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to destroy belongs to the Creator alone, I entirely coincide with Roget and Faraday in the opinion, that any theory which, when carried out, demands the annihilation of force, is necessarily erroneous. The principles, however, which I have advanced in this paper are free from this difficulty. From them we may infer that the steam, while expanding in the cylinder, loses heat in quantity exactly proportional to the mechanical force which it communicates by means of the piston; and that on the condensation of the steam, the heat thus converted into power is *not* given back. Supposing no loss of heat by radiation, &c., the theory here advanced demands that the heat given out in the condenser shall be less than that communicated to the boiler from the furnace, in exact proportion to the equivalent of mechanical power developed.

It would lengthen this paper to an undue extent were I now to introduce any direct proofs of these views, had I even leisure at present to make the experiments requisite for the purpose; I shall therefore reserve the further discussion of this interesting subject for a future communication, which I hope to have the honour of presenting to the Royal Society at no distant period.

Oak Field, near Manchester, June 1844.

LV. *Outline of a New and General Mode of Transforming and Resolving Algebraic Equations. By* James Cockle, *B.A., of the Middle Temple, Special Pleader*.*

1. THE practical application of the following will be found in various papers which I have had the honour of publishing in the Mathematician. The method is, however, here presented in an entirely novel form. Considered generally, its characteristic is, the effecting the proposed reductions by modifying the *roots* of an equation directly. By way of commencement, I have, for this purpose, generalised the assumption of Mr. Murphy† (which is undoubtedly true for equations of the first four degrees), and assumed that the roots of the general equation of the nth degree, in y, are given by a set of expressions of which the type is

$$y_r = \beta_0 + \alpha^r \beta_1 + \alpha^{2r} \beta_2 + \ . \ . \ . \ + \alpha^{(n-1)r} \beta_{n-1}, \quad . \ (1.)$$

where α denotes one of the nth roots of unity. It follows from this, that

$$y_1 + \alpha y_2 + \alpha^2 y_3 + \ . \ . \ . \ . \ + \alpha^{n-1} y_n = n \beta_{n-1}; \quad . \ (2.)$$

and, denoting the left-hand side of (2.) by $\phi(y)$, if $\phi(y) = 0$, $\beta_{n-1} = 0$.

* Communicated by T. S. Davies, Esq., F.R.S., F.S.A.

† Philosophical Transactions, 1837, part I.

2. If it be supposed that, in the roots of an equation of the nth degree in z, $\beta_{n-1} = 0$, then, by taking the roots in a proper order, we obtain, as before,

$$\phi(z) = n\,\beta_{n-2}, \text{ and, if } \phi(z) = 0,\ \beta_{n-2} = 0;$$

and, similarly, $\phi(w) = n\,\beta_{n-3}$, &c.

3. Next, x being the root of the general equation of the nth degree, let $y = \Lambda\, x^{\lambda} + \mathrm{M}\, x^{\mu}$, then, in order that, in the equation in y, β_{n-1} may $= 0$, we have

$$\phi(y) = 0 = \Lambda\,\phi(x^{\lambda}) + \mathrm{M}\,\phi(x^{\mu}), \quad . \quad . \quad . \quad . \quad (3.)$$

since ϕ is a linear function. But ϕ has many values arising from the interchange of the roots one among another; let m of the values of (3.) arising from this circumstance be multiplied together, and we have

$$\Lambda^{m}\,\pi + \Lambda^{m-1}\,\mathrm{M}\,\pi' + \Lambda^{m-2}\,\mathrm{M}^{2}\,\pi'' + \ .\ .\ .\ \mathrm{M}^{m}\,\pi^{(m)} = 0. \ (4.)$$

Now the peculiarity of the quantities π, π', &c. is (see the work above mentioned*), that one is derivable from another by an easy process, and that when one consists of symmetric functions of x, all do; and if we select those forms of ϕ which are included in the expression $u_1 + \alpha^{r}(\phi(u) - u_1)$, giving r every value from 0 to $n - 2$, then, at least for the first four degrees, π is symmetric and (4.) becomes a homogeneous equation of the $(n-1)$th degree, whence $\frac{\Lambda}{\mathrm{M}}$ may be determined.

4. We have thus obtained equations of the 2nd, 3rd and 4th degrees, whose roots are respectively of the forms

$$a^{\frac{1}{2}}, \quad a + b^{\frac{1}{3}}, \quad a + b^{\frac{1}{2}} + c^{\frac{1}{2}},$$

whence a certain convenient relation among the coefficients is obtained (Mathemat. p. 83).

5. To take away another term of the expression for the roots, we must similarly assume $z = \Lambda' y^{\lambda'} + \mathrm{M}\, y^{\mu'}$; this gives us, *primâ facie* at least, the reduction of the biquadratic to the binomial form, and of the equation of the 5th degree to the solvable form of De Moivre, and may be found to throw some light on the difficulties attending those transformations.

6. The assumption indicated for taking away r terms of the root, is

$$y_r = \Lambda^{(r)}\,y_{r-1}^{\lambda^{(r)}} + \mathrm{M}^{(r)}\,y_{r-1}^{\mu^{(r)}}.$$

Devereux Court, March 4, 1845.

* The formal proof will appear in the next July number of that work.