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150. Napier's Rule of Circular Parts

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the same expressions as above, then the following series is in order of magnitude (commencing with  $\infty$  or 2 according as it descends or ascends)

$$\infty \text{ or } 2, n + \frac{(n-2)(n-3)}{n-1}, w, n, \frac{2(w+6)^{\frac{1}{2}}}{(w+6)^{\frac{1}{2}} - (w-2)^{\frac{1}{2}}}, 3.$$

$w$  and  $n$  both lie between 2 and  $\infty$ , and are equal if either of them is equal to 2, 3, or  $\infty$ . F. S. MACAULAY.

**150. [K. 20. f.]** *Napier's Rule of Circular Parts.*

Let  $ABC$  be a spherical triangle right-angled at  $C$ . Then another triangle having the same circular parts may be found as follows.

(1) Take a triangle symmetrically equal to  $ABC$  and place the two together with their sides  $AC$  in contact. They will form an isosceles triangle whose sides are  $2a, c, c$ , whose angles are  $2A, B, B$  and whose altitude is  $b$ .

(2) Take the polar of this triangle. Its sides are  $\pi - 2A, \pi - B, \pi - B$  and its angles  $\pi - 2a, \pi - c, \pi - c$ , and its altitude is the supplement of the altitude of the previous triangle, i.e.  $\pi - b$  (as can easily be shown).

(3) Take the colunar triangle of the last triangle. Its sides are  $\pi - 2A, B, B$ , and its angles  $\pi - 2a, c, c$ , and its altitude  $b$ .

(4) Split this triangle into two right-angled triangles. The angles of these will be  $c, \frac{1}{2}\pi - a, \frac{1}{2}\pi$ , and the sides opposite them  $b, \frac{1}{2}\pi - A, B$ .

If  $A'B'C'$  denote this new triangle, we have therefore

$$a' = b, b' = \frac{1}{2}\pi - A, \frac{1}{2}\pi - A' = \frac{1}{2}\pi - c, \frac{1}{2}\pi - c' = \frac{1}{2}\pi - B \text{ and } \frac{1}{2}\pi - B' = a.$$

The triangle  $A'B'C'$  has therefore the same circular parts as  $ABC$ , moved one place round.

In the penultimate step of this process we had an isosceles triangle formed of  $A'B'C'$  and a symmetrically equal triangle with their sides  $a'$  in contact. By placing two such triangles with their sides  $b'$  in contact and repeating the processes, we obtain a third triangle having the same circular parts moved one place further round, and so on.

This method is not so elegant as the geometric one given in text-books on spherical trigonometry, but I would say that it is easier to remember, as I never find it possible to demonstrate the geometric method to a class without referring to the text-book. G. H. BRYAN.

**151. [P. 3. b.]** *Note on Successive Inversion.*

[The Apollonian circles of a triangle  $ABC$  are the three loci  $b.PB=c.PC, c.PC=a.PA$ , and  $a.PA=b.PB$ . They have for diameters the segments of the sides between the feet of the internal and external bisectors of opposite angles. They intersect in two real points, inverse to the circum-circle, lying on the line joining the circum-centre and symmedian point. The pedal triangles of these points are equilateral.]

Inverting the property that the medians of an equilateral triangle intersect at angles of  $120^\circ$ , it follows that the  $A$ -circles of a triangle are coaxal and cut each other at angles of  $120^\circ$ .

If any point  $P$  be taken on the sides of an equilateral triangle whose centroid is  $O$ , the concentric circle of radius  $OP$  will intersect the sides in five other points, symmetrically situated two on each side. No matter what their number or what their order, successive reflections of  $P$  with respect to the medians give these five points and no more.

Inverting; no matter what their number or what their order, successive inversions of a point  $P$  with respect to the  $A$ -circles of the triangle  $ABC$  give five new points and no more.

The formal proof of the following property presents no difficulty and need only be stated; the pedal triangle of any point is similar to the pedal