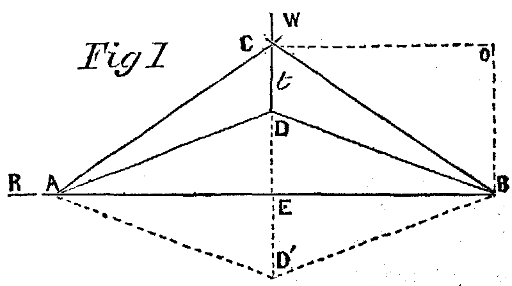
a straight line; and upon a curve of 600 feet radius, it was equal to the gravity on an inclined plane of 1 in 150. Approximately, the resistance due to the curvature might be found, either by dividing the number of lbs. in a ton by 0·224 of the radius of the curve, or by multiplying the deflection angle of the curve by 1·75, to give the resistance in lbs. per ton of train. It should be stated, that the gauge of the line was 5 ft. 6 ins. and that the proper super-elevation had been given to the outer rail on curves.

---

For the Journal of the Franklin Institute.

## *Solutions of a Problem of the Rafters.* By S. W. Robinson, C. E., Detroit, Mich.

I offer the following for publication for the interest manifested in consequence of the numerous solutions by the "triangle of forces," by "moments," by "virtual velocities," or by "the shearing stress." I have met with scarcely another problem whose solution by either of these principles can be made with so nearly equal facility.




Let AC, CB represent the rafters AD, DB or AD′, D′B sub-rafters, or rods joined at their intersection D or D′ with the intersection at C by a tie or strut DC or D′C. And let W be a force acting at C tending to depress C, causing a thrust at A and B, tending to straighten AD, DB or AD′, D′B causing a stress upon DC, or D′C.

*Required the amount of stress t upon* DC, *or* D′C.

When the force W is acting at C, and the combination is in equilibrium, there results a force $t$ acting at C. There is a horizontal thrust at A and B caused by the force at C, which is resisted by an equal and opposite force in AD, and DB. Call this force R.

Now as the forces in any triangle of forces are proportional to the sides of the triangle respectively, we will have from the triangle

AEC, AE : EC=R : (W±$t$) whence $R=(W\pm t)\frac{AE}{EC}$.

Also from the triangle AED, AE : ED=R : $t$, whence

$$R=\frac{AE}{ED}\,t \text{ or } \frac{AE}{ED'}\,t=(W\pm t)\frac{AE}{EC},=W\frac{AE}{EC}\pm t\frac{AE}{EC}$$

by equating; from which

$$t\left(\frac{1}{\text{ED or ED}'} \mp \frac{1}{\text{EC}}\right) = \frac{\text{W}}{\text{EC}};$$

whence $\frac{t}{\text{EC}}\left(\text{EC} \mp \text{ED or ED}'\right) = \frac{t}{\text{EC}}\left(\text{CD, or CD}'\right) = \frac{\text{W}}{\text{EC}}(\text{ED or ED}')$

$$\therefore\ t = \text{W}\frac{\text{ED}}{\text{CD}} \text{ or W}\frac{\text{ED}'}{\text{CD}'}.$$

To make the solution by moments, draw CO and OB parallel to EB and EC.

Then $\quad \text{R}\,.\,(\text{BO}=\text{EC}) = (\text{W} \pm t)\,(\text{CO}=\text{EB})$

and $\quad t\,.\,\text{EB}=\text{R}\,.\,(\text{ED or ED}')$

or $\quad t\,.\,\text{EB}=(\text{W}+t)\frac{\text{EB}\,.\,\text{ED}}{\text{EC}}$

from which $\quad t=\text{W}\,\frac{\text{ED}}{\text{CD}}.$

To make the solution by the principle of virtual velocities, we have from the triangle AEC

$$\overline{\text{AE}}^2+\overline{\text{EC}}^2=\overline{\text{AC}}^2,$$

And from the triangle AED

$$\overline{\text{AE}}^2+\overline{\text{ED}}^2=\overline{\text{AD}}^2.$$

Now if C be depressed through an element of space, A will move horizontally through an element of space in which case AE, EC, and ED become variable, while AC and AD remain constant. Therefore we may differentiate the two equations above and obtain

$$2\,\text{AE}d\,(\text{AE}) + 2\,\text{EC}d\,(\text{EC}) = 0 \text{ and } 2\,\text{AE}d\,(\text{AE}) + 2\,\text{ED}d\,(\text{ED}) = 0.$$

Eliminating $\text{AE}d\,(\text{AE})$ gives

$$\frac{d\,(\text{EC})}{d\,(\text{ED})}=\frac{\text{ED}}{\text{EC}}.$$

The element of space $d\,(\text{EC})$ may be regarded as that through which C passes; and $d\,(\text{ED})$ the element through which D passes. Hence each force multiplied into its displacement gives

$$(\text{W}+t)\,.\,d\,(\text{EC}) = t\,.\,d\,(\text{ED}),$$

or $\quad (\text{W}+t)\frac{d\,(\text{EC})}{d\,(\text{ED})}=t.$

Substituting for the ratio of the infinitessimals its value above, then

$$(\text{W}+t)\frac{\text{ED}}{\text{EC}}=t,$$

from which $\quad t=\text{W}\frac{\text{ED}}{\text{CD}}.$

To make the solution be means of the principle of shearing stress. The shearing stress of a beam with reference to any section, is the amount of force which tends to move one part of the beam over the other, every particle of the beam moving in a direction parallel to this section.

Therefore if the force in AC be P and that in AD be P′, the vertical shearing stress of AC will be

P sin EAC; and as this is the total vertical resistance of AC; it must be equal to the total vertical force acting upon it;

Hence $$P \sin EAC = W + t.$$

Similarly the vertical shearing of AD is

$$P' \sin EAD = t.$$

Taking the horizontal shearing we have

$$P \cos EAC = R, \text{ and } P' \cos EAD = R.$$

Reducing we get

$$R \operatorname{tang} EAC = W + t, \text{ and } R \operatorname{tang} EAD = t,$$

or $$t \frac{\operatorname{tang} EAC}{\operatorname{tang} EAD} = W + t.$$

But $\operatorname{tang} EAC = \frac{EC}{EA}$, and $\operatorname{tang} EAD = \frac{ED}{EA}$.

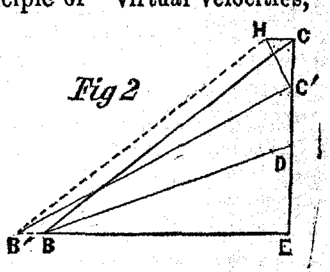
$$\therefore \quad t \frac{EC}{ED} = W + t, \text{ or } t = W \frac{ED}{CD}.$$

Other solutions might be made by some of these four principles. Thus, by the triangle of forces, taking AE to represent R, and EC to represent $(W+t)$, and ED to represent $t$, we would obtain

$$R \operatorname{tang} EAC = (W+t), \text{ and } R \operatorname{tang} EAD = t,$$

which is reduced similarly as the last.

To make a second solution by the principle of "virtual velocities," let C move through an element of space to C′ Fig. 2. At the same time B will move to B′. Draw through B′, B′H parallel and equal to BC′, then HC=B′B and the triangle HCC′ is similar to CEB, which gives BE : EC=C′C : CH=C′C : BB′




whence $\frac{BB'}{CC'} = \frac{EC}{BE}$.

By the principle of "virtual velocities,"

$$R \cdot BB' = (W+t) \cdot CC',$$

or $$R \frac{EC}{BE} = W + t.$$

Similarly we would get from BED

$$R \frac{ED}{BE} = t,$$

which may be reduced as similar expressions above.

If in the result for $t$ we make ED=0, $t$=0. If we make CD=0, $t=\infty$.

If we multiply through by CD and make CD $=\infty$, then ED$=\infty$. Similar conclusions would be drawn from the figure.

If AE and EB, Fig. 1, be taken unequal, we get a similar expression for $t$.

---

## MECHANICS, PHYSICS, AND CHEMISTRY.

---

*The Macrograph.* By Wm. B. Morgan, M. A., Prof. of Math. and Ast. in Earlham College, Richmond, Ind.

To the Editor of the Journal of the Franklin Institute.

The following article is a copy of a thesis which was prepared by Mr. Morgan previous to his taking the degree of Civil Engineer at the University of Michigan, in June, 1863.

I offer it to you for publication on account of its originality, and the hope that it will induce further investigation. It may result in reducing the principles involved to some useful application.

D. V. Wood, Prof. Civ. Eng.

---

This instrument, being as yet without a name, the author of the following article has concluded to call it *The Macrograph*, from its property of describing long curves. (Gr. μακρος, *long*, and γραφω, *to write.*)

The Macrograph is the invention of Oliver Butler, Attorney at Law, Richmond, Indiana.

I. The instrument is represented by Fig. 1, Plate I. It has two cog-wheels, A and B—the latter moving the former—the *number* of teeth in the two being different.

Each wheel carries an arm, M M′, which may be shortened or lengthened at pleasure by means of adjusting screws. At the extremity of the arm, M, is a rotating pivot, P, with a binding screw and a mortise through which the rod R passes. The rod R is attached to the extremity of M′, and also passes through pivot P′, which may be placed any where upon it at pleasure, and fixed there by means of the binding screw. The pivot P′ rotates in the slider S. The latter may be placed any where upon the rod R. This latter rod has holes in it at H H′, &c., into which a pencil may be inserted.

Upon turning the crank of wheel B, while the instrument is in the position indicated, it will describe a curve or rather a series of curves, of which Fig. 2 is a specimen. By placing the pencil in other parts of the same, four figures may be made which differ so much from this specimen, that one would hardly suppose that they were governed by the same law.

It is evident from the construction of the instrument, that if the wheels contained the same number of cogs in each, with any particular arrangement of the pencil, rods, pivots, &c., the pencil must follow in the same path at successive revolutions of the wheels. This