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LXII. *On the Measurement of the Progress of an Eclipse of the Moon with a Sextant or Reflecting Circle.* By T. E. BOWDICH.*

IT is impossible to observe the beginning of an eclipse of the sun or moon on ship-board with precision; but by measuring the progress of either with a sextant, at intervals of five minutes, advantage may still be taken of these phænomena for the determination of the longitude.

This method offers the great advantage of multiplying the angles, and consequently of diminishing the errors by which the partial observations may be affected.

It was first proposed, for eclipses of the sun, by Wales, who thus observed that of 1774; King, who accompanied Captain Cook in 1777, also availed himself of it; but in both instances the mere observations are recorded, without calculation, formula, or result.

Köhler appears to have been the first who recommended, and Humboldt the first who put in practice, the application of this method to eclipses of the moon; the latter thus determined the longitude of Ibagué, within one-fifth of a degree: but as Oltmans, who calculated this observation, has merely given us the result without the formula, and as I do not know of any formula being in print, I thought it might be useful as well as interesting to submit the following, which is general, until a neater one may be discovered.

Let Δ = longitude \mathcal{D} — long. \odot — 180°

λ = latitude of \mathcal{D}

Δ_1 = augmentation of the relative longitude of \mathcal{D}

λ_1 = movement in latitude of \mathcal{D}

d = demi-diameter of \odot

d' = do. . . . \mathcal{D}

p = parallax of \odot

p' = . . do. . . \mathcal{D}

ϵ = enlightened part of \mathcal{D} in minutes

* Communicated by P. BARTLOW, Esq., Royal Military Academy.

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t = the time before or after the instant of calculating the longitude of \mathfrak{D}

T = time for which the above elements (from the Naut. Almanac) are calculated

T' = mean time of the observation.

To determine the relative orbit of the \mathfrak{D} we have

$$\begin{array}{lll} x = ay + b & x = \Delta & y = \lambda \quad \Delta + a\lambda = b \\ & x = \Delta' & y = \lambda' \quad \Delta' + a\lambda' = b \end{array}$$

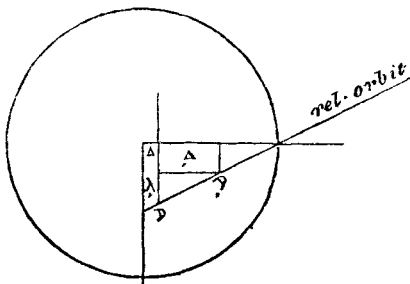
by which latter equations we may determine a and b ; but we need not recur to these equations, observing that when

$$t = 0 \quad x = \Delta \quad y = \lambda$$

$$t = 1 \quad x = \Delta + \Delta' \quad y = \lambda + \lambda'$$

$$t = \frac{1}{2} \quad x = \Delta + \frac{1}{2}\Delta' \quad y = \lambda + \frac{1}{2}\lambda'$$

whence, the general expression $x = \Delta + t\Delta' \quad y = \lambda + t\lambda'$, which gives us the value of x and y in time, and enables us to determine the place of the \mathfrak{D} at each instant.

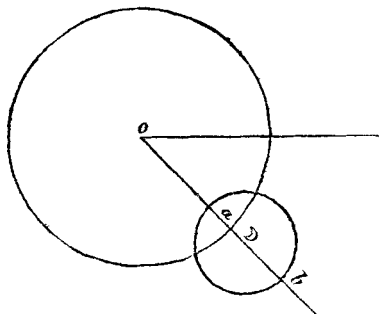


To determine the distance of the centre of the moon and cone of umbra at any moment, we have (calling the radius of the cone of umbra g)

$$D^2 = x^2 + y^2 = (\Delta^2 + \lambda^2) t^2 + 2(\Delta\Delta' + \lambda\lambda') t + \Delta'^2 + \lambda'^2$$

$$0 \mathfrak{D} = D \quad D + \mathfrak{D} b - g = e \quad D^2 = (e + g + d')^2$$

$$(e + g - d')^2 = (\Delta^2 + \lambda^2) t^2 + 2(\Delta\Delta' + \lambda\lambda') t + \Delta'^2 + \lambda'^2$$



To find the value of g we have

$$ST = \frac{TT'}{\tan p} \quad SS' = \frac{TT' \tan d}{\tan p} \quad TR = \frac{TT'}{\tan p'}$$

$$JT : JR :: TT' : RR' \quad JS : JT :: SS' : TT' \quad JS - JT : JT :: SS' - TT' : TT'$$

$$\frac{TT'}{\tan p} : JT :: \frac{TT' (\tan d - \tan p)}{\tan p} : TT' \quad JT = \frac{TT'}{\tan d - \tan p} \quad JR =$$

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The time will be given by the equation

$$t = \frac{-\Delta\Delta_1 - \lambda\lambda_1 + T}{\Delta_1^2 + \lambda_1^2}$$

in which t indicates the middle of the eclipse.

If we would have the beginning and the end of the eclipse, we make $\varepsilon = 2d'$, and the time of each will be given by the equations

$$t = \frac{-\Delta\Delta_1 - \lambda\lambda_1 \pm \sqrt{(\Delta\Delta_1 + \lambda\lambda_1)^2 + (\Delta_1^2 + \lambda_1^2)[(\tau + 2d')^2 - \Delta^2 - \lambda^2]}}{\Delta_1^2 + \lambda_1^2} + T$$

taking the sign $-$ for the beginning and $+$ for the end.

The duration of the eclipse will be given by

$$\frac{1}{2} D = \frac{\sqrt{(\Delta\Delta_1 + \lambda\lambda_1)^2 + (\Delta_1^2 + \lambda_1^2)[(\tau + 2d')^2 - \Delta^2 - \lambda^2]}}{\Delta_1^2 + \lambda_1^2}$$

If we desire the end of the immersion and the beginning of the emersion, we make $\varepsilon = 0$, and the times will be given by the equation

$$t = \frac{-\Delta\Delta_1 - \lambda\lambda_1 \pm \sqrt{(\Delta\Delta_1 + \lambda\lambda_1)^2 + (\Delta_1^2 + \lambda_1^2)(\tau^2 - \Delta^2 - \lambda^2)}}{\Delta_1^2 + \lambda_1^2} + T$$

being after the middle of the eclipse if we take the sign $+$ of the radical and before if we take $-$.

Let I represent the duration of the total immersion of the \mathcal{D} , *i. e.* the time during which she remains completely invisible, and we have

$$\frac{1}{2} I = \frac{\sqrt{(\Delta\Delta_1 + \lambda\lambda_1)^2 + (\Delta_1^2 + \lambda_1^2)(\tau^2 - \Delta^2 - \lambda^2)}}{\Delta_1^2 + \lambda_1^2}$$

Lastly: we have the hour to which any enlightened part ε corresponds, by making ε equal to this part, and deducting the corresponding value of t .

In all these equations we use or repeat nearly the same logarithms, which very much expedites the calculation.

Let us suppose that we have measured the chord of distance between the two horns of the moon, which seems to me to admit of more precision; we have only to make the following additions in the original expressions for the elements,

$c = \frac{1}{2}$ distance of the horns of \mathcal{D}

t' = mean time of the observation of c .

$\tau = \frac{61}{60} (p + p' - d) =$ radius of the section of the *conus umbræ*.

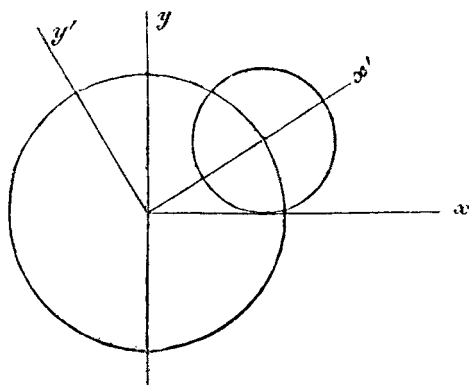
$$\alpha = \Delta t + \Delta \quad \beta = \lambda t + \lambda \quad y = ax \quad \alpha = \frac{y}{x} = \frac{\beta}{\alpha}$$

$$\tan i = a \sin i = \beta \cos i = a$$

$$y = \beta x' + ay' \quad x = ax' - \beta y' \quad (\text{Biot, Geom. Anal. No. 77.})$$

x'^2

$x'^2 + y'^2 = r'^2$ equ. of circle of *conus umbræ*. $y'^2 + (x' - D)^2 = d'^2$ equ. of circumf. of \mathfrak{D} refl. to 2d axes.



$$r'^2 - x'^2 + x'^2 - 2Dx' + D^2 = d'^2 \quad x' = \frac{D^2 + r'^2 - d'^2}{2D}$$

$$x'^2 = \frac{D^4 + r^4 + d'^4 + 2D^2r^2 - 2D^2d'^2 - 2r^2d'^2}{4D^2}$$

$$y'^2 = r'^2 - x'^2 = \frac{-D^4 - r^4 - d'^4 + 2D^2r^2 + 2D^2d'^2 + 2r^2d'^2}{4D^2}$$

making $y = e$ we have

$$4D^2c^2 - D^4 - 2D^2\tau^2 - 2D^2d'^2 = -\tau^4 - d'^4 + 2\tau^2d'^2$$

$$D^4 + 2D^2(2c^2 - \tau^2 - d'^2) = -\tau^4 - d'^4 + 2\tau^2d'^2$$

$$D^2 = -(2c^2 - \tau^2 - d'^2) \pm \sqrt{4c^4 + r^4 + d'^2 - 4c^2\tau^2 - 4c^2d'^2 + 4d'^2\tau^2}$$

$$D^2 = -(c + \tau)(c - \tau) - (c - d')(c + d') \pm 2\sqrt{c^4 - c^2\tau^2 - c^2d'^2 + d'^2\tau^2}$$

$$c^4 - c^2\tau^2 - c^2d'^2 + d'^2\tau^2 = c^2(c^2 - d'^2) - \tau^2(c^2 + d'^2) = (c^2 - \tau^2)(c^2 - d'^2)$$

$$= (\tau + c)(\tau - c)(d' + c)(d' - c)$$

$$D^2 = (\tau + c)(\tau - c) + (d' + c)(d' - c) \pm \sqrt{(\tau + c)(\tau - c)(d' + c)(d' - c)}$$

$$A' = (\tau + c)(\tau - c), \quad B' = (d' + c)(d' - c), \quad D^2 = A' + B' \pm 2\sqrt{A'B'}$$

$$D^2 = \alpha^2 + \beta^2 = (\Delta_i^2 + \lambda_i^2)t^2 + 2(\Delta\Delta_i + \lambda\lambda_i)t + \Delta^2 + \lambda^2$$

whence (making $-\Delta\Delta_i - \lambda\lambda_i = A' \quad \Delta_i^2 + \lambda_i^2 = B$

$$t^2 - \frac{2At}{B} = \frac{D^2 - \Delta^2 - \lambda^2}{B}$$

$$t = \frac{A}{B} \pm \sqrt{\frac{A^2}{B^2} + \frac{D^2 - \Delta^2 - \lambda^2}{B}}$$

The time of the middle of the eclipse is $= T + \frac{A}{B}$

The four roots of the following equation

$$t = T + \frac{A}{B} + \sqrt{\left(\frac{A}{B}\right)^2 + \frac{\tau^2 + d'^2 - \Delta - \lambda^2 \pm 2\tau d'}{B}}$$

$t =$

$$t = T + \frac{A}{B} \pm \sqrt{\left(\frac{A}{B}\right)^2 + \frac{(\tau \pm d')^2 - \Delta^2 - \lambda^2}{B}}, \quad (c \text{ being } = 0)$$

gives the beginning of the eclipse,
the end of the immersion,
the beginning of the emersion,
the end of the eclipse.

$$\text{The duration of the eclipse is } 2 \sqrt{\left(\frac{A}{B}\right)^2 + \frac{(\tau \pm d')^2 - \Delta^2 - \lambda^2}{B}}$$

The shortest distance between the centre of the moon and of the section of the *conus umbræ* (occurring when t is equal to the time of the middle of the eclipse, or $t = T + \frac{A}{B}$) will be given by the value of D derived from the equation

$$\left(\frac{A}{B}\right)^2 + \frac{D^2 - \Delta^2 - \lambda^2}{B} = 0 \quad D = \sqrt{\Delta^2 + \lambda^2 - \frac{A^2}{B}}$$

Lastly: any enlightened quantity of the moon or any distance of the horns will be given by the formula

$$t = T + \frac{A}{B} \pm \sqrt{\left(\frac{A}{B}\right)^2 + \frac{D^2 - \Delta^2 - \lambda^2}{B}}$$

observing that in the former case $D = \tau - d' + \epsilon$.

$$\text{in the latter, } D^2 = A' + B' \pm 2\sqrt{A'B'}$$

$$A' = (\tau + c)(\tau - c) \quad B' = (d' + c)(d' - c)$$

Thus we may have the time t expressed in function of c or in function of ϵ . The longitude of the place will be expressed in time by the formula $L = t - t'$, in which t' represents the time of the observations of ϵ or c ; the longitude being east or west according as L is positive or negative. Finally; substituting the value of t , we have the longitude expressed in the time by the formula

$$\text{Long.} = T - t' + \left(\frac{A}{B}\right) \pm \sqrt{\left(\frac{A}{B}\right)^2 + \frac{D^2 - \Delta^2 - \lambda^2}{B}}.$$

I have calculated M. de Humboldt's observation at Ibaguë* by this formula, and the result would no doubt accord precisely with that in the text, were the elements it contains free from errors; for, after correcting the most palpable, my result differs but 27" from that of M. Oltmans. The following errors cannot be disputed, and other lesser ones certainly exist.

It is impossible that 21^h 20' 45" at Paris, can be the mean time, since the elements are calculated very near the opposition, and this happened, according to the text, at 19° 26' 41". If we suppose for a moment that this latter element is inexact, we may still convince ourselves that 21^h 20' 45" cannot be the

* *Voyage de Humboldt, Astronomie*, (2 vols. 4to.) vol. ii. p. 255.

correct time, by merely observing that the enlightened part of the moon $23^{\circ} 30''$ must be near the end of the eclipse, since both the time and the quantity of the enlightened part of the moon continue augmenting; nevertheless, the time of this measurement is $21^{\text{h}} 0' 13'',9$, *i. e.* less than $21^{\text{h}} 20' 45''$, which is evidently absurd. This error of the text leaves no other alternative than to deduct the time, for which the observation is calculated, from that of the full moon, which gives us $19^{\text{h}} 27' 28'',8$. Another evident error of the text is detected as follows:

Mean time at Paris $21^{\text{h}} 0' 13'',9$ according to the text.
 Ibage $15 \ 50 \ 54,9$

Longitude in time	<u>5</u>	<u>9</u>	<u>19</u>
instead of	5	9	39 given in the text.

Error	<u>20''</u>
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LXIII. *Reply to Captain FORMAN on the Theory of the Tides.*
By Mr. HENRY RUSSELL.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,—**A**S I happen to have a particular veneration for old theories when I consider new ones incapable of exploding them, I shall, with your permission, reply to Captain Forman's communication of last month, in which he so warmly advocates the expansive theory of the tides.

Surely it does not follow, because we admit the expansion of water, that therefore we must, without consideration, relinquish our former theories of the tides! No. Let us call for the assistance of expansion when we can no longer do without it, and reject our present opinions when we discover their retention unnecessary, taking care, as far as we are able, to admit no more causes than are necessarily required to produce the effects.

As Captain Forman has proposed a box of marbles for my instruction, I shall propose for him a better way of trying the experiment. Fill the box at one end with wheat and the other with bran; of course we must consider the wheat the ebb, and the bran the flow: on agitating the box, the wheat will descend, and consequently the bran will be elevated.

Captain Forman next mentions an immense syphon, the existence of which of course I shall not argue; but, those who
 are