



# LXIII. On Mr. Dalby's method of finding the difference of longitude between two points of a geodetical line on a spheroid, from the latitude of those points and the azimuths of the geodetical line at the same

Dr. Tiarks F.R.S.

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LXII. *A Letter from Professor Airy in reply to Mr. Galbraith's Remarks (p. 182.) on some late Computations of the Earth's Ellipticity.*

*To the Editors of the Philosophical Magazine and Annals.*  
Gentlemen,

IN an article which appears in your Journal for September last, Mr. Galbraith expresses himself much astonished at the difference between the value of the earth's ellipticity which he has obtained, and a result at which I had arrived. I think that any person who reads my paper on this subject will see the ground on which such a difference might have been looked for; but for those who do not, a single line may serve to point out the state of the case. Mr. Galbraith's calculations proceed on the assumption that the earth is known to be an ellipsoid: mine, on the supposition that this is not known. It is manifest that to satisfy the observed curvatures in different places we shall have different proportions of the axes, accordingly as the meridian is supposed to be an ellipse, or to be some other figure.

I am by no means disposed to consider his hypothesis to be better founded than mine: but more pressing employments compel me at present to abstain from the discussion of this "much-vexed" topic, to which he so obligingly invites me.

I am, Gentlemen, yours, &c.

Observatory, Cambridge.  
Oct. 17, 1828.

G. B. AIRY.

LXIII. *On Mr. Dalby's Method of finding the Difference of Longitude between two Points of a Geodetical Line on a Spheroid, from the Latitude of those Points and the Azimuths of the geodetical Line at the same.* By Dr. TARKS, F.R.S. &c.

THE ingenious method first suggested by Mr. Dalby, of deducing the difference of longitude between any two points on a spheroid, from the latitude of these points and the inclination of the geodetical line connecting them to their meridians at these points, is founded on a remarkable property of spheroidal triangles formed by geodetical lines, which may generally be thus enunciated: The sum of the three angles of any spheroidal triangle formed by geodetical lines is a function of the latitudes of the angular points and their differences of longitude only, and is altogether independent of the eccentricity of the spheroid. This sum, accordingly, is the same as  
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the sum of the three angles of a spherical triangle whose angular points have the same relative situation to a particular diameter, which is considered as the polar axis; that is to say, the same latitudes respectively, and the same differences in longitude. The particular case of this general proposition which is employed by Mr. Dalby, is the one in which two of the geodetical lines are meridians, and where consequently one of the angular points is the pole of the spheroid itself: but it will be easily seen, that from the demonstration of the particular case the truth of the general proposition may be immediately derived. The method used in the Trigonometrical Survey requires, that if two points on a spheroid having respectively a certain latitude on meridians forming a certain angle are connected by a geodetical line, the sum of the angles of this line with the meridians of the points should be the same, whatever the ellipticity of the meridians may be; and, accordingly, that it should be equal to the two angles of the spherical triangle the sides of which are the co-latitudes of the points, and the inclosed angle the inclination of the two meridians. The inclination of the meridians of the spheroid or their difference of longitude is then derived from the two sides and the sum of the angles opposite to them: viz. the co-latitudes, and the sum of the azimuths. Mr. Dalby's method was first published by General Roy in the *Phil. Trans.* for 1790, in his own words and with his own demonstration. It would appear that this demonstration has not given general satisfaction; for I have observed that the want of success in the application of the method which is, indeed, acknowledged on all hands, has sometimes at least partially been ascribed to its incorrectness; whereas the principle on which it is founded is not only perfectly correct, but neither limited by the length nor the position of the geodetical line to which it is applied. Before I had seen Mr. Dalby's demonstration, I had convinced myself of the correctness of the method, with which I became acquainted through that part of the *Trigonometrical Survey* published in the *Phil. Trans.* for 1795, by a demonstration which, although perhaps substantially the same as Mr. Dalby's, yet differs in some respects from it. I hope that this demonstration will not be considered as perfectly useless at the present moment, and I shall add to it a few remarks on the cause of the failure of the practical application which has hitherto been made of this method.

Let 1. be the great semiaxis of the oblate spheroid or the radius of the equator.

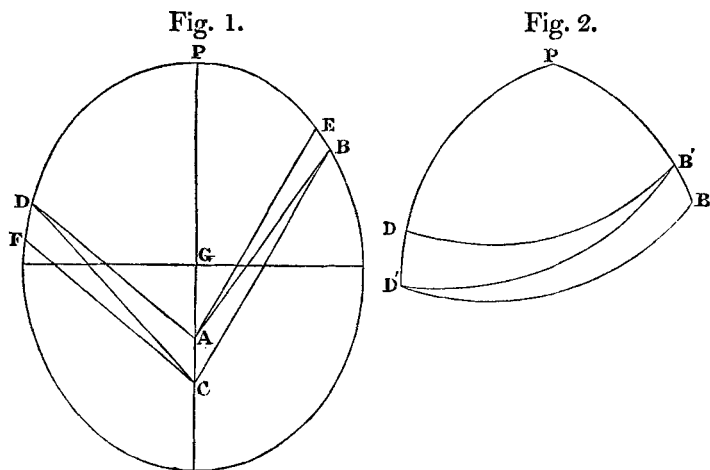
$e$  the excentricity of the elliptical meridians.

$\omega$  the

- $\omega$  the angle formed by the planes of the meridians of the two points on the spheroid B and D. (fig. 1.)  
 $\lambda, \lambda'$  their latitudes; and let  $\lambda$  be greater than  $\lambda'$ .  
 $l, l'$  the reduced latitudes, or the angles dependent on  $\lambda, \lambda'$  by this equation  $\text{tang. } l = \sqrt{(1-e^2)} \text{ tang. } \lambda$ .  
 $m, m'$  the angles formed at the intersection of the geodetical line and the meridians between the former and each of the latter.  
 $\mu, \mu'$  the angles in a spherical triangle, two sides of which are  $90^\circ - \lambda$  and  $90^\circ - \lambda'$ , and angle between them  $= \omega$ ;  $\mu'$  being opposite to the side  $90^\circ - \lambda$ , and  $\mu$  opposite to the side  $90^\circ - \lambda'$ , and  
 $\beta$  the third side of the spherical triangle opposite to  $\omega$ .

Let PDF and PEB represent the two meridians whose inclination to each other is  $= \omega$ , and let the lines BC and DA be perpendicular to the meridians at B and D. Draw AE parallel to CB and CF to AD, and join A and B and C and D by straight lines. Let CB be  $= r$ , and AD  $= r'$ ; the angle EAB = ABC =  $y$ , and the angle FCD = CDA =  $x$ .

It is then clear, that the inclination of the plane BCD to that of the meridian PEB is  $= m$ , and that the inclination of the plane BCF to the plane of the same meridian, is  $= \mu$ , and that  $\mu > m$ . If we now assume that the three lines CB, CP, CF determine on a sphere described about C as a centre, the angular points of the spherical triangle PB'D' (fig. 2.), and



that on the arc PD', DD' is made equal to  $x$ , we shall have

have  $PB' = 90^\circ - \lambda$ ,  $PD' = 90^\circ - \lambda'$ ,  $PB'D' = \mu$ ,  $PB'D = m$ ,  $PD'B' = \mu'$ ,  $D'B' = \beta$ . We have, therefore,

$$\begin{aligned}\sin D : \cos \lambda &= \sin m : \cos (\lambda' + x), \text{ and} \\ \sin \beta : \sin D &= \sin x : \sin (\mu - m), \text{ hence}\end{aligned}$$

$$\sin (\mu - m) = \frac{\cos \lambda \cdot \sin m}{\sin \beta} \cdot \frac{\sin x}{\cos (\lambda' + x)}.$$

In the same manner it will be seen that the angle of the geodetical line with the meridian of D is the inclination of the planes PAD and DAB =  $m'$ , and that the corresponding  $\mu'$  is the inclination of the plane DAE to that of the meridian PAD. If we, therefore, assume a sphere whose centre is A, the lines AE, AD and AP will determine on that sphere the angular points of the triangle PD'B' (fig. 2.); and if we produce PB' to B so that BB' =  $y$ , we shall have  $PD'B' = \mu'$ ,  $PD'B = m'$ , and consequently,

$$\begin{aligned}\sin B : \cos \lambda' &= \sin m' : \cos (\lambda - y), \text{ and} \\ \sin \beta : \sin B &= \sin y : \sin (m' - \mu'), \text{ hence,}\end{aligned}$$

$$\sin (m' - \mu') = \frac{\cos \lambda' \cdot \sin m'}{\sin \beta} \cdot \frac{\sin y}{\cos (\lambda - y)}.$$

From the triangles ACB and CAD (fig. 1.) we shall easily derive  $\frac{\sin x}{\cos (\lambda + x)} = \frac{AC}{r'}$ , and  $\frac{\sin y}{\cos (\lambda - y)} = \frac{AC}{r}$ , and as AC

$$= GC - AG = \frac{e^2 \sin \lambda}{\sqrt{(1 - e^2 \sin \lambda^2)}} - \frac{e^2 \sin \lambda'}{\sqrt{(1 - e^2 \sin \lambda'^2)}},$$

$$\text{or nearly} = \frac{e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda')}{\sqrt{(1 - e^2 \sin \lambda^2)} \cdot \sqrt{(1 - e^2 \sin \lambda'^2)}}, \text{ and}$$

$$r = \frac{1}{\sqrt{(1 - e^2 \sin \lambda^2)}}, \quad r' = \frac{1}{\sqrt{(1 - e^2 \sin \lambda'^2)}}, \text{ we have}$$

$$\frac{AC}{r} = \frac{e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda')}{\sqrt{(1 - e^2 \sin \lambda^2)}}, \text{ and}$$

$$\frac{AC}{r'} = \frac{e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda')}{\sqrt{(1 - e^2 \sin \lambda'^2)}}.$$

Substituting these values of  $\frac{AC}{r'}$  and  $\frac{AC}{r}$  for  $\frac{\sin x}{\cos (\lambda + x)}$ , and  $\frac{\sin y}{\cos (\lambda - y)}$  in the above equations, we obtain

$$\begin{aligned}\sin (\mu - m) &= \frac{\cos \lambda \cdot \sin m \cdot e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda')}{\sin \beta \cdot \sqrt{(1 - e^2 \sin \lambda^2)}} \\ &= \frac{\cos \lambda \cdot \sin m}{\sin \beta} \cdot e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda') \quad \text{and} \\ &\quad \sin\end{aligned}$$

$$\sin (m'-\mu') = \frac{\cos \lambda' \cdot \sin m'}{\sin \beta} \cdot \frac{e^2 (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda')}{\sqrt{(1-e^2 \sin \lambda'^2)}}$$

$$= \frac{\cos \lambda' \cdot \sin m'}{\sin \beta} e^2 \cdot (\sin \lambda - \sin \lambda') (1 + \frac{e^2}{2} \sin \lambda \cdot \sin \lambda').$$

But by the characteristic property of the geodetical line we have  $\cos l \sin m = \cos l' \sin m'$ ; and it is therefore evident that  $\mu - m = m' - \mu'$  or  $m + m' = \mu + \mu'$ , and the sum of the three angles  $\omega + m + m' = \omega + \mu + \mu'$ , independent of the ellipticity of the meridians. If we now conceive any three points on the spheroid connected by geodetical lines, and draw their meridians, the comparison of the angles of the three triangles thus formed, each by two meridians and a geodetical line, with the analogous ones on the sphere, will immediately prove the general proposition.

*Example.*—Professor Bessel has accurately deduced from the latitude of the observatory at Seeberg =  $50^\circ 56' 6''.7$  ( $\lambda'$ ), the length of the geodetical line from Seeberg to Dunkirk (whose logarithm = 5.47830314) and the inclination of that line to the meridian of Seeberg =  $85^\circ 38' 56''.82$  ( $m'$ ) (the last two as resulting from General Müffling's great measurement), the following results, supposing  $le = 8.9054355$  and log. semi-polar-axis = 6.51335464:

Latitude of Dunkirk .....	= $51^\circ 2' 12''.719$ ( $\lambda$ )
Inclination of the geodetical line to the meridian of Dunkirk.....	} = $87 51 15.523$ ( $m$ )
Difference of longitude between Seeberg and Dunkirk.....	
	} $8 21 19.04$ ( $\omega$ )

These quantities, therefore, certainly belong to the same spheroid, whether right or wrong as to the places named. I find from  $\lambda$ ,  $\lambda'$  and  $m + m'$  in the spherical triangle

$$\mu = 87^\circ 51' 25''.78$$

$$\mu' = 85 38 46.61$$

$$\beta = 5 15 28.44$$

$$\omega = 8 21 19.09. \quad \text{And,}$$

$$\text{next: } 2e^2 \cdot \frac{\cos \lambda' \cdot \sin m}{\sin \beta} \sin \frac{\lambda - \lambda'}{2} \cdot \cos \frac{\lambda + \lambda'}{2} \left( 1 + \frac{e^2}{2} \sin \lambda \sin \lambda' \right)$$

=  $10''.26$  and  $\mu - m = 10''.257$  as it ought to be. The equation for  $\sin (\mu - m)$  is a complete check upon geodetical calculations of this nature; for if the parts used do not strictly belong to the same spheroid, the difference between the calculated value of  $\mu$  and the given value of  $m$  will vary considerably from the value of the same difference by the formula.

The operations of the Trigonometrical Survey alluded to may be thus represented.—The quantities  $\lambda$ ,  $\lambda'$ ,  $m$ ,  $m'$  referring to Beachy Head and Dunnose, were assumed to be exactly known, and

and  $\omega$  was correctly derived from these quantities without the employment of any other quantity. But in the assumption of the correctness of these quantities, and by the use made of them in the further calculations, the ellipticity of the spheroid to which these quantities belong was implied. This is disguised by the introduction of the length of the geodetical line, but the same ellipticity may be obtained as accurately without this line. From the equation of the geodetical line it is easily proved, that making  $\tan \psi = \left( \frac{\sin m}{\cos \lambda'} \right)^2$  and  $\tan \psi' = \left( \frac{\sin m'}{\cos \lambda} \right)^2$

$$\text{we have } \frac{e^2}{1-e^2} = \frac{\sin(\psi' - \psi)}{\cos \psi \cdot \cos \psi' \cdot \sin(m - m') \sin(m' + m')}.$$

Now the calculations of the Survey lead to the value  $e$  contained in this equation, and, as conducted, could not possibly lead to any other. Not having logarithmic tables to more than seven decimals at hand, I cannot determine the angles  $\psi'$  and  $\psi$  as accurately as it might here be required. I find from the data given in the Survey the following quantities;  $\psi = 67^\circ 46' 45''.37$ ,  $\psi' = 67^\circ 46' 47''.37$ ,  $\log e^2 = 8.1252235$ , and the ellipticity about  $\frac{1}{149.4}$ , nearly the same as in the Survey. Next in the spherical triangle  $\omega = 1^\circ 26' 47''.93$  as in the Survey,  $\mu = 96^\circ 58' 23''.27$ ,  $\mu' = 81^\circ 54' 27''.82$ ,  $\beta = 0^\circ 55' 28''.52$ , consequently  $\mu - m = 2' 25''.27$ ; and as a proof that these values are correct, and that the quantities of the Survey belong to the spheroid here deduced, it will be found that  $2e^2 \sin \frac{\lambda - \lambda'}{2} \cdot \cos \frac{\lambda + \lambda'}{2} \sin m \cdot \cos l \cdot \left( 1 + \frac{e^2}{2} \sin \lambda \sin \lambda' \right) = 2' 24''.78$ . The small difference  $0''.49$  arising from the imperfect determination of  $(\psi' - \psi)$  above stated, shows that the excentricity is a little greater than the one I arrived at; namely, about  $\frac{1}{148.9}$  ( $\mu - m$  being nearly proportional to the excentricity).

It will, therefore, be clear that the values used in the Trigonometrical Survey belong to a spheroid of an ellipticity equal to about  $\frac{1}{149}$  and to no other, and that the corresponding difference of longitude was correctly derived. The employment of the geodetical line gave only the *linear* dimension of the spheroid, the figure of which was determined without it. But it will be seen how much this figure will change by a slight alteration of the data used for finding it. As  $e^2$  is about  $\frac{1}{75}$ , the numerator of the fraction  $\frac{e^2}{1-e^2}$  is to its denominator nearly as

1: 74, and the change in the value of  $\psi' - \psi$  will, therefore, principally determine the value of  $e^2$ . Now the logarithmic tables will show, that a change of  $1''$  in  $m'$  and  $\lambda$  will produce a change in the logarithmic tangent  $\psi'$  (to seven decimals) of 6 and 51.6 and an equal variation of  $m$  and  $\lambda'$  will change the logarithmic tang  $\psi$  5.2 and 51.2, and a change of 60.2 in the logarithms of tang  $\psi$  and tang  $\psi'$  will change these angles  $1''$ . The difference of  $\psi$  and  $\psi'$  is  $2''\cdot00..$  and therefore a change of  $1''$  in the value of  $\psi' - \psi$  will reduce the ellipticity to one half or increase it by one half of its value; that is to say, will change it from  $\frac{1}{149}$  to  $\frac{1}{298}$  or to  $\frac{1}{99}$ . This circumstance is no doubt one of the principal causes of the failure of the method in its application to small geodetical lines; and however correct in theory, such an application of it must clearly always lead to erroneous results.

If we change the conditions of the problem, and assume the ellipticity of our spheroid or the length of any one of the axes, the length of the geodetical line together with  $\lambda$  and  $m$  of one place will give those of the other place their difference of longitude, and the linear dimensions of the spheroid very nearly correct, as has been satisfactorily proved by Mr. Ivory.

There can be little doubt at present that the difference of longitude between Beachy Head and Dunnose, does not much differ from  $1^\circ 27' 5''$ ; and this proves, as Professor Airy has correctly observed, that there is an error of about  $13''$  in the sum of the azimuths. A new determination of the azimuths at these places would certainly be desirable, and might lead to a decision of the question, whether local attraction has had any effect in producing these erroneous measurements.

Oct. 13, 1828.

J. L. TIARKS.

#### LXIV. Notices respecting New Books.

*Elements of Algebra: being a short and practical Introduction to that useful Science; on a new Plan; including a Simplification of the Rule for the Solution of Equations of all Dimensions.* By ROBERT WALLACE, A.M. late Andersonian Professor of Mathematics, Glasgow. London, 1828; 8vo: pp. 60.

WE extract from this work the table of contents, and the simplified rule for solving equations of all dimensions; the latter involves some interesting particulars respecting part of the modern history of Algebra.

Contents: Definitions—Characters or symbols of operation—Less common symbols of operation—Terms—Equations—General Rule to obtain an equation—Axioms—Addition—Equations to be resolved by