Railway Journal (8th November 1913, Vol. 42, page 1021). Large users of gear-wheels under severe conditions, such as tramway companies and departments, are now giving considerable attention to the wear of gear-teeth under various conditions, and are keeping systematic records of measurements taken by special gear-tooth micrometers at regular intervals.

Sir C. A. Parsons (Inst. Naval Arch., 1913) states that greater attention is being given every year to accuracy in dividing or pitching out of gear wheels when being cut.

Mechanical Engineer, 19th December 1913, page 577: Modern hardened and lubricated gears are said to be able to carry 3,000 lb. per inch of width.

In the *Electric Railway Journal* (20th December 1913, page 1299) are given some particulars of tests on tool-steel wheels and pinions on the Memphis Street Railway. One set had already run 183,759 miles, and showed the wheel-teeth only polished and worn one-third perhaps as much as could ultimately be obtained as against a total life for soft or untreated wheels of 156,000 and pinions of 32,675 miles. (See *Science Abstracts*, B. 1914, Vol. 17, page 216.)

Vincent Gartside: Trans. Manchester Assoc. of Engineers, 1912–13, page 133. [Page 357.]

American Machinist, 1913, Vol. 38, pages 539–40. An editorial reference to discussion of Marx's Paper.

W. R. Stults: *American Machinist*, 1913, Vol. 38, pages 999–1000. Suggestive criticisms on Marx's Paper.

Electrical Review: July 1913, Vol. 73, page 84. Treatment of tram-car gear-wheels and pinions.

J. H. Parker: Off. Rep. Nat. Mach. Tool-Builders' Assoc., October 1913, pages 176–190. Heat treating and case hardening of gears for machine tools.

A. C. Gleason: *Idem.* Also in *Iron Age*, November 1913, Vol. 92, pages 1020–3. Also in *Industrial Engineering*, February 1914, pages 71–75.

G. L. Colburn: *American Machinist*, 27th November 1913, Vol. 39, pages 895–6. Testing strength of teeth by drop hammer.

A. C. Gleason: *American Machinist*, 1913, Vol. 39, page 1039. Six tables of strength of gear-teeth, hardened and soft. Also in *Machinery*, January 1914, Vol. 20, page 388. And in *American Machinist*, May 1914, Vol. 40, page 830, a description of a spur-gear testing machine at the Gleason works.

F. W. James: *Mechanical Engineer*, October 1913, Vol. 32, pages 409–10. A chart for determining horse-power of spur-gears.

Charles Fair in reply to the Discussion (Proc. Amer. Inst. Elect. Eng., 1914, February, page 320) gives 4,200 feet per minute for cloth gears and 5,000 feet per minute for "herring-bone" teeth.

John Parker (Trans. Amer. Soc. Mech. Eng., 1913, page 785) wrote on Gearing for Machine Tools, dealing with different materials and best methods of treatment for different conditions of service.

T. V. Converse (*American Machinist*, 1914, Part 1, Vol. 40, page 182) advocates increasing the angle of obliquity.

Wilfred Lewis: Trans. Amer. Soc. Mech. Eng., 1914, Vol. 36, pages 231–7. Gear testing machine. Also see *American Machinist*, July 1914, Vol. 41, pages 41–2.

S. Trumpy: *American Machinist*, 1914, Vol. 40, page 956. Letter as to the safe working stress for heat-treated gears.

Practical Engineer: July 1914, Vol. 50, pages 76 *et seq.* General discussion of Lewis formula and Marx's experiments.

W. L. Allen: "Recent Developments in Railway Motor Gearing," *Electric Journal*, October 1914. Follows through the various

materials used for railway gearing up to case-hardened steel, the cost of which is rather excessive, owing to the care necessary in the heat treatment. Also describes a new process of heat treatment by which a surface corresponding to that of case-hardened steel could be obtained at a much lower cost.

J. Parkinson and Son: *American Machinist*, October 1914, Vol. 41, page 743. Gear testing machine described and illustrated.

E. S. Sawtelle: *Electric Railway Journal*, November 1914, Vol. 44, pages 1157–8. Tool-steel pinions and mild-steel gear-wheels.

Practical Engineer: March 1915, Vol. 51, pages 120–3. Strength of teeth in relation to shape.

H. F. Moore and F. B. Seeley: "The Failure of Materials under Repeated Stress," American Society for Testing Materials, June 1915. [Page 380.]

A. A. Ross: "Operating Conditions of Railway Motor Gears and Pinions," *General Electrical Review*, pages 249–258, April 1915.

Marx and Cutter: Amer. Soc. Mech. Eng., Sept. 1915, "The Strength of Gear-Teeth." [Fig. 15, page 373.]

E. A. Suverkrop: *American Machinist*, 1915, Vol. 42, pages 725–730. Heat-treating equipment and methods.

Iron Age: 16th September 1915, Vol. 96, page 629. Heat treatment of gears.

APPENDIX II.

Amplified Abstract from Article by Lasche in Zeit. des Ver. deut. Ing., 1899, Vol. 43, pages 1528 *et seq.*

In this article it is shown that the excess loads due to irregularities in pitch vary as the square of the pitch-line velocity, and may be of high values in comparison with the transmitted load.

Referring to Fig. 17, the error in the pitch of one tooth of the driven wheel much exaggerated is represented by the distance Δp. In consequence of this error the teeth a and b begin contact at a point X, instead of at the point Y on the theoretical line of action.

Assuming that the teeth *a* and *b* remain in contact during the arc of action, and that the angular velocity *v* of the driving wheel remains constant, it is obvious that the angular velocity of the driven wheel must vary, attaining a maximum when *a* and *b* come into contact,

FIG. 17.

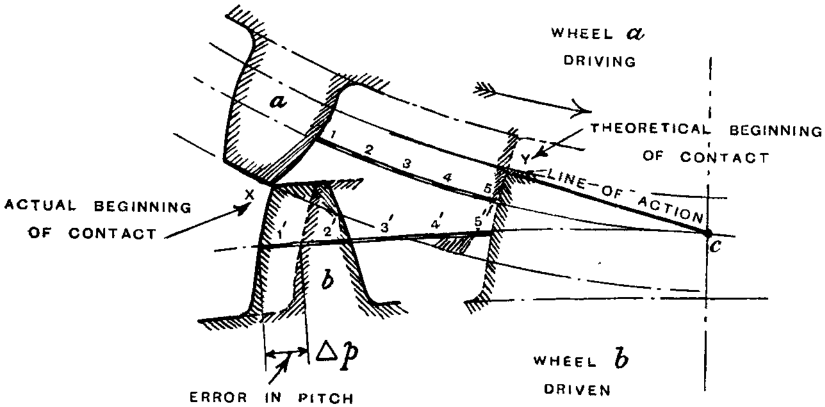
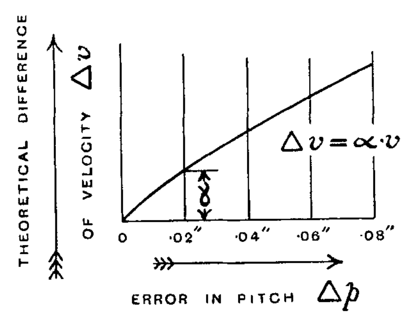


FIG. 18.



and gradually diminishing to normal as *b* approaches the line of centres. The shaded areas 1′, 2′, 3′, etc., on the pitch circles of the driven wheel show the movement of the latter corresponding to the uniform movements 1, 2, 3, etc., of the driving wheel. The difference between the lengths 1′ and 1 will be proportional to Δv, the difference in pitch-line velocity of the wheels when contact first takes place. This difference Δv can be ascertained for different errors in pitch and plotted against the latter for any required pair of wheels, as shown in Fig. 18.

From such a curve it is possible to read the difference in velocity Δv corresponding to any given error in the pitch, and, knowing the magnitude of Δv, the acceleration pressure P can be calculated, providing certain assumptions are made.

The following are the assumptions made :—

(1) That at the beginning of contact between the teeth *a* and *b* (Fig. 17) the driven wheel is only accelerated by one-half the difference of the pitch-line velocity Δv of the two wheels, the driving wheel being retarded by a similar amount.

(2) That the equalization of velocity does not take place instantaneously but extends over a period of time corresponding to the time taken for the tooth *b* to move from the actual beginning of contact at X to the theoretical beginning of contact at Y.

Now let t = time taken to equalize the velocity—that is, time taken for tooth *b* to move from X to Y.

And let Δv = difference of pitch-line velocity at the point of contact X.

Let I = moment of inertia of largest wheel.
,, M = mass of largest wheel.
,, k = radius of gyration of largest wheel.
,, R = radius of pitch circle of largest wheel.
,, $\dot{\omega}$ = angular acceleration of wheel.

Then couple necessary to produce the acceleration $\dot{\omega}$

$$= \mathrm{I}\dot{\omega} = \frac{\mathrm{I}.\Delta v}{2t\mathrm{R}} = \frac{\mathrm{M}k^2.\Delta v}{2t\mathrm{R}}.$$

But couple = acceleration pressure P × pitch circle radius of wheel R = P × R.

$$\therefore \quad \mathrm{PR} = \frac{\mathrm{M}k^2.\Delta v}{2t\mathrm{R}}.$$

$$\therefore \quad \mathrm{P} = \frac{\mathrm{M}k^2}{\mathrm{R}^2} \times \frac{\Delta v}{2t} \quad . \quad . \quad . \quad . \quad (1)$$

The ratio of the length 1′ in the figure to the length 1 is obviously independent of the velocity of the wheel, and simply depends upon the amount of error in the pitch and the numbers of teeth in the two gear-wheels considered. It follows, therefore, that instead of Δv we can substitute $\alpha \times v$ where α is a constant and v is the normal pitch-line velocity of the wheels. Further, for any two given wheels with a given error in pitch, the proportion which the distance XY bears to the circumference of the wheel will also be constant, and we may write $t = \frac{\beta}{v}$ where β is a constant and v is as above. Substituting for Δv and t in (1) we have

$$P = \frac{M.k^2}{R^2} \times \frac{\alpha}{2\beta} v^2 \quad . \quad . \quad . \quad (2)$$

which shows that, providing the assumptions made are reasonably correct, the acceleration pressure increases as the square of the velocity.

The following example, taken from page 1530 of Lasche's Paper, shows the calculation of the acceleration pressure by means of formula (1).

Two gear-wheels having 35 and 136 teeth respectively were used to transmit 75 h.p., the smaller wheel being the driver and running at 570 revolutions per minute. The pitch circle diameters were 16·5 inches (420 mm.) and 64·25 inches (1,632 mm.) respectively; the pitch was approximately $1\frac{1}{2}$ inch (37·7 mm). The measured error in pitch was found to be 0·02 inch (0·5 mm.). The corresponding value of Δv was 3·15 feet (960 mm.) per second and t was 0·00106 second. The normal pitch-line velocity v was 41 feet (12·5 metres) per second.

The mass of the driven wheel considered as a fly-wheel acting at the pitch circle $\frac{M.k^2}{R^2}$ was 612 lb.

Substituting the above values in formula (1), the following value is obtained for acceleration pressure P:—

$$P = 612 \times \frac{3 \cdot 15}{2 \times 0 \cdot 00106} \text{ poundals}$$

(the poundal being that unit of force that will produce an acceleration of 1 foot per second per second on a mass of 1 lb. and equals $\frac{1}{g}$ or $\frac{1}{32 \cdot 2}$ lb. weight approximately, from Newton's Second Law of Motion).

$$\therefore P = \frac{612 \times 3 \cdot 15}{32 \cdot 2 \times 2 \times 0 \cdot 00106} \text{ lb. weight}$$
$$= 28{,}200 \text{ lb.}$$

But peripheral load due to power transmitted

$$= \frac{75 \times 550}{41} = 1{,}006 \text{ lb.}$$

Showing that in this instance, and on the assumption made, the acceleration and retardation of the wheels caused by the inaccuracy in the spacing of the teeth produce a pressure approximately 28 times that required to transmit the power alone.

Retaining the same assumptions, the following values are yielded for an ordinary pitch-line velocity of, say, 9·83 feet (3 metres) per second. The corresponding value of Δv was 0·75 feet (230 mm.) per second, and t was 0·0044 second.

Substituting again in formula (1) as above, gives

$$P = 612 \times \frac{0 \cdot 75}{2 \times 0 \cdot 0044} \text{ poundals,}$$

$$\text{or } \frac{612 \times 0 \cdot 75}{32 \cdot 2 \times 2 \times 0 \cdot 0044} \text{ lb. weight}$$

$$= \frac{459}{0 \cdot 28336} \text{ lb.}$$

$$= 1{,}623 \text{ lb.}$$

Now the ratio of 28,200 to 1,623 is as 17·33 to 1, which is the same as the ratio between 41^2 and $9 \cdot 83^2$, showing again how the acceleration pressure (on the assumptions made) increases as the square of the velocity.

Discussion in London.

The PRESIDENT, in moving that a very hearty vote of thanks be accorded to the Author for his interesting and useful Paper, said that the teeth of wheels played a very large part indeed in the work of engineers, and any improvements which could be made in them were of considerable importance.

The Resolution of Thanks was carried by acclamation.

Mr. H. HUBERT THORNE, in opening the discussion, said that to all engineers the subject of gearing was one of great interest, but it was particularly so to himself, because he had spent a large amount of time in his engineering experiments in dealing with the problem. The Author referred at the commencement of his Paper to there being no need to give much consideration to the question of outline, on account of the very exhaustive work which had already been carried out by different engineers in the past. It seemed to him, however, that the consideration which the Author had given to the subject clearly proved that it was by no means as yet exhausted. The relative merits of the various standards which had been put forward by different firms were still eagerly debated by those firms, and there was still a great deal to be said about them. He was sorry the Author had confined himself to a consideration of straight-toothed gears, because, with the rapid advance which had been made during the last few years so far as the question of double helical gears was concerned, it seemed to him that the subject was hardly complete without some reference to that matter.

The Author referred to Bruce as advocating a greater pressure angle in order to increase the strength of teeth. One of the reasons why an advantage was to be gained by an increase in the strength of teeth was stated to be that the engineer was thus able to use a finer pitch and consequently more teeth, which meant that a greater number of teeth were in engagement at one and the same time. That was quite correct, but it seemed to him that those

(Mr. H. Hubert Thorne.)

who had adopted the double helical form of tooth rather than the straight tooth had sought to improve engagement conditions in a very much better way, in that in the double helical tooth, by increasing the face width, it was possible continually to bring in more teeth. With straight-cut teeth the engagement conditions of the teeth could not be improved by increasing the face width, no matter to what width the face was extended, whereas with double helical teeth, by extending the width of the tooth, the load could be distributed over an increasing number of teeth.

He was very pleased to see that the Author referred (page 361) to Lasche's recognition of the effect of the rate of curvature of the surfaces in engagement on the breadth of the line of contact. So far as he had been able to judge, that was a part of the subject which had not received the consideration of engineers generally which it deserved. He thought it must be apparent to all engineers that Lasche was quite correct in his contention that with small curves the lubricant which was used was readily displaced, and some consideration should be given to that fact in designing gears. Where very small pinions were used, and consequently very small curves, the face width of the gear should be increased in order to allow for that factor. His own firm took that into consideration in the design of gears, but he did not think that it was, generally speaking, sufficiently recognized.

Mr. G. GERALD STONEY said he quite agreed with the last speaker that it was a great pity the Author had not dealt with the modern developments of gearing, which, as exemplified in the application of geared turbines especially for marine work, had made enormous progress during the last few years; at the present time powers of over 10,000 h.p. per pinion, with pitch velocities of between 6,000 and 7,000 feet per minute, and pressures up to 700 or 800 lb. per inch, were in common use. It had only been possible to make that enormous development by the use of very accurately-cut helical gearing; in fact, the introduction of such gearing had completely revolutionized the use of gears. Such powers as those to which he had referred were unheard of formerly. For smaller powers, such as for gearing pumps or for crane work, helical gears, accurately cut, were being very largely used instead of raw-hide pinions, which had hitherto been employed to a considerable extent. He thought that helical gears would be exclusively used in the future for a great deal of work, instead of straight-cut gears.

Mr. DANIEL ADAMSON inquired what was the probable inaccuracy in the pinions to which Mr. Stoney had referred with pitch velocities of between 6,000 and 7,000 feet per minute.

Mr. STONEY replied that the probable inaccuracy was exceedingly small. He could not state the exact amount, but it was something of the order of $\frac{1}{1,000}$ inch. This was the maximum deviation at any point of the pinion from the truth, the inaccuracy between any two teeth being vastly smaller. The possibility of getting such gears to run silently at these high speeds depended wholly on exceedingly accurate cutting. Considerable difficulties had been experienced in getting these gears to run silently, but by the use of very highly accurately-cut gears, such as were now made by several firms in the country, the difficulties had been completely overcome.

Mr. ADAMSON inquired what pitch was used.

Mr. STONEY said the usual pitch was about 0·815 inch.

Mr. WALTER J. IDEN noticed that the Author had not referred to the question of the grinding of gears to their true form, after they were hardened. That process was now being used in motor construction.

Mr. ARCHIBALD SHARP said the Author had referred to a Paper he (Mr. Sharp) communicated to the Institution of Civil Engineers some time ago.* His knowledge of the subject was practically

* Proceedings, Inst. C.E., Vols. 113 and 121.

(Mr. Archibald Sharp.)

confined to the mathematical side. The Paper was entitled "Circular Wheel-Teeth," and it was an attempt to deal with the knowledge of the subject he had derived from the text-books of their President, Dr. Unwin, and to tabulate a series of circular arcs that would be of practical use to the draughtsman in laying out such gears as the Author had dealt with in his Paper.

On again looking at his own Paper, he might generalize his notions on the subject as to the accuracy of the form of the tooth, by saying that the tooth form was of very little importance compared with accuracy in carrying out the work. In other words, to make a mathematically accurate pair of wheels a tooth outline of correct theoretical form was required, as satisfied by the well-known involute and cycloidal forms. It was necessary that each tooth should be an exact replica of every other tooth, and for all teeth to be at exactly the same pitch. He had investigated the errors due to substituting circular arcs for the true curves, and he desired to quote an example taken more or less at random from the Tables that formed the Appendix of his Paper. Taking a pair of wheels geared one to three, with 16 teeth in the smaller wheel, the maximum error due to the circular arc outline, instead of the theoretically accurate one, was 0·38 of 1 per cent. In another part of the Paper he found that an error of 1 per cent. in the velocity ratio corresponded to an inaccuracy at one part or other of the tooth outline of less than $\frac{1}{1,000}$ inch. He ventured to tell Mr. Stoney that the accuracy of the teeth of the wheels to which he had referred must be much finer and closer than the limit he stated; personally, he thought it would be measured in $\frac{1}{10,000}$ parts of an inch, or a fraction of $\frac{1}{10,000}$ inch.

The Author had referred to the work of Herr Lasche on the errors due to displacement of tooth outline. It was a well-known fact that the greater the number of teeth in a pair of wheels, the more smoothly they would work. One result of his (Mr. Sharp's) Paper was that, in the teeth forms he had discussed, the maximum error in velocity varied inversely as the cube of the number of the teeth. For example, in the pair of wheels to which he had just referred, with

16 teeth in the pinion, if the number of the teeth were increased to 32, the error was reduced to $\frac{4}{100}$ part of 1 per cent. There he thought a mathematical accuracy was obtained of greater accuracy than the errors due to the best results with even mechanical grinding.

With regard to the question of the general lay-out of the teeth outlines of a pair of wheels—say for mill gearing—that had to be designed *de novo*, he did not think engineers had paid sufficient attention to the dictum laid down by Mr. Michael Longridge, namely, that nearly all teeth were made too long in the addenda. If engineers would be content with shorter teeth, each tooth, acting as a cantilever, was in a much better form to resist the bending stresses at its root. A smaller tooth of smaller height meant less arm for the bending moment; the bending moment being reduced, the thickness and the pitch could be reduced and the number of teeth could be increased. As he had just pointed out, the magnitude of the errors of velocity ratio that were affected by the number of teeth varied inversely as the cube of that number, a point to which he thought greater attention should be paid. It would make a most valuable supplement to the Paper if Mr. Gerald Stoney or some other member could give particulars of the tooth outlines and other particulars relating to geared turbines of very high power.

The conclusions derived in the speaker's Paper "Circular Wheel-Teeth" as to errors in velocity ratio might be extended to all cases of error in tooth outline. Such errors in tooth outline might exist in all cases of cut gearing, except those generated correctly from a rack with straight-sided teeth. For example, in the standard sets of gear-cutters, one cutter served to cut wheels with from 17 to 20 teeth, another cutter served for wheels with 21 to 25 teeth, etc. The form of the cutter might be correct for one particular number of teeth in the wheel, but was then theoretically incorrect for any other number. In cutting double helical gears the same theoretical errors of tooth outline arose. In practice these errors were small, and it seemed not unreasonable to assume that they would all be of the same kind as those arising

2 D 2

(Mr. Archibald Sharp.)

from circular arc outlines. If the driving-wheel of the pair were supposed to revolve with absolutely constant speed, the periodically varying angular speed ω of the driven wheel might be expressed by the formula

$$\omega = \omega_0 \left\{1 + \frac{k}{N^3} \sin N\theta\right\} \quad . \quad . \quad . \quad (1)$$

where ω_0 is the average angular speed, from which ω should only differ by a negligibly small amount, N is the number of teeth in the smaller wheel of the pair, θ is the angle described by a point on the smaller wheel, and k is a constant depending on the speed-ratio and the angle of obliquity. The formula is derived directly from formulæ (23) and (24) of his Paper "Circular Wheel-Teeth." Evidently from the form of (1), when $\theta = \frac{2\pi}{N}$, or any multiple thereof, $\omega = \omega_0$. The periodic maximum excess or deficiency of ω above or below ω_0 is $\frac{k}{N^3}$, and, as already remarked, was very small.

Differentiating (1), we get the periodic acceleration ϕ of the driven wheel:—

$$\phi = \frac{d\omega}{dt} = \omega_0 \left\{\frac{k}{N^2} \cos N\theta\right\} \frac{d\theta}{dt}$$

$$= \omega_0^2 \left\{\frac{k}{N^2} \cos N\theta\right\} \quad . \quad . \quad . \quad . \quad (2)$$

The maximum value of ϕ is $\frac{\omega_0^2 k}{N^2}$.

That is, the periodic acceleration is proportional to the square of the speed, and inversely proportional to the square of the number of teeth, the former agreeing with the result of Herr Lasche, as quoted by the Author.

In helical gears where the width of face was such that there were always at least two pairs of teeth in contact, the arc of nominal contact of each pair of teeth should be set off at half the pitch, the angle of obliquity being in this case chosen greater than when only one pair of teeth was in contact.

Mr. WALTER PITT (Member of Council) said that with reference to the contour of teeth, he would be glad to know whether or not it was a geometrical fact that the condition that the same rack should generate all the wheels of a set, so that any two wheels of that set should gear together with uniform velocity ratio, did not necessitate that the rack contour should consist either of two identical cycloidal curves, generated by equal rolling circles for both the tops and the bottoms of the teeth; or else be straight-sided, as in the case of involute teeth.

Young engineers used to be taught that about 20 was the limiting number for the teeth of an involute pinion, and it was apparent from Fig. 4 (page 356) that when an endeavour was made to make a straight-sided rack gear with a pinion with too few teeth, fouling occurred. It had been stated that the curves used for the cutters made by one firm were a trade secret. Was that not merely a "fudging" of the curve to get rid of that fouling? The modified or "fudged" curve would no longer be quite a straight line and might cause a slight departure from the condition of uniform velocity. Was it not possible that this might be a contributing cause where inertia difficulties were experienced?

One speaker remarked that with helical gearing more teeth were in contact than with straight teeth. Was that statement quite correct? Would not a series of sections taken parallel to the face of the wheel show exactly the same successive displacements of contour in each case?

Mr. W. E. SYKES said there were some remarks in the Paper which called for a certain amount of criticism. Probably many of the statements made in the Paper were justified at one time, but conditions had altered so much recently that one's ideas had entirely changed on the subject. Dealing with the question of wear of teeth, the Author seemed to imply that short teeth were considerably better than long ones. That was generally considered to be correct at the present time, but personally he was rather in doubt whether shorter teeth were in most cases an advantage. With a pressure-angle of $22\frac{1}{2}°$ he had not found it necessary in practice to alter the length of teeth below the

(Mr. W. E. Sykes.)

standard proportions for double helical gears, which might be mentioned as shorter than the Brown and Sharpe standard. Mr. Archibald Sharp had mentioned that gears with large numbers of teeth ran more smoothly than those with small numbers. He did not believe Mr. Sharp intended that to apply to double helical gears, because pinions with from 7 to 15 teeth were frequently used, and it was found that they ran as smoothly as could possibly be desired.

The question of pressure-angle was a subject which justified considerable investigation and discussion. It was well known that for a considerable number of years $14\frac{1}{2}^\circ$ had been considered the correct angle. During the last few years the 20° pressure-angle had been advocated and adopted to a considerable extent. There were not many people, however, who had so far adopted as a general thing a larger pressure-angle. A pressure-angle of $22\frac{1}{2}^\circ$ had often been used, and, according to all published reports, with satisfaction. It might interest the Meeting to know that, for some little time past, turbine-gears of such powers and run at such speeds as Mr. Stoney had indicated had been made with a pressure-angle considerably more than $22\frac{1}{2}^\circ$; in fact, approaching $28\frac{1}{2}^\circ$.* Those gears had been transmitting power and working constantly at their designed load for something like twelve months with every success, so it seemed that, as far as pressure-angles entered the question, they might be increased with advantage.

He noticed that the Author quoted statements to the effect that revolving masses should not be connected to pinion-shafts. It might interest the Meeting to know that for some considerable time it had been the practice of one well-known firm, at least, to couple fly-wheels to the pinion-shafts of very heavy rolling mills to overcome the peak loads. They had been applied to mills up to 500 mean horse-power, and such gears had been running quite successfully since 1911. No doubt when that statement was made, the art of gear-cutting had not reached its present stage. It seemed to have been indicated at the Meeting that in the minds of many people the question of the curves of gear teeth was rather

* *See* Author's Reply (page 448).

misunderstood. It used to be commonly held that teeth should either be cycloidal or involute. He desired to refer those interested in the subject to Grant's extremely interesting book, which fully explained the matter. It was stated in that book that, for teeth to be correct, they merely required to be what Grant termed " odontoidal." That particular condition might be met either by cycloidal, involute, or many other forms of teeth. An involute tooth was adopted at the present time, because it had a curve that could be comparatively easily generated by mechanical means.

The Paper, and the discussion so far as it had gone, had not dealt with the question of the mechanical generation of teeth. The turbine-gears now made were for the most part generated by what was known as the hobbing process, with excellent results. The hobbing process might be considered theoretically accurate, and for all practical purposes it answered so far as immediate requirements were concerned. Recently, however, other processes had been invented which, instead of using the milling process, which it might be said a hob worked on, employed a shaping process. In the first place, by using the shaping process, teeth on cutters could be accurately produced—that is, after the cutters were hardened, the teeth could be ground mathematically accurate. Some five years ago he took up the question of generation by means of shaping tools, and since that time he had developed quite a new process, which would shortly be described in the technical Press. For the benefit of the Meeting he might say that the cutters used were in the form of a single helical pinion, and they were ground, after hardening, by a particular mechanical method which undoubtedly produced the involute curve without the slightest modification. The helical teeth produced had pointed apexes, and were theoretically correct in every respect. No doubt those interested in the question would pursue it when the details were published.

One speaker had raised the question of departing from the involute. It appeared to be understood by many people that straight-sided racks could not be used, but must be departed from. That was incorrect, as there was, as far as he could discover after exhaustive investigation, no reason why in generating processes

(Mr. W. E. Sykes.)

the straight-sided rack should not be used. The same speaker raised the question of the number of teeth in engagement with increase of face width in helical gears. There was no doubt whatever that, in the case of helical gears, a very large number of teeth in engagement could be obtained by merely increasing the width of the teeth.

Mr. M. HOLROYD SMITH said that "old times were changed, old manners gone." He felt like an ancient landmark in rising to speak on the question of gearing, because the modern, wonderfully thought-out and constructed instruments for making gears had robbed the old millwright of the skill and handicraft he had to use in days gone by to produce the wheels that the former "Daniels" used. He was glad to see that another Daniel was advocating gears that would not have been permitted by Mr. Daniel Adamson, Sen.

There were two points in the Paper that had given him a little personal satisfaction, the first being the sentence which said: "It is very evident that durability depends upon accuracy of construction as much as upon correctness of design." In his earlier days, accuracy of construction was an exceedingly difficult thing to obtain, and in order to get wheels fairly well made he adopted the somewhat heterodox plan of eliminating as far as possible all those wonderful decimal points that figured so much in modern books. He liked to reduce things to some definite measure found on the ordinary mechanic's foot-rule. He remembered having to make some wheels which it was desired to run smoothly and noiselessly at a high speed, and, though admitting the cycloidal tooth to be more theoretically correct, he decided to use the involute form as far and away preferable. He believed he was one of the earliest advocates of involute gearing in real practical work. When he came to describe an involute tooth according to the mathematical theories and formulæ contained in the books that then existed, he saw it was practically hopeless to get any pattern-maker that he was then acquainted with to follow the somewhat complex curve accurately. He found, however, that after an involute curve had been set out, it was possible to find some point near the pitch circle from which a single curve could be struck with a pair of compasses that approximated so nearly to the theoretical curve as to be acceptable. As a result, he was able to make perfectly successful running wheels. He believed if he could find the old drawing of the wheels he made more than forty years ago, it would be seen that the shape and proportions of the teeth would be exactly like those in the drawing, Fig. 8 (page 359).

At one time he was acting professionally for a firm that was carrying out some electrical pumping work, and that firm would insist upon putting in its own wheels, with long fingered cycloidal teeth, though he tried to persuade them to put in wheels of the single-curve type as he drew them. As a result, the machinery had not been running for more than a fortnight before an injunction was obtained by people in the neighbourhood, owing to the nuisance of the noise that was made, and the pumping had to be stopped. He was then asked if he would supply his own design. He consented to do so on the condition that he was allowed to go into the pattern shop and set out the wheels himself. That was agreed to, but he experienced the greatest difficulty in getting the pattern-maker to follow instructions. The wheels were, however, made according to his plan. They did not have a correct involute curve, but it was so like it that it would be difficult to tell the difference. The curve was set out with a pair of compasses. The wheels were subsequently put to work, and never a single murmur was afterwards heard from the neighbours in regard to noise made by the machinery. His method could not well be described without a drawing, and it was not now necessary, because since then Brown and Sharpe's book had been published, in which were given carefully worked-out rules that arrived at a like result, set out in a better manner than he could do. He would, however, express the opinion that no gear-wheel should have less than 15 teeth, especially when designing a set to intergear.

Reference had been made in the Paper to raw-hide pinions and mortise wheels, but hardly sufficient had been said in favour of the desirability of elasticity in the teeth of one of a pair of wheels, and

(Mr. M. Holroyd Smith.)

no mention had been made of invert wheels, probably because of a prejudice that seemed to exist in the minds of many engineers against them, possibly due to the difficulty in cutting the teeth. Referring to Fig. 12 (page 366) when the reduction of speed was as much as shown in the third, fourth, and fifth examples, instead of the train of spur-wheels, it would, in his opinion and experience, be much better to employ a worm and worm-wheel. Surely the delusions upheld in the technical books previously referred to ought by now to have disappeared from the practical mind, and it ought to be realized that when the ratio was at or about eleven to one, the loss in transmission would be less with a properly constructed worm and wheel than with a train of spur-gears. He had proved this as far back as 1885, but it took the engineering world a long time to realize it, and some were still unconverted.

Nowadays so much could be obtained from specialized shops that there was little need for the personal resourcefulness that was previously necessary; but occasions sometimes arose, and with the view of possibly helping some one in out-of-the-way parts he would mention that, once being unable to get the pair of wheels he wanted, he made a pattern and had a gun-metal pinion cast and constructed a lantern wheel, the "teeth" being round pegs of oak crossing from flange to flange. They were only half-inch diameter, but it was remarkable the load they withstood, and how long they lasted and how quietly they ran. They acted as an elastic medium. The advantage of these round pegs over the ordinary mortise tooth was the facility of construction and renewal, for there was no need to remove the wheel; the old teeth could be punched out and a new peg driven in whilst the wheel was in place. The oak became saturated with the wheel grease, tightened in the flanges, and never worked loose. The idea might sound rudimentary, but for installations in the Colonies far away from home supply, it was worth remembering.

Mr. WALTER PITT (Member of Council) said he desired to state that he was quite aware that a rack might be made of any shape, and that a true mating curve could be found to give the wheel a uniform velocity. If a wheel A were cut from such a rack and also another wheel B, both would gear correctly with the rack, but his query was, whether the condition that A should gear with B, combined with the condition that the normals to the points of contact should pass through the "pitch-point," could be fulfilled by other curves than the cycloid and involute.

Mr. DANIEL ADAMSON, in reply, after thanking the members for the kind manner in which they had received his Paper, and especially those who had taken part in the discussion, said that owing to the shortness of time he would reply only to the salient points that had been raised, and deal with any other points afterwards by correspondence. Mr. Hubert Thorne considered that the Paper gave so much attention to the question of outline that this part of the subject could hardly be dismissed as closed, but the general tenour of the discussion showed that the involute form was adopted almost universally, and for the reason (as given in the Paper) that this shape lent itself readily to accurate generation from a cutter with straight-sided rack teeth. Mr. Thorne had referred to the question of helical gears. He (the Author) intentionally did not refer to that subject in the Paper, because it was not his object to provoke a discussion on the pros and cons of helical *v.* straight teeth, but to try and show the correct principles, and once these were thoroughly understood they could be applied equally to inclined teeth as to straight teeth.

Mr. Gerald Stoney emphasized the essential need of accuracy for gearing to run satisfactorily at high speeds, thus confirming what the Author had endeavoured to express in actual figures. Mr. Stoney had referred to velocities of 6,000 feet per minute, which were very much higher than those given in the diagrams in the Paper, but, as Mr. Sharp had pointed out, Mr. Stoney must have been in error in the figure he gave for the probable amount of inaccuracy.

Mr. STONEY said he desired to add that the figure of $\frac{1}{1,000}$ inch, to which he had referred, would be between any two parts of the wheel and not between adjoining teeth. In a large wheel up to

(Mr. Stoney.)

14 feet diameter there would be no part more than $\frac{1}{1,000}$ inch from the truth.

Mr. DANIEL ADAMSON said that that was very different, but the calculations quoted from Lasche referred to the error between adjacent teeth. He had found that the *average* error between adjacent teeth on a cast wheel of $1\frac{3}{4}$-inch pitch was only in the neighbourhood of $\frac{7}{1,000}$ inch or 0·4 per cent. of the pitch, whereas the *maximum* error on the same wheel would be $\frac{80}{1,000}$ inch or 5 per cent. of the pitch. On an ordinary machine-cut wheel he had found the average error to be 0·2 per cent. and the maximum error about 1 per cent. of the pitch. The corresponding figures for such "generated" wheel-teeth as he had lately measured were 0·01 per cent. and 0·08 per cent. respectively. Mr. Walter Iden had referred to the question of the grinding of gears, which would imply the use of the involute shape advocated in the Paper, so that the sides of the teeth could be ground with a straight-sided wheel corresponding with the rack cutter mentioned on page 356.

Mr. Sharp had referred to possible errors in the angular velocity ratio that might follow from the adoption of his circular arcs, and had correctly expressed the fact that the error due to the adoption of the circular arc would be very much less than some of the errors quoted in the Paper. As the Author had already said, the advantage of the involute shape was that it allowed the correct shape to be generated geometrically without the interference of the human element. Mr. Sharp had also mentioned heavy gearing. Within the last few days an engineering friend of his had shown him some teeth he had developed for his own practice of $7\frac{1}{8}$-inch pitch, 20° involute, with an addendum of $1\frac{7}{8}$ inch, and he (Mr. Adamson) was very pleased to be able to show that the same proportions were the basis of the Table on page 364 of the Paper, where he gave the variation in duration of contact for such teeth.

Mr. Pitt had asked a question with reference to cycloidal and involute teeth. As he understood it, those were the two theoretically correct shapes, and the only usual departures from them were in the direction of circular arcs which had been so carefully explained by Mr. Sharp. He believed that Willis was the first to advocate circular arcs to approximate to the correct geometrical shapes, and his system was improved upon afterwards by Grant (*see* page 357). Mr. Pitt asked whether other shapes than the involute or the cycloidal could be satisfactorily generated from a rack, and the Author thought the answer to this would be found in the Appendix (page 384) in the reference to Grant's book. Mr. Pitt asked what was the minimum number of teeth in pinions of involute shape. The limiting number of teeth of a pinion of pure involute shape depended upon the angle of obliquity and the length of the addendum (page 360). As was suggested at the top of page 360, the minimum limits were in the neighbourhood of 12, 13, or 14. The inertia difficulties referred to by Lasche were due to errors in the pitch rather than errors in the theoretical shapes, because it was much more easy to measure the error in the pitch than the error in the theoretical shape. As dividing mechanisms became more accurate, then attention was given to generating the correct shape (page 355). Mr. Pitt had also referred to the fact that increase in the width of helical pinions could not increase the number of teeth in contact. That seemed to be correct, because, although as the width of the pinion was increased, more teeth would engage, yet this was not an "increase" in the number of teeth in contact in the sense that was conveyed by the same expression used in connexion with straight teeth.

Mr. Sykes referred to the question of short teeth *v.* long teeth, expressing the opinion that long teeth were preferable. He gave his reason indirectly for that opinion later on, when he advocated an angle of obliquity very much in excess of anything suggested in the Paper. Of course, if an angle of obliquity of $28\frac{1}{2}°$ were used, the length of the teeth must be increased, otherwise too short a duration of contact was obtained, as might be seen from an inspection of Fig. 4, where, if the angle of obliquity were increased to $28\frac{1}{2}°$, all interference would be done away with, even with less than a 12-tooth pinion, and the duration of contact would still be greater than unity. He thought the limitation to 20° was made with the desire to obtain a universal system. If

(Mr. Daniel Adamson.)

only one pair of wheels was going to be made, as in turbine reduction, or the number of sets of wheels to be made was limited to a small range, the angle of obliquity or any other particular detail could be controlled to suit the engineer's convenience. But if, as was usually the case, the engineer desired to develop a universal system of gear-teeth running from a 12- or 13-tooth pinion up to a rack, then he thought one was bound to limit the angle of obliquity. Mr. Sykes also said that pinions with small numbers of teeth ran satisfactorily, and misquoted the remark Mr. Sharp made. Mr. Sharp quite distinctly said that the possible error *in his system* of setting out teeth would vary inversely as the cube of the number of teeth, so that it was an advantage to have a greater number of teeth in the pinion. Mr. Sykes' practice was no doubt to use very high angular velocities, or, in other words, a high number of revolutions per minute. If there were a large number of teeth in a pinion making a large number of revolutions per minute, a very high pitch-line velocity would be obtained. He was very much obliged to Mr. Sykes for the reference he had made to the application of fly-wheels near to pinions. That was certainly an improvement upon the examples quoted by Lasche about eighteen years ago.

Mr. Sykes had quoted Grant as stating that certain odontoidal shapes would be quite as satisfactory as cycloidal or involute. He thought all that Grant would say, just as years before Professor Willis had said, and as Mr. Sharp and Mr. Holroyd Smith had said, was that it was possible to approximate the theoretically correct shape by circular arcs and get quite satisfactory results. Mr. Sykes went on to agree with him as to the advantage of the involute shape, in that it could be so readily generated. The hobbing process had been referred to by Mr. Sykes, and it was without doubt an improvement on the previous methods; but latterly "shaping processes" or systems with similar names were being used, in which the cutters could be ground to correct shape after hardening. That implied a straight-sided cutting edge, so that it could be geometrically reproduced with accuracy and ease. It would be very interesting to all the members to see the published particulars of the new method evolved by Mr. Sykes, but methods were already in use in which a shaping process was used, such as Mr. Sykes had described, employing a straight-sided rack cutter.

He was much obliged to Mr. Holroyd Smith for his reference to past history, but he was not quite correct in saying that he was one of the earliest advocates of involute teeth, because it would be noticed in Appendix I (page 381) that Euler was supposed to have been one of its first advocates, in a book published in St. Petersburg, about 1760. Mr. Holroyd Smith, however, qualified his statement by saying that it might have been in the books, but it was not in the pattern shops.

The Author was very sorry that no members had accepted the invitation given on page 381 of the Paper to give particulars of actual examples of successful or unsuccessful practice, particularly the latter.

Discussion in Manchester.

The CHAIRMAN (Principal J. C. M. GARNETT) said that Mr. Adamson had, in addition to his own contribution, reviewed the work already done on spur-gearing, and had presented much of it in a more convenient form than any hitherto available. In particular, he had made extensive extracts from Herr Lasche's work, which contained much valuable information. Appendix II showed why the permissible load on the tooth decreased as the speed increased.

From Appendix I it would be seen that American engineers had given a great deal of attention to the subject. Nevertheless, British engineers, especially in the early stages, had contributed most valuable information concerning the teeth of wheels. Camus and Sang were practically unknown to the present-day engineer, with the exception of a few who have made a special study of gearing; yet "The Teeth of Wheels," published by Sang in 1852,

(Principal J. C. M. Garnett.)

was to-day the only scientific investigation of the qualities which such gears ought to possess. Sang was not only a skilled mechanician, but a first-rate mathematician. His investigations were, however, too exhaustive and his writing too pedantic to attract the attention which his work deserved. Among other matters, Sang investigated the effects due to differently shaped contact paths, and their effect on the maximum and minimum obliquity of driving thrust, the number of teeth simultaneously engaged, the smallest number of teeth in pinion for practical shape, the undercutting of flank, the work spent in friction, the abrasion of surfaces, and the liability to wear out of true shape.

The short tooth adopted by Herr Lasche was advocated by Mr. Michael Longridge as far back as 1887. In the annual report of the Engine and Boiler Insurance Company for the year 1891 Mr. Longridge explained the cause of several breakdowns, and went on to say:—

Unfortunately these cases are not singular; rather are they types of many others where 1,000 i.h.p. and upwards are transmitted through one pair of wheels. The damage in such cases is nearly always attributable to concentration of pressure upon the ends of the teeth, or to shocks produced by the teeth of the wheel and pinion being driven against each other as they come into gear. In the one case the teeth are too wide in proportion to the length of the crank-shaft; in the other they are too long in proportion to the pitch.

In his annual reports for 1887 and 1888 Mr. Longridge stated that:—

The only chance of transmitting great power with safety at the speeds required in modern mills lay in reducing the length of the teeth so much as to limit the arc of contact on each side of the pitch point to little more than half a pitch. And this he believes is the principle that will have to be adopted if such accidents are to be avoided. So far but little progress has been made in the direction indicated. The majority of wheel-makers still make teeth from two-thirds to three-quarters of the pitch in length. They object to short teeth on æsthetic no less than on utilitarian grounds; the shape, they say, is strange and ugly, and the surface insufficient to withstand the wear. Their sense of fitness needs development. Could they but see it, a high-speed driving wheel with elongated teeth is just as unbecoming as a racehorse with an elongated tail. There are some cases in which stumpiness is more expressive of a highly cultivated taste than even Hogarth's lines of beauty. This is one. And as to wear, mere outside beauty is proverbially frail. The sinuous curves wherewith the inexperienced are seduced do not retain their pleasant shapes. Long teeth must be chipped ere they be polished to the points; stumps, on the contrary, hold the grease and keep their shape because they do not grind against each other.

He (the Chairman) thought that Mr. Adamson, by calling attention to Lasche's results, had pointed out how much there was in the prophecy Mr. Longridge made twenty-five years ago.

Mr. ALFRED SAXON said that members in Manchester who had not had the pleasure of hearing the Paper read and discussed in London were somewhat at a disadvantage, because they might be covering the same ground. He would like to congratulate the Author, who was a colleague of his in connexion with other research work, because he knew by that experience the great amount of trouble he was prepared to take in order to deal thoroughly with any subject he took in hand.

The first point he wished to refer to was the title of the Paper. Mr. Adamson, it appeared to him, had chosen the title somewhat unconsciously. It might be because he (Mr. Saxon) had had a millwright's training—he was brought up in an engineering and millwright's works—and they spoke of spur-gearing in a millwrighting sense, as being an absolutely different thing from bevel-gearing. He inferred that the Author, when referring to maximum speeds (pages 374–6) and quoting examples of bevel-wheels, meant to cover all kinds of plain tooth-wheels. If that was intended, and he thought it was, then the title might be a little misleading.

The second point was, that the Author referred to the introduction of moulding machines and gear-cutting machines as they affected the use of the cycloidal or involute shapes of teeth. He had made some inquiries upon that point, because it must not be forgotten that there were a great number of gear-wheels made in this country of cast-iron and cast-steel from patterns and moulding machines, quite apart from those produced from gear-cutting machines. The general practice for cast-wheels was to use the cycloidal form of tooth. There was a firm in Manchester—

(Mr. Alfred Saxon.)

known probably all over the world—which made large quantities of wheels both by gear cutting and also in the cast form, and their practice in gear cutting was to use the involute form of tooth up to 3 inches pitch, and the cycloidal form for pitches above 3 inches. In heavy mill-gearing and main wheel-drives the cycloidal form of tooth appeared to give the best result, with the grease method of lubrication usually adopted.

The Author stated (page 359) that he adopted a modified form of Brown and Sharpe tooth, and in that connexion he (Mr. Saxon) was in a position to support what was said with regard to the length of cog used. He remembered very well the advice Mr. Longridge gave, in the warm terms to which the Chairman had called their attention. Both with regard to bevel-gears and spur-gears, his firm put in several examples according to Mr. Longridge's odontograph; but as these were not very successful, a modified length of cog, first of all of $\frac{9}{16}$ and later $\frac{5}{8}$ of the pitch was introduced, and the latter length of cog has done exceedingly well in practice, which was a compromise between the Brown and Sharpe tooth and Mr. Adamson's revised tooth. His firm had adopted $\frac{5}{8}$ of the pitch for cast-wheels as well as for machine-cut wheels. Of course, the length of cog in Mr. Adamson's case was only intended to be used in cut wheels, and the speaker was quite satisfied that for cut wheels the Author had adopted a good standard.

He wished to ask the Author a few questions. Reference was made (page 375) to the fact that E. Graves (in the Proceedings of the Engineers' Club of Philadelphia) gave particulars of some cast-steel bevel-wheels, but they were not told whether those were cut or not. Then the Author gave another example, later on, by Christie, and it was quite evident those wheels were machine cut, because the pinion was made from a forging, though it did not state so. Neither were they told whether the New York subway wheels to which reference was made were cast or cut wheels. A further example was given on page 376 (E. and G., Bolton, November 1910) of a pair of machine-cut bevel-wheels, but it was not stated whether those were cast-iron or cast-steel, or from forgings.

He had many records of gear-wheels, but would content himself with quoting examples connected with the driving of a cotton mill in the Oldham District, Lancashire. The main driving-wheel in the engine-house was fitted with a spur rim, and this rim and pinion were of cast-steel, plain teeth unmilled, 170 and 52 teeth respectively, $4\frac{3}{16}$ inches pitch by 19 inches broad, running a rim speed of 2,600 feet per minute, and transmitting, after deducting engine friction, a load of about 1,600 i.h.p.

The bevel-wheels at the bottom of the upright shaft were of cast-steel, plain teeth unmilled, 51 and 45 teeth, $4\frac{1}{2}$ inches pitch, 13 inches broad, running a rim speed of 2,380 feet per minute, and transmitting a load of about 1,200 i.h.p.

The bevel-wheels on the upright driving a cross-shaft connected to the line-shaft in the main ring spinning-room were of cast-steel, plain teeth unmilled, 51 and 35 teeth, 4 inches pitch, 12 inches broad, running a rim speed of 2,390 feet per minute, and transmitting about 1,000 i.h.p.

The bevel-wheels originally fixed on the cross-shaft driving the line-shaft in ring room were of cast-steel, plain teeth unmilled, 44 and 30 teeth, $3\frac{3}{4}$ inches pitch, 11 inches broad, running a rim speed of 2,820 feet per minute, and transmitting about 1,000 i.h.p. These gave way after three years' working, and were replaced by cast-steel wheels, unmilled, with double helical teeth, of similar pitch and breadth, in October 1911, which were still working and giving every satisfaction. A spare pair of plain teeth unmilled, cast-steel wheels had been prepared for this position, with 44 and 30 teeth, 4 inches pitch, 12 inches broad, which would run a rim speed of 2,990 feet per minute; but up to the present these had not been required. The teeth of these wheels, especially the helical wheels, had all been carefully pitched and trimmed.

Mr. Joseph Butterworth said the Paper was very well reasoned and well balanced, took note of all valuable information on the subject, and brought it up to date. In common with other Papers on gearing, it assumed an absolute accuracy which as yet was unattainable in engineering practice; for instance, it assumed that

(Mr. Joseph Butterworth.)
the wheels or their pitch circles should run in absolutely perfect circles, and that their centres should be fixed.

First of all, the shafts must be perfectly aligned, the connexions between those shafts carrying the two wheels absolutely rigid, and no slack in the bearings. Until those conditions were obtained, a great part of the Paper was theoretical; without them one got unequal pressures in the teeth, also wear and vibration. In the case of the latter faults of accuracy, they got on the angle of approach a pushing apart of the centres, and in the angle of recess they got pulling back or "plucking," due to friction between tooth surfaces, which altogether altered the conditions under which the wheels worked. Büchner said (page 364) that the involute tooth would tend to become cycloidal in practice. That was exactly what his remarks came to. They got the plucking action just below the pitch-line in the angle of recess, and it at once began to alter the shape of the tooth from the involute, which was one curve, to the cycloidal, which had two curves. Of course, that upset the whole thing as regards the accuracy of working.

There was another point mentioned in the Paper: that even the pressure of the teeth on each other caused a certain amount of deformation of the surface. It must be small; something like a five-hundredth or a thousandth part of an inch would be quite excessive deformation due to pressure. On the other hand, any inaccuracy owing to slackness in bearings, etc., would cause far more difference in the working of the wheels owing to the plucking action. When the teeth were in the angle of approach, they pushed the centres apart, but in line of recess they tended to draw the centres together, with the result that motion between the tooth surfaces was momentarily stopped, and then suddenly accelerated or "plucked," causing abrasion or wear at a particular part of the tooth, generally below pitch-line.

Another point occurred to him. The ordinary form of calculating the strength of the tooth was upon the assumption that it pressed on the corner. If that were the case, it altered the stress in the proportion of 1·4 to 1, making it that much weaker. Owing to working conditions, this was avoided in motor-cars. One never saw a sliding wheel with square corners in the gear-box of a motor-car. They were approximately rounded off so as to avoid the conditions he had named. Why should not the ends of the teeth in all cases be bevelled off so as to do away with that weak point, and then one would be free from the unsightliness of a wheel with the corners of the teeth here and there knocked off, although the rest of the teeth were perfect and the wheel worked all right? Tool-makers in general should take account of that.

The reason he mentioned these things, as regarded want of accuracy in the conditions, was that in machine-moulded wheels, of which he had had a very large experience during the last 12 or 15 years, they thought they were doing very well if they got within $\frac{1}{64}$ inch in pitch, especially in large wheels, yet it was found that rigidity of the wheels and their centres had far more influence in successful running than an accurate pitch, especially in large wheels of 20 to 40 feet diameter. In such sizes quite an appreciable spring could be obtained in the rim and in the arms, all tending to make the working conditions deviate from theory. Unless engineers went into the matter carefully and considered the working conditions as well as what he might call the theoretical conditions, they would never arrive at a real approximation to what was applicable in teeth that were working together.

In reply to the Author, Mr. Butterworth added that, in the case he had cited, the pitch was $3\frac{1}{2}$ inches, the velocity 2,000 feet, and 500 h.p.

Mr. William G. Gass said he could answer the point raised by Mr. Saxon in reference to the large wheels mentioned on page 376, as they were made by his firm. The pinion was of forged steel and the large wheel of cast-steel, both cut. The vibration arising from the $3\frac{1}{2}$-inch pitch was very great, and it was owing to this that the finer pitch-wheels were put in, with a view to smoother running, and this was accomplished to a large extent. With the $3\frac{1}{2}$-inch pitch the teeth in contact varied for an instant from one only in contact to two, and this was supposed to cause the vibration, but

(Mr. William G. Gass.)
with $2\frac{1}{2}$-inch pitch there were always two in contact. These ran satisfactorily for three or four years and were afterwards replaced by double helical gears of the Citroen make, of the finer pitch of $2\frac{1}{2}$ inches. A peculiar effect was noticed on the pinion of this pair, as the metal flaked off the teeth in patches after about six months' working, but this did not materially affect the running.

Another point was the question of coarseness or fineness of pitch. For the majority of gearings, the finer the pitch that could be used the better. Also the teeth should not exceed $\frac{5}{8}$ths of the pitch in length. A case in point was a large planing-machine in his own works. The reversing wheels were 6 inches diameter, 4 inches width of face. They were originally put in 1-inch pitch, and they reached a very high speed and gave a great deal of trouble from vibration and noise. He took them out and substituted wheels $\frac{1}{4}$-inch pitch made of steel, which had been running night and day for about six or seven years and showed no signs of wear. They were transmitting the power of a 15 h.p. motor, which was generally loaded up to the limit. That proved to him that the fine pitch was better than the coarse wherever it could be put in. But he did not think the fine pitch was good with cast-iron, as the metal did not seem to stand. It was necessary to have sufficient strength in the metal itself, and that was the reason why he did not think fine-pitch wheels were advisable in cast-iron, though they worked all right in bronze or steel.

For cast-gear it had been his practice to make the teeth with the points thinned off materially and the roots strengthened a corresponding amount, the length of the tooth being 0·6 of the pitch. This was neither epicycloid nor involute, but gave very good results when running. The bearing surface on each side of the pitch-line when they were started up was only about 0·1 of the pitch, but very quickly gave an excellent bearing surface over about two-thirds the length of tooth. In his opinion a great deal of trouble arose in cast teeth, of either the involute or the epicycloidal shape, from the thickness of the tooth at the point.

Mr. DEMPSTER SMITH said the Author admitted that our knowledge of this branch of mechanical engineering science was very imperfect. They were greatly indebted to him, not only for his own contribution, but for compiling an excellent summary of the work which had been done on spur-gearing. On account of the difficulty in developing the profiles of the teeth, and the loss of rolling motion due to inaccurate setting or wear of the bearings, the cycloidal form of tooth had been almost entirely displaced by the involute form. In the involute gears the angle of obliquity commonly used was $14\frac{1}{2}°$. Why that angle was adopted was difficult to understand, except that it was about the mean angle of pressure in ordinary cycloidal gears. With such an angle and addendum of about 0·32 pitch, there would be interference between a rack and all pinions having less than 30 teeth. Such interference might be overcome by "faking" the tooth, as was done at present, by increasing the angle of obliquity of action, or by decreasing the addendum of the tooth.

The Brown and Sharpe tooth, now almost exclusively used, was a "faked" tooth and not a true involute. It was the outcome of a great deal of experimenting, and as the Author pointed out, it was their exclusive property. Whilst credit was due to that firm for its enterprise, it was not to the interest of engineers generally that this information should remain entirely in the possession of one concern. To adopt a pure involute tooth, however, appeared to be a more rational procedure. With an angle of obliquity of $22\frac{1}{2}°$ and an addendum $\frac{1}{4}$ pitch, a pinion having 12 teeth would engage with a rack without interference. They had then something definite and known to every one. The increased angle of obliquity of action would increase the pressure on the bearings about 5 per cent. Such a tooth could be cut by a simple cutter, hob or generating machine. It was stronger than that at present in use; the obnoxious wear at the point was got rid of, and the net result of a tooth so formed was equal to a cycloidal under the best conditions.

In order to obtain quietness when running, one firm in the neighbourhood of Manchester had adopted a very long tooth, whilst a leading machine-tool maker in the Midlands preferred the Brown and Sharpe standard form, but had about 40 cutters to the

(Mr. Dempster Smith.)
set instead of the usual 8. Lasche appeared to have been equally successful with a short tooth. From these it would seem that success in this direction was due to the accurate formation of the teeth rather than to the proportion of the same.

With regard to the allowable load on the teeth, the values given on page 372, and commonly attributed to Lewis (it was stated on page 374 that credit for these was really due to E. R. Walker), followed the law of $f = \frac{85{,}000}{\sqrt{\text{speed}}}$ for cast-iron gears and speeds over 100 feet per minute. For low speed, Lewis's values gave too great a pitch and too small a pitch for high speeds. The values given by Lasche were little better.

If a common profile and proportion of tooth had been agreed upon, it was highly probable that the strength and durability of spur-gears would have received more attention than had been given to them.

Dr. F. H. BOWMAN said he saw no reference in the Paper to the ringing of the wheels, so that they practically ran right on the pitch-line and were only about half the length of the pitch.

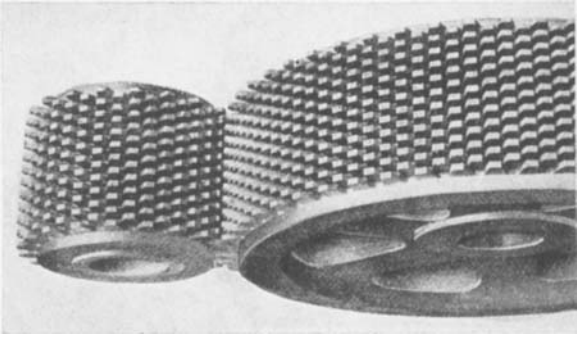
A MEMBER: Shrouding the teeth up to the pitch-line.

Dr. BOWMAN agreed. He had known wheels break, but when treated in that way, even with the same form of tooth, they had run for years without any difficulty. He believed it arose from the fact that the teeth were elastic to a certain extent, and as a consequence they were really moving when they were working. When strengthened by giving them support in that way, wheels, which, with the same form of teeth, were always breaking gave no trouble. In fact, he had increased the velocity from 50 to 75 revolutions per minute, and had a great deal less trouble than before.

Mr. J. DRUMMOND PATON said the Author had asked for information about recent developments, and in one section of the

Paper had made special reference to shock absorption. To absorb shock one must have a resilient body, and if they were dealing with case-hardened teeth or any severely hardened material, they had a very slight and low resilient function in the tooth. The result was that if the impact passed the resilient function, some portion of the tooth must go. The fault would arise possibly on one side of the tooth, as in Dr. Bowman's case, and eventually develop right across the face and snap off. By means of side flanges, whilst they increased the strength of the tooth, they diminished the resilient

FIG. 19.—*Laminated Gear.*



function, and on the top of this added the impossibility of machining the teeth.

He had pleasure in submitting to their notice one of the latest developments in gearing. The teeth were resilient but also of a naturally hard steel, namely, 0·60 per cent. carbon, and similar to that used in the best saw material. The wheels were built up on a centre by assembling several rings of steel, shown in Fig. 19. After assembly they were cramped up under heavy pressure, and some of the holding screws were drilled through. The assembled plates were then hobbed, afterwards reassembled with a thin disk of zinc or other shock-absorbing material between each lamina of steel. The grouping gave every alternate tooth the half pitch of the hobbing, and by this means back-lash was diminished; a helix was

(Mr. J. Drummond Paton.)
formed, and while the wheel could be considered a multi-helix, the driving face was at right angles to the line of transmission, avoiding displacement trouble, which arose with the helical teeth. The shock absorption was dependent on the back-lash dimension and the velocity of the wheels at the time of impact. When a reversal took place, the dimension of the impact depended on the amount of back-lash which had to be made up and the distance of the driven tooth from the driver. The whole was on a basis of MV^2, and where this impact exceeded the resilient function or the capacity of the teeth, rupture took place. In this case they had a high resilient capacity and practically no noise when running under even the most extreme conditions.

With regard to Mr. Gass's reference to a tooth which flaked away, he thought a photomicrographic examination of the metal on the face of the tooth would be interesting. His opinion was that this wheel had been cast with chills on the face, which produced a laminar structure of metal at the wearing point, and this tended to flake. In the manner of assembly in laminated gears, instead of wearing on the flake, they were wearing on the end of the fibre, and this was an apparent advantage.

Regarding the question of lubrication, the interstice between the laminæ admitted of the lubrication being carried through, and not as in the case of a full-faced tooth being carried out to the end and thrown off. The structure of the teeth also enabled a large amount of lubrication to be retained, and up to certain critical velocities, where centrifugal action became excessive, it was unnecessary to mount these laminated gears in oil-boxes. The limiting velocity had not yet been defined in practice, but they were already running at much greater velocities than the general matter which had been discussed in the Paper.

Mr. E. A. POCHIN said that probably all who had carefully studied the question of tooth-form had by now come to the conclusion that a standard form based upon a larger angle of pressure than $14\frac{1}{2}°$ and of reduced full depth would be advantageous. The fact that all gears down to a 12-tooth pinion could be generated by a straight-sided tooth-cutter of rack or worm (hob) form gave the $22\frac{1}{2}°$ stunted involute form an enormous advantage over other interesting but sometimes decidedly hybrid forms which had from time to time been suggested. It happened very frequently that pinions with as few as 12 teeth were necessary, and in order to obtain the requisite strength in the pinion-teeth, the present standard form had to be departed from, causing an expense which would otherwise not be incurred.

With reference to the tooth length proportions, he noted that the Author made his addendum and dedendum ratios of the circular pitch. He (Mr. Pochin) suggested that those proportions should be ratios of the module. The use of the module was the simplest method of obtaining the pitch diameter of any wheel of circular pitch and the larger diametral pitches. The Author's proportions agreed with his own ideas very closely, but an addendum of 0·8 and a dedendum of 0·95 module would be a real convenience where many gears were designed and made. The outside diameter would then be, module (No. of teeth + 1·6), and the full depth 1·75 module. Compared with the Author's proportions they would be as follows:—

—	1-inch Pitch.		1-inch Diametral Pitch.	
	Author.	As suggested.	Author.	As suggested.
	Inch.	Inch.	Inch.	Inch.
Addendum .	0·25	0·2546	0·7854	0·8
Dedendum .	0·32	0·3024	1·0053	0·95
Full depth .	0·57	0·557	1·7907	1·75

The dedendum was not quite as large as that of the Author, but it was large enough to allow reasonable clearance, and added slightly to the strength of the tooth.

The references to Lasche's Paper of 1899 were new to him, and very interesting—particularly the calculation of the stresses set up by inaccurate dividing. One was quite prepared to find that those

(Mr. E. A. Pochin.)
stresses were very large, especially at high speed, though the example worked out and showing stresses 28 times above the normal load, due to momentary acceleration, would be quite an abnormal case, an error of 0·02 inch in 1-inch pitch being, he hoped, most unusual. But the formula took no account of the excessive friction which would be present on the arc of approach—which might become a gouging out of the flanks of one set of teeth by the point of the other. He did not know how such a stress could be calculated, but it would not be negligible.

The five examples of transmission, however, illustrated in Fig. 12 (page 366) with their data, were not of great value except to warn one against certain local conditions, such as damp and inaccurate workmanship. The admission of errors of workmanship, amounting to 0·02 inch in a wheel or pinion of $1\frac{1}{2}$-inch pitch, running at 2,460 feet per minute, prepared one for the inevitable result. It would have been interesting had the Author been able to tell them the result of replacing the cast-wheel in the first example by a cut wheel of equal width of face. He altered two conditions at once, and one could not say to which alteration the improvement was due. Given gearing of reasonable accuracy, carefully and correctly assembled on rigid housings, most of the examples should have been successful according to his experience, which led him to remark how often gearing was not given a fair chance. Even when used by engineers themselves, it was surprising how often, in a motor-drive, the motor and driven shaft were not rigidly braced together. He had himself seen a pair of spur-gears where the pinion was keyed to the armature shaft as usual; the motor underhung from the middle of two joists about 8 feet long, and the wheel keyed to $2\frac{1}{4}$-inch shaft nearly half-way between the hangers, which would also be about 8-foot centres. That was an isolated and extreme case, but very frequently there was insufficient tying together of wheel and pinion shafts—of motor bed-plates and machine, etc. In the same way, wheels carelessly fitted to shafts and insufficient provision for elimination of end-play of shafts fitted with bevel-wheels caused endless trouble attributed unjustly to gearing.

Generally speaking, he found the loading of the wheel teeth, both of the five examples, and as given in the first schedule on page 368, to be on the light side compared with everyday practice and the Lewis formula. For instance, in the first example, which failed to transmit 50 h.p., the gearing by Lewis's formula should have been capable of transmitting 95 h.p. if a machine-cut wheel had been employed. The second, A, he should call a good and well-designed drive, since the torsion in the drifting-shaft would probably be considerably less than that of the three-throw crank-shaft, and its failure led one to think there were local conditions not specified, except under the admission of damp, that were responsible for the failure. The success of the substitution of one pinion only of 9 inches diameter, 15¾ inches face without an outer bearing, was as surprising as the failure of the first design. With reference to the third, fourth, and fifth examples, their value was discounted considerably by the admission that neither the accuracy in construction nor installation was all that might be desired, but by the Lewis formula they were capable of transmitting 200 per cent., 27 per cent., and 35 per cent. more than called upon to do. Similarly all the hide drives scheduled on page 368 allowed a considerable margin on the Lewis formula.

Turning to the question of material, he said he cordially agreed with the Author that, in metal-to-metal gearing, the pinion should always be of a harder material than the wheel. The ideal combination would be that in which each wore equally and both were worn out in the same time. It was, of course, most inadvisable to put a new pinion to work with a wheel of worn tooth form. That sounded very obvious, but it was far too often done. It did not apply to gearing where a raw-hide or paper pinion was used, as the wheel usually retained a good tooth form.

He had hoped to collect certain data of successful and some unsuccessful drives to present to the Meeting, but unfortunately he had not had time. He might say that as a general rule they did not find any difficulty at all in making satisfactory metal drives with steel pinion and cast-iron wheel running up to 1,200, 1,300, and 1,400 feet per minute. What the maximum speed with

(Mr. E. A. Pochin.)
raw-hide pinions was, he did not know at present, but anything up to 2,500 feet per minute they did not consider excessive.

Mr. VINCENT GARTSIDE said there were several points mentioned in the Paper with which he agreed. There was the question of the short tooth, which he had advocated for a considerable time. He believed that in his Paper read before the Manchester Association of Engineers, referred to by Mr. Adamson, he mentioned that by shortening the tooth a great amount of noise was obviated. He made a pair of gears as nearly accurate as possible, standard 8 pitch gears. They were, comparatively speaking, fine pitches, but they were rather noisy until he took about $\frac{1}{16}$-inch off the tops of the teeth. That supported the argument about rounding the points off, but he did not quite see the advantage of doing so. If the teeth were cut away to make them run quieter by rounding the points, the latter might as well be cut off altogether. That was what he advocated some time ago.

A point that struck him in the Paper was that one seemed to get confused about cast-gears and cut-gears, and, in his opinion, it would be better if they could be separated. When dealing with, say, cast-gears up to 4, 5, or 6 inches pitch, and at the same time considering fine pitches down to say $\frac{1}{4}$ inch, it seemed to him that what applied to one could not possibly apply to the other, owing to the vast difference in the methods of production and the accuracies which could be obtained. One speaker mentioned the question of the shaving off of the teeth in flakes. He might state an experience he had had with a pair of large gears. The teeth were 3 inches pitch, about 7 inches wide on the face, and gears of mild steel. The particular thing which happened was flaking off of the teeth on the spur-gear. He investigated that very carefully and came to the conclusion that it was owing to the wrong shape of the teeth. On testing their shape, he found that the pinion was not the correct shape. The wheel was fairly good, but owing to the wrong shape of the pinion there was an excessive pressure on the teeth at certain places on their face, which caused abrasion, and parts came away from the wheel in flakes. These gradually got into the teeth, but part of the flakes were filed away and the teeth brought down to something like the correct shape. It was seven or eight years since the machine was put in, and as nothing further had been heard about it, he gathered that the wheels were now running satisfactorily.

He begged to differ from Mr. Dempster Smith's remarks about the Brown and Sharpe shape being almost exclusively used in the machine-tool trade. If anyone cared to investigate what shape of tooth was being cut at the present time by different gear-cutters and tool-makers, he would find that there were several shapes apart from the Brown and Sharpe standard. The hobbing of gears had brought in various modifications to get over interferences and little inaccuracies, and it would be found that makers of hobs had various standards of their own, and each gave a different type of tooth. His own firm had their own standard tooth for hobbing, and they tried at the time to get as near as possible to the Brown and Sharpe shape. They endeavoured to make a wheel that would gear with gears cut with a Brown and Sharpe cutter, but they found that it was not possible. They got the nearest approximation they could, and had kept to it, but they found that other firms had their own ideas as to what was the correct shape of tooth for hobs, and the result was that they got various shapes of gears from different firms.

With regard to Mr. Dempster Smith's remarks about the tool-makers in the Midland district using a long tooth, he had not himself heard about it. It only went to prove that tool-makers were not all using the standard Brown and Sharpe shapes. But he did know that one of the largest motor-car firms in the Midlands was using a long tooth. He had never been able to find out the real reason, but he knew they were going to considerable trouble in having special grinding machines for grinding the teeth after they were hardened—which was a very interesting part of the subject—and the gears that had been ground had all long teeth. He would like to know whether the practice had been adopted in this country of roughing the teeth out, subsequently having them heat-treated or oil-hardened, and then cutting or finishing them afterwards. He had seen this process in France in

(Mr. Vincent Gartside.)

some of the motor-car shops, where the cutting speed on the tools was something like that used for chilled iron, running about 10 or 12 feet a minute. It seemed a very slow process, but he believed that good results were obtained. He had not heard of that practice being followed in this country.

He was rather interested in the laminated gears, Fig. 19 (page 421), but he could not quite follow the reasoning in halving the back-lash. It looked as though it might be possible to arrange the laminations in such a way as to take out the back-lash altogether. In that case they would, of course, only drive one-half the number of teeth. This reminded him of a case of a pair of driving wheels which were made 10 or 12 years ago for driving a copper band turning lathe at high-speeds. It was driven direct by a variable-speed motor through one pair of gears. Owing to the high speed, and anticipating trouble through the inaccuracy or probable inaccuracy of the gears, he had arranged the pinions with a broad driving portion, and then a narrow portion by the side of it which was coupled to the wheels through a spiral spring; this caused the narrow portion to press on the back of the teeth. The wide portion turned the driving wheel in the forward direction, and the narrow portion was arranged to press on the back of the teeth in the reverse direction by means of the spring, with the idea of preventing the flying forward of the wheel owing to any inaccuracy of the teeth, thus reducing the hammering. He was glad to say that the result was very encouraging. It was proved that when the machine was run without the narrow wheel, the wheels made a great deal more noise than they did with the retarding wheel on, which showed that the noise was caused by the vibration of the teeth, one against the other. If they sprung it tight enough to keep out the vibration, no rattle or hammering noise was heard, but only a humming which was more musical to the ear.

Mr. J. P. BEDSON said he did not profess to be an authority on wheels. The Paper had been very interesting so far as it went, but he came to learn, and it did not go as far as he wished. To him the point of chief interest in connexion with spur-gearing was the peripheral speed. Since the application of the electric motor and the steam-turbine, one's ideas on the question of gearing and the speed of gearing had altered considerably. He observed that in the Paper 3,900 feet per minute was about the speed-limit of the examples mentioned. For a considerable time he had been running large cast-iron gears machine-moulded at 3,360 feet a minute, and since the application of motors he had gone as high as 6,600 feet a minute. The question was whether one could go further than that and reach 10,000 feet a minute, which was being done in turbine boats. If they wanted a ready application of the high-speed motor and to bring it down to a commercial speed, this question of the high peripheral speed must be investigated.

They were greatly indebted to the Author for the information he had given, and their debt would be increased if the Paper, which was exceedingly interesting, led to a further investigation of the high speeds at which they could run the double helical cut-gears of to-day, and brought out something to guide engineers who had to run tools and machines. He was referring to the running side entirely. If the Paper suggested anything in that direction, it would be of double value not only to tool-makers but to engineers who had to apply these high peripheral speeds, which, he believed, would come increasingly to the front as the use of electricity extended. He had machine-cut wheels running at very large reductions of 12 to 1, and nothing could be better. They were well put together and well housed; they were tied together and ran in gear-cases, oil being liberally used in their running. Some of those wheels had been running for four or five years; and there was hardly any wear on them at all. Nobody could do better than Mr. Adamson if he liked to carry his investigations further, and let them know at how high a speed wheels could be run.

Mr. DANIEL ADAMSON said it was difficult to reply satisfactorily at the moment to every speaker, but if he failed in that respect he would take the opportunity of elaborating his remarks when the discussion came before him in a printed form.

He was much obliged to the Chairman (Mr. Garnett) for his

(Mr. Daniel Adamson.)

references to the work of Sang, and would recommend any member who was sufficiently interested in the subject to make a careful study of the book mentioned. It would, for example, provide the answer to the question asked by Mr. Walter Pitt in London as to the conditions governing the interchangeability of wheels generated from a given shape of rack tooth.

Mr. Alfred Saxon raised the point as to whether he was speaking of bevel-wheels or spur-wheels. He had quoted in the Paper several examples of bevel-wheels, because he had particulars of them that were pertinent to the subject, but it was his intention to confine himself to the principles governing the design and construction of the straight-toothed spur-wheels, many of such principles being, however, equally applicable to bevel and helical wheels.

Mr. Saxon mentioned that the cycloidal tooth form was used for cast-gears. He (Mr. Adamson) had tried to point out that the involute form had been largely adopted in recent years, because it lent itself to accurate reproduction in the machine-cut form. Mr. Saxon said he had been told that the cycloidal form was more suitable for the larger pitches. Within the last week (since the Paper was in print) an engineering friend had shown him examples of pinions for rolling mills $7\frac{1}{8}$ inches pitch, 15 teeth, $1\frac{7}{8}$ inches addendum, involute shape, 20° obliquity. He was pleased to find that these proportions agreed almost exactly with the figures given on page 364, and they showed that the involute tooth was found to be quite suitable for heavy gearing. Mr. Saxon also said that Mr. Michael Longridge advocated teeth half pitch long. His own recollection of what Mr. Longridge said was that the duration of contact should extend to about half pitch on each side of the pitch-point, and he thought the illustrations published by Mr. Michael Longridge in 1891 showed teeth of even less than half pitch long on large wheels. The length of the teeth was to be governed by the desired duration of contact. The teeth of the Graves wheels at Niagara, quoted in the Paper, were carefully cut to involute shape —addendum 0·318 of circular pitch. The New York Subway wheels had also machine-cut teeth of involute form, both $14\frac{1}{2}$° and 20° obliquity being used. Mr. Gass had answered Mr. Saxon's question as to one example quoted in the Paper, and said the pinion was of forged steel and the wheel of cast-steel.

Mr. Butterworth suggested that, unless the wheels were perfectly mounted, much of the Paper lost its value. Although it was very desirable to have accurate mounting, some departures were unavoidable, and he thought provision was made for those in the large factor of safety—called by some the factor of ignorance—which was usually allowed in engineering structures. A factor of safety of about 3 or 4 was suggested in the Paper. Mr. Butterworth said the deformation of the tooth surface must be very small. It was certainly very small if measured by the ordinary units, but it was that very deformation which brought about the destruction of the surface and eventually caused the abrasion and the back-lash which gave rise to these higher stresses.

Mr. Butterworth referred to the calculation of the strength of the teeth across the corners. That was an old assumption dating from the times of cast gearing when they could not depend upon the load being equally carried across the whole width of the face. About twenty-five years ago it was the general understanding that the strength increased as the square of the pitch. This was quite true if the load were assumed to come on one corner of the tooth, but with the cut tooth they could depend upon the tooth bearing across the whole width, and accordingly the strength varied—as the pitch multiplied by the width. Previously, no matter what was the width, it was assumed as varying as the square of the pitch.

Mr. Butterworth mentioned an error of $\frac{1}{64}$ inch on a 3-inch pitch as being suitable for 2,000 feet per minute velocity; $\frac{1}{64}$ of an inch on a 3-inch pitch came out at about half per cent. If reference were made to the diagram, Fig. 16 (page 373), it would be seen that it ran up to 5 per cent. error in pitch. He would like to say that 5 per cent. error was very great; one could see it without measurement. But while in cast-wheels he found the maximum error was 5 per cent. in bad cases, in cut wheels it came out about 1 per cent. and in generated wheels about $\frac{1}{10}$ per cent., measured from tooth to tooth. He mentioned that for comparison with Mr. Butterworth's figure of half per cent.

2 F 2

(Mr. Daniel Adamson.)

Mr. Gass had kindly answered Mr. Saxon's question, and also referred to the Citroen gears. It was unfortunate that the gears mentioned had not continued in regular use during the last two years; otherwise more experience of their running might have been obtained. Mr. Gass recommended the finer pitches as being very often advantageous when compared with the coarser pitches; this was exemplified by planing machines. That was a very common experience. If other conditions were suitable for the finer pitch, quieter running would result from the change.

Mr. Dempster Smith confirmed his impression that involute teeth were largely superseding the cycloidal. Incidentally he raised the mysterious question as to why $14\frac{1}{2}°$ obliquity was adopted. Wilfred Lewis told the Author once in conversation that he had considered that question, and could find no reason, except that the sine of $14\frac{1}{2}°$ would work out at $0 \cdot 25$. What particular advantage that was he could not say, but it might simplify the setting out. Possibly another reason was that it was about equal to the mean obliquity of the cycloidal teeth then in use, and that when the involute tooth came, the angle of obliquity was adopted which compared approximately with what was already in use. The report of the U.S.A. Committee mentioned by Mr. Smith was referred to on page 386. The information he had from New York was that it was never published, and he thought the reason was that the Committee did not agree.

Mr. Dempster Smith mentioned 40,000 for $\frac{PN}{eb}$ for cast-iron. He understood that to refer to machine-tools. All these empirical factors had to be considered in relation to the use that the wheels would be put to, because, as Schäfer mentioned, different machinery ran varying number of hours per week.

Dr. Bowman mentioned shrouded teeth as an advantage. There was no doubt it was an advantage in cast teeth, but the shrouding had to be discarded when machine-cut teeth were adopted (Appendix I, page 384).

Mr. Paton had put before them examples of laminated wheels. The thanks of the members were due to him so far as his remarks were of general application to designers adopting ordinary designs of wheels. The remarks upon lubrication he could not quite follow. It had always seemed to him that a long surface was better than a short one, because when the oil got in, it could not get out, and that when between the teeth of ordinary gear-wheels it had a very much better chance of remaining in than if it was between narrow edges or faces like the laminated gears Mr. Paton had shown.

Mr. Pochin's remarks were very valuable, and it was to be hoped he would add his opinions with regard to Fig. 16 (page 373), and also to the durability factor. Mr. Pochin frequently referred to the Lewis formula. He (Mr. Adamson) had been trying to establish something more rational than that formula.

In reply to Mr. Vincent Gartside there was no confusion in the mind of the Author as to the difference between cast-gears and cut-gears because, so far as the Paper was concerned, they differed only in degrees of accuracy; the greater the inaccuracy, the slower the wheels must be run if quietness and safety were to be considered.

Mr. Bedson implied that only helical gears were suitable for high velocities, but the Author was not satisfied yet that the best had been got out of straight-toothed gears, and believed that if as much care were given to their accurate construction and mounting they would be as satisfactory and more durable than helical gears. Then there was the question of durability and inaccuracy, and he thought that was the direction in which Mr. Bedson must look for some guidance as to the possible limits to speeds. The speeds mentioned by Mr. Bedson were much higher than he had run his diagrams out for, but he believed the same law would hold good, and that, as the accuracy was improved, they would be able to increase the speed to an extent limited only by the centrifugal forces which would then be brought into play. The reason he had not mentioned such speeds was that, as far as he knew, there were no published data on the subject; they were largely special questions which were only known to the firms dealing with those high-speed gears. Still, he was hopeful that some members of the Institution or their friends would do something in the way of supplying information upon these questions. In conclusion, he

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thanked the speakers for the very careful consideration they had given to the Paper.

On the motion of Mr. EDGAR WORTHINGTON (Secretary), a vote of thanks was passed to the Engineers' Club for their kindness in placing their meeting-room at the disposal of the Institution.

Communications.

Mr. FRANCIS J. BOSTOCK wrote that, although spur type of gear was largely used at the present time, there was a feeling that it was gradually being displaced by the double helical form of tooth, so that the subject under discussion was mainly from an academical point of view. The question of the tooth form was naturally one upon which great controversy rested, and although the cycloidal form, or the other special shapes, such as Mr. Sharp's or Professor Smith's, should have distinct advantages, they were naturally very little used, on account of their not lending themselves to easy generation.

The old form of gear cutting by means of rotary cutters or formers had, as the Author stated, been superseded by generating systems. The involute form of tooth was based upon a straight-line system, and therefore lent itself to easy cutter manufacture and generation. The writer considered that perhaps the best generating method was that of the use of a straight-sided rack-cutter based upon the Sunderland system. There were undoubtedly defects with the involute form, mainly on account of the fact that in the passage of one tooth across its mate a certain amount of sliding took place. The rotary cutter method of cutting gears depended upon the condition that all gears belonged to an interchangeable series, and consequently had equal addenda. It was difficult to say why popular prejudice was against varying the addenda of the two mating gears in order to reduce the amount of sliding, but very little progress seemed to have been made in that direction. In some cases there might be as much as 60 to 80 per cent. of the contour of the tooth sliding across the one in contact with it, but by the simple adjustment of the addenda to suit the working depth, the number of teeth, and the ratio of the pair, it could often be reduced to 20 per cent. This sliding was obviously a function of the wear characteristic, and he was of the opinion that all gears should be designed to suit the wear characteristic as well as the strength.

If one obtained the ratio between the amount of sliding and rolling that took place, terms could easily be defined to suit various conditions. Thus, for ordinary gears this ratio might be equal to unity, but for high-speed turbine-gears, which often ran at 8,000 to 10,000 feet circumferential speed per minute, this value should be about 0·20 or 0·25. Naturally, for the latter, it meant the employment of a fine pitch, at the same time making the addendum of the pinion greater than that of the wheel, to suit certain mathematical conditions. He appended (pages 436–7) empirical formulæ which were based upon such calculations, showing the proportions and numbers of teeth that could be used in pairs of gears for ordinary purposes, and wherein the above factor was unity.

An important point developing out of the above correction was that the strength of the teeth differed very radically from the "Y" in Lewis's formula. Full consideration should be given to the number of teeth in contact, the distribution and position of the load on the tooth. By regarding all these factors it was found that the "Y" value, say for a rack, could be increased from 0·154 (for involute 20° pressure angle) up to 0·210, and that for a corresponding pinion in mesh with it, say for 24 T., it could be increased from 0·108 to 0·248, and so on throughout the whole series. Again, it was found in practice that Lewis, in obtaining his "S" factor, appeared to have overestimated the effects of high velocities and perhaps underestimated low velocities. The writer considered that the factor "S" should vary as $V^{-\frac{1}{3}}$, which appeared

(Mr. Francis J. Bostock.)
to be commensurate with the high velocities obtained in turbine gear.

There was a direct relation between the strength of the teeth, the amount of sliding, and the wear characteristic, and by formulating these amounts one would readily see that certain maximum pitches were advisable, namely, for spur-gears consisting of a cast-iron wheel and mild-steel pinion $4\frac{1}{2}$ inches to $5\frac{1}{2}$ inches, according to the ratio, should be the maximum pitch, and with a cast-steel wheel and mild-steel pinion $1\frac{7}{8}$ inch to $2\frac{3}{8}$ inches was the corresponding maximum pitch. For the narrow-faced double helical gear, which, as before mentioned, was rapidly superseding the spur-gear, the corresponding maximum pitches appeared to be $3\frac{1}{4}$ inches to $4\frac{1}{2}$ inches and $1\frac{1}{2}$ inch to $1\frac{7}{8}$ inch respectively, whilst for turbine-gears it should be limited to $\frac{7}{8}$ inch to 1 inch. If these pitches were exceeded, there was every probability that more or less rapid wear would take place. Of course, when finer pitches were used, longer life was thereby obtained. The whole of the matter with reference to the tooth design, proportions, speed, etc., together with the development from the spur to the double helical and to turbine-gear, involved so much consideration that it was impossible in the discussion to do more than outline the direction of development, upon which gear manufacturers were now working in order to obtain higher satisfaction in gearing.

INVOLUTE TOOTH CORRECTION FOR SPURS, ETC. (PLANED OR HOBBED).

Wherein equal sliding and rolling obtains.

NOTE :—Working Depth = 0·6366 Pitch.

N = Gear Teeth.

n = Pinion Teeth.

θ_N = Spiral Angle of N.

θ_n = Spiral Angle of n.

Ordinary Gears.

$$\text{Addendum of Pinion} = a = \left(\frac{8N}{5N + 3n}\right) \text{Ordinary addendum.}$$

$$\text{Addendum of Gear} = A = 2\left(\frac{N + 3n}{5N + 3n}\right) \text{Ordinary addendum.}$$

Rack and Pinion.

Addendum of Pinion $= A_r = 1 \cdot 6$ Ordinary addendum.

Addendum of Rack $= A_R = 0 \cdot 4$ Ordinary addendum.

NOTE.—For BEVELS, use N^2 and n^2 in place of "N" and "n," also "Ratio" $= \left(\frac{N}{n}\right)^2$.

For HELICALS, use "Normal Pitch" in place of "Circular Pitch."

Ratio		1:1	2:1	3:1	4:1	5:1	6:1	7:1	8:1	9:1	10:1	∞:1
Number of teeth in pinion	14½° pa	33	26	22	20	19	18	17·5	17	16·5	16·5	13·5
	20° pa	20	16	13	12	11·5	11	10·5	10·5	10	10	8

Mr. CLAUDE W. HILL wrote that the subject of spur-gearing was occupying a good deal of attention at the present time, so that the Paper came at an opportune moment, and the thanks of the profession were due to the Author for the information and suggestions he had put before them.

The most important portion of the Paper appeared to be that dealing with the stresses in the teeth of wheels and the effect of inaccuracies in increasing these stresses at high speeds. Inaccuracy in the gear caused unsteady running of the driven parts, so that as the tooth passed through the arc of action they were alternately accelerated and retarded. The changes of speed took place in a very short space of time, and consequently, although the actual amount of change might not be great, the rate of change might be very considerable. To impart the acceleration, the teeth had to exert a force in the same direction as the driving force, so that the stresses in the teeth were due to the sum of these two forces. If the driven parts contained heavy masses, the stresses due to acceleration might greatly exceed those due to the driving force.

It had consequently been the custom in designing toothed gear to allow lower unit stresses (or, what amounted to the same thing, a higher factor of safety) in high-speed gearing than in gearing intended for slow speeds. In place of this the Author suggested

(Mr. Claude W. Hill.)
that the accuracy with which wheels were cut should increase as the speed at which they were intended to run increases, and he gave a curve showing the percentage of allowable error with increasing speed which would keep the acceleration forces within a given limit, so that higher unit stresses might be allowed in designing. The writer understood that the error which the Author had in mind was error in the pitching of the teeth.

Unsteady driving could, however, also be set up by departure from the true involute form, so that it would be necessary to apply the limits to the whole face of the teeth and not only at the pitch-line. He believed an instrument was now made which would measure any departure from the true involute form. Speaking as a purchaser, he thought that the adoption of a specification embodying the Author's limits of error would undoubtedly lead to the construction of better, more durable, and more silent gear for high speeds. There might, however, be difficulty at first in getting wheel-cutting firms to accept such a specification, and it would probably be an advantage if the Engineering Standards Committee would take the matter up.

Some years ago, finding there was considerable divergence of opinion among different authorities on the strength of gearing, the writer collected data of a number of cast-iron and steel gears which were giving satisfaction. From these data he calculated the stresses and plotted them as curves showing the relation of stress and speed. These curves were given in the chapter on Toothed Gearing in his book on "Electric Crane Construction" published in 1911 (a copy of which was presented to the Institution Library). He also gave the following formula for calculating the strength of gears,

$$p = \sqrt{\frac{\mathrm{KH}}{\mathrm{S}dfx}},$$

in which

p = Pitch in inches.

K = A constant depending on the number of teeth. It is 2,110,000 for a 12-tooth wheel and 762,000 for a 30-tooth wheel, with intermediate values for wheels having 13 to 29 teeth.

H = Horse-power.
S = Revolutions per minute.
d = Pitch diameter in inches.
f = Stress in lb. per sq. in. at root of teeth taken from the curves mentioned.

$$x = \frac{\text{Width of wheel}}{\text{pitch}}$$

This formula he had used in his own work with satisfactory results.

There still seemed to be considerable divergence of opinion on the subject of strength of gearing among those who presumably were to be regarded as experts. A short time ago he designed a train of gear to transmit 50 h.p. When sending out the inquiries full particulars were given of numbers of teeth, pitch, width of wheels, materials, and speeds, and each firm was asked for its opinion as to what power the driving pinion could transmit. Four firms replied as follows :—

Firm A stated that the pinion could transmit	20 to 25 h.p.
,, B ,, ,, ,, ,, ,,	30 h.p.
,, C ,, ,, ,, ,, ,,	60 h.p.
,, D ,, ,, ,, ,, ,,	120 to 130 h.p.

The wide divergence among these figures pointed to the necessity of engineers coming to some agreement as to a standard formula for the strength of spur-gearing.

Referring to the second example (A) in Fig. 12 (page 366), the same arrangement had been used in this country in several cases with similar results. The arrangement B (in the same Fig.) did not seem the best solution of the difficulty, as it doubled the torque on the crank-shaft and threw the majority of the wear to the one end. A better plan would be to retain the two pinions and use a simple equalizer to divide the load equally between them.

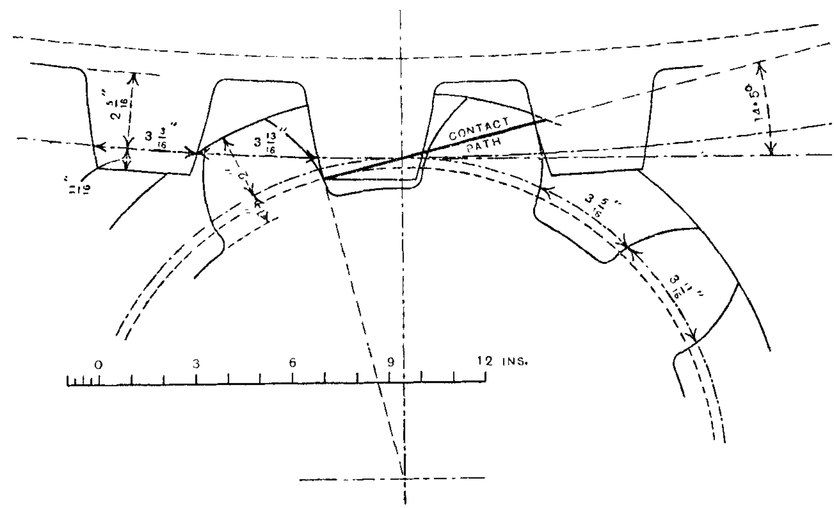
In designing the machinery of the "Flip Flap" which was constructed for the Franco-British Exhibition in 1908, he used a similar arrangement for driving the towers. A large spur-wheel was bolted to each side of each tower, and each was driven equally so as to avoid putting a twisting force on the tower. The speed of

(Mr. Claude W. Hill.)

the motor being 500 revolutions per minute and that of the main wheels being 0·2 revolution per minute, the reduction ratio was 2,500 to 1, so that it was necessary to provide a train of gear leading up to each main wheel. These trains of gear were driven through differentials, so giving equal force on each main wheel. As the main pinions and wheels were of somewhat special design, the particulars of them might be interesting, and they were given on the drawing Fig. 20. The gear was of cast-steel. The

FIG. 20.—*Large Spur-Gearing at Franco-British Exhibition*, 1908.

Pinion 9 Teeth, 7″ Pitch. 20·05″ Pitch diam. Base Circle of Involutes 9·7″ Radius.
Wheel 86 ,, 7″ ,, 191·60″ ,, ,, ,, ,, ,, ,, 92·7″ ,,
Pitch circles = dot-and-dash curves. Base Circles of Involutes = dotted curves.



pinions had 9 teeth of 7 inches pitch and were double-shrouded to the tops of the teeth. The width of the wheel teeth was 8 inches. The angle of incidence was 14½°, and, in order to make the arc of action slightly exceed the pitch, the pitch-line was placed near the root of the pinion teeth and near the top of the wheel teeth, as shown in the Fig. The teeth were 3 inches deep, so that they were stronger than the Brown and Sharpe standard form, which for 7 inches pitch would be 5 inches deep.

The load on these gears was very variable. With maximum loading and driving against a wind pressure of 20 lb. per square foot, the force at the pitch-line of each wheel was 51 tons. With minimum loading and a 20 lb. wind moving in the same direction as the tower, there was a reverse pressure at the pitch-line of 42½ tons. These extreme pressures only occurred very seldom, the average load being probably 10 to 15 tons, but it might be mentioned that the "Flip Flap" had frequently been worked when the wind was so high as to blow the cars off the rails on the Mountain and Scenic Railways. With 51 tons pitch-line pressure, the pressure per inch width of wheel was 6·4 tons. Such an extreme pressure was, of course, only permissible where it was applied occasionally and at very slow speed, as in this case. Taking the 51 tons at the top of the tooth, the stress at the root was 14,390 lb. per square inch. The gearing had worked five seasons and showed very little wear.

Reverting to the question of tooth stresses, apart from stresses set up by inaccuracy in the gear, very heavy stresses might be thrown on the teeth, in the case of electrically-driven machinery, by badly designed starting switches which set up an excessive rush of current through the motor during the starting period.

Mr. MICHAEL LONGRIDGE (Vice-President) wrote that the field covered by the title of the Paper was so extensive that he felt unable to survey more than a small corner of it, but the conditions in that particular corner differed so radically from the conditions prevailing outside it that the rules which applied to the design of spur-gearing generally were not applicable to the particular class of gearing to which his remarks would be confined, namely, heavy gearing for transmitting power from the main shaft of a steam-engine. Here the cordial co-operation between theory (as taught in the text-books) and practice, which the Author of the Paper found so interesting, ceased; and it ceased because the axis about which the driving wheel revolved was movable and not fixed, as the text-books postulated. The main shaft of a steam-engine, and especially that of a pair of engines, moved vertically, horizontally,

(Mr. Michael Longridge.)

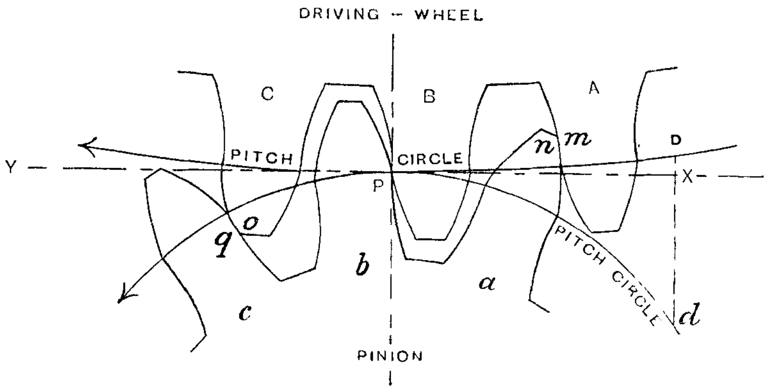
and longitudinally, and often revolved with variable angular velocity; moreover, unless the teeth were cut, they were liable to irregularities of form. He was not referring exclusively to the old style of spur-wheel made up of segments cast from patterns, and pitched and trimmed by a class of millwright who had unfortunately almost disappeared, but also to machine-moulded wheels and specially to cast-steel wheels. Teeth were not always of exactly the same pitch and thickness, nor were their faces always exactly parallel to the axis of the wheel. Pitch-lines, especially where rims were cast with arms, were not always true circles; and wheels could not always be geared exactly to the pitch circles corresponding to the number of teeth in them, because, especially in the case of steel, contraction was not always what it was intended to be. From these facts resulted two very obvious consequences, which designers of hard gear for transmitting power from the main shafts of steam-engines would do well to bear in mind.

The first was that it was impossible to prevent the whole load being carried sometimes by a single pair of teeth, and therefore it was useless, and indeed detrimental (seeing that a tooth was really a cantilever), to lengthen teeth with the object of having more than one pair in gear at any one time. The second was that it was impossible to ensure uniform distribution of the load over the whole width of the tooth-face, and therefore that the strength of a tooth was a very long way from being proportional to its breadth.

To mortise gear, of course, this reasoning did not apply; there, both length and breadth of teeth were advantageous. So far his remarks applied to all forms of teeth of hard wheels of the kind to which he was referring, but there were other cogent reasons for shortening the teeth of such wheels to the utmost, where those teeth were, as had hitherto been the almost universal practice, of *quasi* cycloidal form. He used the word *quasi* with intention, because he wished to emphasize the fact that here theory and practice, instead of being in co-operation, actually were at war.

To make heavy gearing to transmit power from the crank-shaft of an engine at a high peripheral speed with cycloidal teeth, set out according to the text-books, was simply to court disaster. He knew it by experience. The makers of such wheels recognized it by their practice. They invariably set out the profile of the teeth so far inside the true cycloids that, when newly-geared, the teeth "bore on" only in the immediate vicinity of the pitch-line, and when through wear the bright bands crept out towards the points they chipped the teeth, or, in other words, reduced their effective length. A glance at the diagram, Fig. 21, would show the reason for this. A, B, C were three teeth of a spur driving-wheel, *a*, *b*, *c* three teeth of a driven pinion, supposed to be of correct cycloidal form. As

FIG. 21.—*Cycloidal Teeth showing Period of Contact.*
(From Annual Report of Engine and Boiler Insurance Co. 1887.)



the wheels revolved, contact commenced between some point *m* on the flank of the wheel-tooth A and the extreme point *n* of the face of the pinion-tooth *a*, and ceased when the extreme point *o* of the face of the wheel-tooth C left some point *q* on the flank of the pinion-tooth *c*.

If, owing to slackness in the crank-shaft bearings, the wheel A, B, C moved towards the pinion *a*, *b*, *c*, the point *n* of the pinion-tooth or the point *o* of the wheel-tooth, or both, must do one of two things—either they must penetrate into the flanks of the teeth on which they bore, or they must retard the motion of the wheel and accelerate that of the pinion shafting attached thereto, either of

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which would cause enormous pressures acting at the greatest possible leverage, namely, the full length of the teeth. If, on the other hand, the contact took place only in the neighbourhood of the pitch-line where the surfaces of the teeth were practically nearly flat, the effect of a movement of a wheel towards the pinion would be practically nil. In the same way the effect of fulness at the points of teeth and, if he might coin a word, of "uncircularity" of pitch circles would be minimized by confining contact to the neighbourhood of the pitch-line, while irregularities of pitch, though still destructive, would have less effect, because the leverage at which the pressure would act would be reduced by shortening, not only the addendum, as was done when teeth were chipped, but also the dedendum whose depth was dependent on it.

On the subject of wear, he would merely point out that the component parallel to the line of centres of the relative sliding was the difference between X*d* and XD, which increased rapidly as the distance from the pitch-point P increased. On all accounts, therefore, teeth long enough to give a path of contact only a little longer than the pitch were preferable for the heavy spur-wheels of mill-engines. He had held these opinions for many years, but it was in 1889 or 1890, as far as he could fix the date, that he first put it to the proof—in a case where a succession of breakages had led to the adoption of teeth, as far as he could remember, 6 inches pitch and 4·2 inches length. When these broke in their turn, he replaced them by others $3\frac{7}{8}$ inches pitch and $1\frac{1}{4}$ inch length, which ran well until the engines were pulled out some years later.

In ordering uncut wheels, he thought it important to have pitch circles turned on the ends of the teeth, with radii equal or proportional to the radii marked on the drawings, and thus to ensure equality in the pitch of the two wheels. Whether these pitch circles coincided with the circles marking the junctions of the epi- and hypo-cycloidal curves was a matter of secondary importance so long as the teeth were short. Accuracy of shape acquired importance in proportion to the square of the distance of the point of contact from the pitch-point.

With regard to stresses, two formulæ were in very general use:—

$$(1)\ p = \frac{33{,}000 \times \text{i.h.p.}}{v} = \Sigma \frac{bt^2}{6h},$$

$$(2)\ p = \frac{33{,}000 \times \text{i.h.p.}}{v} = \Sigma \frac{t^2}{3},$$

in which

p = pressure teeth in lb.

v = velocity of pitch-line in feet per minute.

b, h, t = breadth, length, and thickness of tooth in inches.

Σ = stress in lb. per square inch.

The first assumed that the pressure would be uniformly distributed over the whole width of the tooth, and was applicable to mortise gear. The second, suggested by Rankine, assumed that the whole load would be concentrated on one extreme corner. This, or some modification of it, should be used in connexion with hard wheels. The proper value of Σ depended on the quality of the workmanship, the nature of the load, and speed of the pitch-line. His (Mr. Longridge's) experience led him to think the following to be fair average values:—

Material.	Cast-iron.	Cast-steel.	Cast-steel cut.	Mortise-wheel hard pinion.
v	2,000	2,200	2,300	2,400
$p \div l$	900	900	1,000	650
Σ in (1)	—	—	—	2,000
Σ in (2)	5,000	7,000	8,000	—

In conclusion, he would like to add a few words about double helical teeth. There was a widespread impression that such teeth were stronger than straight teeth when used on wheels carried on engine-shafts. This was a pure delusion; the contrary was the truth. If the stress could be evenly distributed on both the helices, it would be exactly the same as in a straight tooth on a

(Mr. Michael Longridge.)

rim of the same width as the rim carrying the two helical teeth, because, although the sum of the two b's of the double helical teeth would exceed the single b of the straight tooth, the sum of the normal pressures upon the two helical teeth would exceed the pressure on the straight tooth in exactly the same proportion.

As a matter of fact, with a moving crank-shaft the whole pressure was thrown upon one of the helices, and not distributed between the two; so that the stress was just double that in a straight tooth of equal pitch and height, and far more than double if the momentum of a heavy fly-wheel and the inertia of a heavy pinion and lengths of second-motion shafting were taken into account. Indeed, to keep such gearing safe, it was often necessary to provide sliding couplings on each side of the pinion, to allow it to move sideways under the lateral pressure on the teeth without having to move the whole length of the second-motion shafting and the wheels upon it. Straight teeth were stronger and more satisfactory.

Mr. T. MOHN wrote that no reference was made in the discussion at Manchester to the question of safe tooth-pressure, regarded from a surface-pressure point of view only, and not with regard to strength of teeth. A case-hardened wheel might not be actually stronger than a wheel made of mild steel, but it would withstand the wear better. Similarly, wheels with internal and external gears worked with less wear than gears with external teeth on both wheels. A few years ago the writer gave this subject some attention, and came to the conclusion that the curvature of the tooth had a great deal to do with the question. He thought it should be investigated on similar lines to the surface contact pressure between a cam and a roller, where the radius of the surfaces was of importance. Line contact per unit length or profile contact should be measured by the reciprocal of the relative curvatures. On the strength of this he worked out a chart from an article in the *American Machinist*,* but he had not had sufficient experience to say if the loads were safe.

* *American Machinist*, 1st February 1908, page 95.

Another point not mentioned in the Paper was the advisability of having a jet of oil playing on the wheels at a point where they approached each other. He also thought that wheels should be forced on to their shafts. Wheels only keyed on were very liable to run out of truth. He would be glad of the Author's opinion as to whether it was advisable to touch the teeth with a file after they had been running for some time. He (Mr. Mohn) thought that the wheels should be left alone if they were running quietly, even if one tooth were not bedding quite satisfactorily, but they should be examined for running true both diametrically and sideways.

One of the most interesting gears he had seen was the "Pedersen Three-speed Bicycle Gear," * in which the stresses were extremely high.

With regard to the length of teeth, some shapes of teeth in use at the Vulcan Shipyard, Stettin, were published in *Traction and Transmission*.†

Mr. RICHARD SHAW wrote that both the Author of the Paper and Mr. Dempster Smith had referred to the advantages of involute gears with an angle of obliquity of 20° or 22½° and a short addendum, yet one did not hear of this form of tooth being used to any great extent. Messrs. Wm. Sellers and Co. had adopted a 20° obliquity; but the writer believed that the addendum was about the same as Brown and Sharpe's, and the Author had told them that he had adopted gears with an addendum of 0·25 P. but with an obliquity of 14½°. The following experience, therefore, although limited, might be of interest.

In 1905 the firm of De Bergue and Co., Ltd., decided to abandon all wheel patterns and adopt machine-moulded gears. The class of work was mainly punching and shearing machines of several types. Pitches of gears varied from 1 inch to 3½ inches, and the maximum peripheral speed was about 1,100 feet per minute, although only a small portion of the gears were required to run above 800 feet per minute. Involute gears were decided

* *Engineering*, 11th May 1906, page 631 (illustrated).

† *Traction and Transmission*, 1903, Vol. 7, page 123.

(Mr. Richard Shaw.)

upon. The same depth of tooth was adopted as used by the Author, namely: addendum = 0·25 P., and dedendum = 0·32 P. The bottom clearance of 0·07 P. made it possible to have a radius equal to 0·125 P. at the root of the tooth. Two angles of obliquity were adopted—20° where the smallest pinion would have 14 T, and $22\frac{1}{2}°$ where pinions down to 11 T would be used. If a decision had to be made to-day, he did not doubt that $22\frac{1}{2}°$ would be used in all cases. He was not able to give any percentage of accuracy, but no pains had been spared to produce correct gears. The results had been highly satisfactory. Noise would appear to be reduced to the minimum for metal gears. In calculating the strength, the whole width of tooth was taken into account. Pinions were generally flanged up to or beyond the top of the teeth. It was impossible to say anything yet with regard to the durability. There had been a few breakages, and of these, wheels had failed in the boss or arms; pinions—where the bore had been relatively large—had split through the keyway; but the writer was not aware of any case where a tooth had broken away from the rim.

Mr. DANIEL ADAMSON wrote, in reply to the Communications, that Mr. F. J. Bostock advocated a displacement of the pitch inch, or in other words, a variation of the addendum as between the wheel and the pinion; but while this had many advantages, it did not lend itself to interchangeable sets of gear-wheels, any two of which must gear together.

In reply to Mr. Claude W. Hill, the intention was to consider all errors that would affect the running, as reduced or referred to the one factor, namely, error in pitching the teeth for purposes of comparison.

Mr. Hill was the only critic who discussed the part of the Paper that was intended to be original, namely, that the accuracy must improve as the velocity was increased; before a definite rule could be laid down connecting these two factors, we should require to know more about the actual errors of wheels that had run successfully and of those that had failed, if such information could be obtained.

Mr. Michael Longridge's communication was very valuable, and it was gratifying to the Author to see that the greatest living authority on the particular section of the subject he dealt with confirmed the reasoning of Lasche and the statements in the Paper as to why it was better to shorten the teeth (p. 444). Mr. Longridge's reference to the want of co-operation between theory and practice led the Author to quote here an extract from the Appendix to Camus on "The Teeth of Wheels," published in 1837:— "There is a lamentable deficiency of the knowledge of principles and correct practice in a majority of these most respectable houses in forming the teeth of their wheel work." Then follows a list of fourteen of the most eminent firms of that period. Mr. Longridge's remarks on double helical teeth were important, in view of the very frequent reference to this type in the discussion, although not dealt with in the Paper.

In reply to Mr. T. Mohn, the fact that wheels were running quietly was sufficient evidence that all was well, and they were best left alone when in that condition.

It was interesting to learn that Mr. R. Shaw, in 1905, not only adopted an addendum of 0·25 pitch but also a bottom clearance of 0·07 pitch, these being the same as the Author chose in 1899, but it was not until 1912 that he departed from the generally accepted angle of obliquity of $14\frac{1}{2}°$ and began to use 20°.

Since the discussion was closed, Mr. J. Pickering had sent the following notes of his practice for over ten years when specifying for gearing for sugar-cane crushing mills, stating that they had been adopted as standard by some of the principal sugar-machinery makers:—

The material to be Siemens-Martin steel, having a tensile strength of 38–42 tons with an elongation of 10–14 per cent. in two inches; two test-bars to be cast on each piece, and the chemical analysis to be as follows: carbon 0·38—0·42 per cent.; silicon 0·30—0·35 per cent.; sulphur and phosphorus not to exceed 0·07 per cent. The teeth to be machine-moulded and shrouded to pitch circle. The teeth to be of involute form, addendum 0·3 pitch, clearance 0·155 pitch. The errors in pitch not to exceed 1 per cent., and the maximum variation in thickness of teeth not to exceed 2 per cent.

(Mr. Daniel Adamson.)

Mr. W. E. Sykes (page 401) had written subsequently regarding the $28\frac{1}{2}°$ mentioned in the discussion (page 402), and added that this high angle of pressure had been adopted for turbine-gearing for ship propulsion, in order to obtain teeth of maximum strength in view of the large ratios of reduction (15 to 1 or 20 to 1), and the consequent small number of teeth in the pinions. Such gearing had given excellent results, the height of tooth being 0·485 inch, circular pitch 0·816 inch, and normal pitch 0·575 inch.