



## LX. Memoir on a new and certain method of ascertaining the figure of the earth by means of occultations of the fixed stars. With notes and an appendix by Francis Baily

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LX. *Memoir on a new and certain Method of ascertaining the Figure of the Earth by means of Occultations of the fixed Stars.* By A. CAGNOLI. With Notes and an Appendix by FRANCIS BAILY\*.

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The following memoir originally appeared in the Transactions of the Italian Society (*Memorie di Matematica e di Fisica della Società Italiana*, Tom. vi. Verona 1792): but, although it has been so many years before the public, I do not find that the subject of it has been taken up by any persons either here, or on the continent. In fact, I believe that it is not generally known in this country: and it is with a view of drawing the attention of astronomers more closely to it, that I now present them with this translation. The acknowledged talents and abilities of the author must at all times ensure it an attentive perusal; but particularly at this period, when a more than ordinary degree of interest is excited towards the subject of the *true figure of the earth*. I am aware that there are some apparent difficulties in the method pointed out in the memoir, yet I am not without hope and expectation that they may be eventually overcome: and that the mode herein proposed may at least be brought *in aid* of the other methods at present adopted for determining that difficult problem.

To the memoir itself I have subjoined some notes connected with the subject; which, however, may be distinguished from those of the author by the addition of the letter B. And to the whole I have added an Appendix, wherein I have attempted to illustrate the views of the author; and ventured to propose such methods as may best tend to carry his object into execution.

Gray's Inn, May 20, 1819.

FRANCIS BAILY.

MEMOIR, &c.

§ 1. **T**HE truth of the Newtonian theory of universal gravitation is proved by the wonderful agreement in all the celestial phenomena, which have hitherto been submitted to the tests of calculation and of observation. There probably is not any astronomer of the present day who is not convinced that our planet is compressed, or flattened, at the poles, and protuberant at the equator. In fact, if we combine this theory of Newton with the hypothesis of the elliptical form of the earth, we shall find that the precession of the equinoxes and the nutation of the earth's axis are sufficiently accounted for. Other arguments indeed, in favour of the elliptical form, may be derived from the rotation of the earth, which is now generally admitted: and others again from the differences in the length of the pendulum vibrating seconds in different latitudes. Still, however, we want some theory of the internal density of the earth; which, it is feared, will ever remain hid from human investigation. But if, to the want of

\* The present memoir has been already printed, but not for sale. It is now reprinted with the polite permission of the author.

this

this essential knowledge, we add the irregularity of the figure of the earth, as shown by the measurements of the degrees on its surface ; if, to the doubts which some persons may still entertain of the truth of the compression of the poles\*, we add the discordant opinions as to the quantity of that compression ; we may surely assert that it will be no small advantage to point out a method (independent of all hypothesis) for determining, with facility and with the greatest precision, the differences between the terrestrial radii at an indefinite number of points on the earth's surface.—Such indeed is the object of the present memoir.

§ 2. It has always been said, when speaking of the compression or flattening of the poles of the earth, that the *parallax of the moon* would afford the best means of ascertaining it ; provided the variations, arising therefrom, were of sufficient magnitude to be observed with perfect accuracy. But, since, by supposing this compression not to exceed  $\frac{1}{3000}$  of the semidiameter (which is the most received opinion at the present day) there would be a difference of not more than 9'' between the moon's parallax at the equator, and her parallax at a high latitude, for instance 60° ; so indeed it is but too true that a difference so small might be easily concealed under the possible errors of observation. It was on this account that Manfredi and Maupertuis in vain suggested the determination of the compression by direct observations of the moon's parallax : nor has any advantage been hitherto obtained by this method.

§ 3. But this is not the only instance in which a quantity, extremely minute on one side, has been found to produce effects that are quite discernible on the other. The attention of the observer should be directed to those points where they are most sensible. This has been the object of my research ; and I hope not entirely without success. For, there are circumstances in which scarcely a *second* in the parallax may cause a difference of 15, 20, or 30 seconds of time, or even more, in the duration of the occultation of a star by the moon. But, occultations, more particularly when the immersions take place behind the dark limb of the moon, can easily be observed without committing an error of a single second of time † : so that opportunities exist not only of removing all doubt as to the reality of the compression of the poles, but also of such frequent occurrence that the gradation of the compression, (or variation of the curvature) at different lati-

\* See Lorgna "*Principj di Geografia &c.*" § 31.

† The *duration* of an occultation cannot probably be so well observed when the immersions take place behind the dark limb of the moon, as when they take place behind the illuminated side ; since the *instant of emersion* cannot be so well ascertained. B.

tudes, may be accurately distinguished. The importance of the subject induces me to treat it more fully in detail.

§ 4. I shall not take up the time of the reader in demonstrating that no hypothesis is required to deduce the variation of curvature from the observation of the parallax. For, since the variation of curvature is nothing more than the inequality of the terrestrial radii (and that is nothing else than the difference of the parallax), it is evident that finding by observation the variation of parallax at different latitudes is in fact finding the variation of curvature by immediate observation. One involves the other without any intermediate help. The only difficulty consists in freeing from uncertainty those observations which show the unequal parallax: and this is precisely the special character of the particular kind of occultations which I have in view. It is true that astronomers, in calculating occultations, have made the parallax vary in conformity with that quantity of compression which they have adopted as the true theory. Moreover, these variations do not in general produce differences which are discernible in the relative duration of the occultations observed in different latitudes. Those differences, which *are* discernible, arise only in certain circumstances, to which hitherto no particular attention has been paid. It is on this account that occultations have not, as yet, served at all to determine the amount or quantity of the compression of the polar axis.

§ 5. Before I proceed to describe what these favourable circumstances are, it will be proper, in order to appreciate their utility, to define the precise degree of accuracy of which the *computation* and *observation* of occultations are capable. With respect to the *computation*, it is evident that the apparent distance between the star and the moon cannot be accurately determined unless we know exactly not only the apparent place of the star, but also that of the moon and its diameter. These elements are obtained by calculating the phænomena from observations made in a place (the longitude of which is known) under circumstances where the alteration of the parallax, on account of the variation in the curvature of the earth, does not produce a perceptible difference in the duration of the occultation: conditions not uncommon, nor difficult to be obtained.

§ 6. Let us then suppose that the moon's diameter and the place of the star are well determined (in which elements the least uncertainty exists); and that any small errors, which may occur, be thrown on the place of the moon. Let us, moreover, endeavour to discover (by comparing the calculation with the observation) the errors of the tables in the longitude and latitude of the moon, mixed (as we have already stated) with the preceding errors.

errors\*. In the present state of astronomy I do not see any great danger to be apprehended from this union of the two errors, notwithstanding the delicacy of the investigation in question. At any rate we may arrive at a sufficient degree of accuracy by verifying the place of the star by a sufficient number of observations, and also the diameter of the moon by means of the telescope which is used for the occultations.

§ 7. Having corrected the errors of the lunar tables, we shall have accurate elements wherewith to represent the state of the heavens: and there can then be no doubt that we may arrive at great exactness in our calculation of the duration of an occultation (or of the moment of immersion and emersion) for the place in which we wish to ascertain the curvature of the earth: since the geographical longitude of the place has been determined by means of occultations under circumstances in which (and they are the greater part) the variation in the curvature is not perceptible.

§ 8. With respect to the accuracy of observations of this nature, I have already said that the *immersions* behind the dark limb are not subject to the least error. Those on the illuminated side may be liable to some slight uncertainty if the star be not of the first or second magnitude, or if the power of the telescope be too small. The *emersions* from the dark limb are in general to be depended upon: whilst those from the illuminated side are the most doubtful of all†. But, even supposing that the error, in the last-mentioned case, may amount to 8 or even 10 seconds of time, will it be sufficient to conceal the variation of curvature altogether in the circumstances I have contemplated, and where the difference that it will produce may be ten or twenty times greater than the presumed error? Can we, indeed, expect to obtain more certain observations‡? since a single observation

\* See the Memoir which obtained the prize from the Academy of Copenhagen: "*Méthode pour calculer les longitudes géographiques.*" *Vérone: chez Ramanzini.*

† It appears that, in some occultations, the immersions and emersions take place wholly on the dark side of the moon. See an account of the occultation of  $\beta$  Virginis observed by Mr. Troughton on May 22, 1801. *Connaissance des Temps, Année xiii.* page 324. Sometimes an occultation may be observed when the moon itself is not visible: as in the occultation of Venus on May 13, 1801, between 8 and 9 o'clock in the morning; the moon being then only a few hours old. *Ibid.* page 417. A favourable occultation may also occur during a total eclipse of the moon, when the immersions and emersions of stars of the 6th or 7th magnitude may be distinctly observed. *Ibid. Année ix.* page 335. B.

‡ I find it difficult here to give a faithful translation of the author's words: the original runs thus, "Senza che ci possiamo attenere alle osservazioni più sicure: ed una sola fase, &c. &c." B.

of this kind accurately made in the place, for which the variation of curvature is sought, is sufficient to discover it by means of the comparison of the *calculated* with the *observed* moment.

§ 9. I shall now point out what are, in fact, the circumstances in which the differences of the parallax between two latitudes may be shown in an undoubted manner: and, in order virtually to embrace all the cases, I shall take mean quantities in the elements which enter into this investigation. Let us therefore suppose, first, that the moon's apparent semidiameter is  $15'.45''$ , its equatorial parallax  $57'.40''$ , and its horary motion  $32'.56''.5$ : secondly, that the occultation is observed in north latitude  $60^\circ$ , in which parallel there are three celebrated observatories, viz. Petersburg, Stockholm and Upsal: lastly, that the apparent height of the moon is  $10^\circ$ . If there be no variation in the curvature of the earth's surface the parallax of height will be  $56'.47''.4^*$ : but, if the polar compression amount to  $\frac{1}{360}$  of the earth's radius, the same parallax will be  $56'.38''.9^\dagger$ . The parallax would therefore under these circumstances experience an alteration of  $8''.5$ ; a quantity certainly too small to be verified with accuracy by means of observations of the height of the moon. But, this slight alteration produces, in certain cases, effects that are very visible: a fact which escaped the observation of Maupertuis; who, it is true, speaks of occultations as a mean of discovering the flattening of the earth $\ddagger$ . But, he speaks of them generally; and so slightly as to class them with appulses, as being *equally* fit to determine it. Now, as appulses are far from being observable with such certainty, in regard to time, as the instantaneous disappearance and reappearance of stars in occultations, it is evident that Maupertuis can never have had in view the particular cases which I am about to point out, and which differ materially from ordinary ones.

\* Let  $p$  = the horizontal parallax of the moon; and  $h$  = the height of the moon: then  $\sigma = p \cdot \cos h$  = the parallax of height. B.

† Let  $a$  = the polar compression of the earth, supposed =  $\frac{1}{360}$ ;  $\lambda$  = the latitude of the place; and the radius of the equator equal to unity: then we shall have the length of any other terrestrial radius =  $(1 - a \cdot \sin^2 \lambda)$  nearly; which, being multiplied by  $\sigma$ , will give  $\varpi = \sigma (1 - a \cdot \sin^2 \lambda)$  = the parallax of height on the supposition that the earth is an oblate spheroid. Whence we

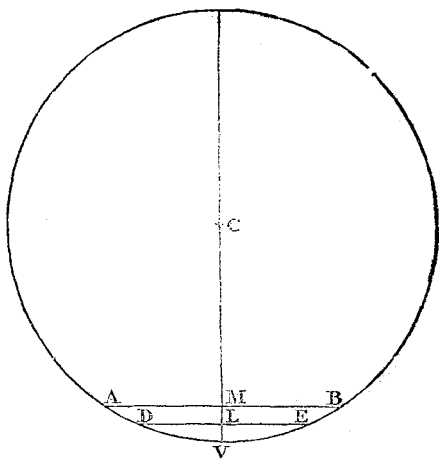
also have  $\frac{\varpi}{\sigma}$  = the length of the terrestrial radius at any given latitude.

And if  $\delta$  denote any given increase of the parallax, we shall have  $\frac{\delta}{\sigma}$  for the corresponding increase in the length of the earth's radius. B.

‡ *Préface au discours sur la parallaxe de la lune.*

§ 10. Let

§ 10. Let us suppose AVB a portion of the circumference of the moon's disc opposite to the observer; C the moon's centre; and that the radius CV, which divides the arc AVB into two equal parts, coincides with the vertical circle of the place of the observer. Let us further suppose that the line MV (or the ver-sine of the arc) is equal to  $60''$ ; and that, during the occultation, the chord AB of the moon is that which the star would apparently describe if the earth were spherical; or the chord DE that which it would apparently



describe if the axis of the earth were compressed  $\frac{1}{300}$ . Then ML will be equal to  $8''.5$  as before mentioned: and, computing from the mean rate of the horary motion of the moon, it will be found that the duration of the occultation behind AB will be  $20'.7''$  of time; and, behind DE,  $18'.41''^*$ . We therefore see how considerable is this difference of  $1'.26''$  of time; and how well such observations are adapted not only to convince those who doubt the reality of the compression of the earth's axis, but likewise to show with very great approximation to truth the relative length of the earth's radii in different latitudes. For, in the case just mentioned, the variation of a twenty-thousandth part of that length  $\dagger$  will cause a difference of one second of time in the duration of the occultation.

§ 11. Moreover, it is evident that the effect, of which I have

\* For, joining C,A, and C,B, we shall have  $CV=CA=CB=15'.45''$ , and  $CM=(15'.45''-60'')=14'.45''$ ; consequently  $AM=MB=\sqrt{(AC-CM)}$ .  $(AC+CM)=5'.31''.4$ ; which, being multiplied by  $\frac{60'}{32'.56''.5}$ , in order to reduce this distance into time, will give  $10'.3''.6$  for the time of the star's passing from A to M: therefore the time of the star's passing from A to B will be equal to  $2 \times (10'.3''.6)=20'.7''.2$ . But, if the depression of the earth's axis cause a variation in the apparent height of the moon equal to  $8''.5$ , then will  $CL=14'.53''.5$ , consequently (as  $CD=AC$ )  $DL=LE=\sqrt{(AC-CL)}$ .  $(AC+CL)=5'.7''.7$ ; which, being also multiplied by  $\frac{60'}{32'.56''.5}$ , will give  $9'.20''.5$  for the time of the star's passing from D to L: therefore the time of the star's passing from D to E will be equal to  $2 \times (9'.20''.5)=18'.41''$ . B.

$\dagger$  Or, about 1000 feet. The mean radius of the earth being 20,898,240 English feet. B.

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been speaking, may be much greater. It increases greatly as the line MV (or the ver-sine of the arc) becomes smaller: if, for example, it were 30'', the duration of the occultation under AB would be 14'. 20''; and, under DE, 12'. 10'', being a difference, in the duration, of 2'. 10'' of time\*. In such a case, a difference of 500 feet in the length of the earth's radius might be rendered evident: since such a variation would produce a difference of one second of time in the duration of the occultation†.

§ 12. I am well aware that it scarcely ever happens that the apparent path of the star is rectilinear, and at the same time perpendicular to the vertical circle which passes through it, at the moment of the middle of the occultation, as in the case I have supposed. But such a supposition may be readily conceded, as being a mean between the possible cases. For, if the actual phenomena do in some cases deviate therefrom, so as to render the effect of the compression *less* upon the duration of the occultation than when calculated on the above hypothesis, it will on the other hand often occur that they are *greater* ‡. Hence I have no hesitation in asserting that the means, which I have pointed out, are not dependent on any hypothesis, and are the most certain of any that have yet been suggested for determining the variation in the curvature of the earth's surface.

§ 13. With respect to the *pendulum*, the differences in its length are not only too minute and too difficult to be measured with precision, but are also too much influenced by the unknown internal density of the earth to be brought at all into comparison

\* In this case,  $CM = (15'.45'' - 30'') \div \frac{60'}{32.56',5} = 15'.15''$ ; consequently  $AM = 3'.56',2$ : which being multiplied by  $\frac{60'}{32.56',5} \times 2$ , as in the preceding cases, will give 14'. 20'', 4 for the time of the star's passing from A to B. But, supposing the depression the same as in the former instance, we should have  $CL = 15'.23',5$ ; consequently  $DL = LE = 3'.20',4$ : which, multiplied also by  $\frac{60'}{32.56',5} \times 2$ , will give 12'. 10'', 2 for the time of the star's passing from D to E, as mentioned in the text. B.

† According to my calculation, a difference of only 400 feet in the length of the earth's radius would produce a difference of one second of time in the duration of the occultation. But, if the line MV were only 15'', a difference of less than 200 feet would produce a difference of one second of time in the duration. And, agreeably to these principles, it will be evident that the duration of such occultations will vary according to the elevation of the spot (from which they are observed) above the level of the sea; and will sensibly differ when observed in valleys, or on the tops of high hills and mountains. A fact which perhaps has not been sufficiently attended to, either in observations of occultations, or in calculations wherein the parallax of the moon is involved. B.

‡ I confess I do not see the accuracy of this conclusion: for it appears to me that the *effect* of the compression, on the duration of the occultation, must be *greatest* when the apparent path of the star is perpendicular to the vertical circle. B.

with



with the great differences, above mentioned, in the duration of occultations.

§ 14. The different *measurements of the degrees of the meridian* are likewise so discordant amongst themselves, that the respectable author (whom I am about to quote) asserts that the greater number of degrees that are measured, the more uncertain does the figure of the earth seem to be \*. Let us moreover bear

\* This indeed appears to be the case from some of the late measurements. With respect to the meridian of France, M. Delambre states (*Traité d'Astronomie*, vol. iii. page 567) that "it was hoped that the measurement of that arc (divided nearly into two equal parts by the parallel of Evaux) would have given the quantity of the compression of the earth's axis. The calculation, however, makes it =  $\frac{1}{37}$ ; a quantity much too great. The whole arc, compared with that of Bouguer at Peru, makes it =  $\frac{1}{334}$ ; which is too small." And he goes on to observe that, from a revision of the calculations of Lacondamine, and taking a mean between him and Bouguer, he has deduced the quantity  $\frac{1}{889}$ ; which he considers as the most probable one, and which he proposes to be adopted in all future calculations. Nevertheless he afterwards states (*Ibid.* page 572) that a "compression of  $\frac{1}{880}$  represents very well the arcs between Greenwich and Paris, between Dunkirk and Paris, between Greenwich and Evaux, between Greenwich and Barcelona, and between Carcassonne and Mount-Jouy. It appears then (he adds) that, without much variation, the arc of the meridian from Greenwich to Barcelona, indicates (in the whole and in its several parts) a compression which differs very little from  $\frac{1}{880}$ ." And the conclusion, which he draws from the whole, is that "the arc of France indicates a compression more considerable than that of the globe in general." It is worthy of observation that Maupertuis, who was one of the astronomers that measured the arc of the meridian in Sweden in 1736, makes the compression =  $\frac{1}{78}$ , by comparing it with the arc then measured in France. But, M. Svanberg who, in 1805, remeasured the same arc, deduces the compression =  $\frac{1}{337.4}$  when compared with the arc recently measured in France; and =  $\frac{1}{323.3}$  when compared with that of Bouguer at Peru. This is however on the presumption that the standard measure, used in the survey, was equal to the double metre at zero of the thermometer: but, if it was so only at the temperature of  $+16\frac{1}{4}$ , the corresponding compressions would be  $\frac{1}{328.9}$ , and  $\frac{1}{331.4}$ . Other results are obtained according to the different tables of refraction made use of in the calculations; so that, after all, considerable doubts exist as to the exact quantity of the compression, as deduced from that survey. M. Delambre, from the measurements, given by Major Lambton, of an arc in the East Indies, stated the compression =  $\frac{1}{205.7}$ : but, from the subsequent operations of the same gentleman, he afterwards deduces it =  $\frac{1}{328}$ . Major Lambton himself, however, from a more recent measurement has stated the compression to be =  $\frac{1}{370}$ , but, that on comparing the whole of his arc with the whole of the French arc, the compression would be =  $\frac{1}{333}$ . A remarkable discordance in the results. The measurement of Lacaille, at the Cape of Good Hope, compared with that at the equator, gives  $\frac{1}{880}$ . Lastly, I shall add that the recent measurement of the arc, in England, seems to indicate a *prolongation*, instead of a *compression* of the polar axis. These discordant conclusions leave a very unsatisfactory impression on the mind, notwithstanding the acknowledged

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bear in mind the enormous expense and labour required in measuring a degree of the earth's surface. Let us recollect the great uncertainty that may attend such a measurement from the deviation of the plumb-line made use of in the instruments employed for the astronomical observations, occasioned by mountains and valleys, subterraneous cavities and other irregularities in the upper strata of the earth : and, of what great importance such anomalies may be, we may learn in the celebrated work of Boscovich\*.

§ 15. It may perhaps be objected that the two conditions necessary for my purpose may not frequently occur together ; namely, that the altitude of the moon†, and the chord of the lunar disc which passes over the star, must be small. This is probably the reason why this method of ascertaining the compression of the earth's axis has not hitherto been practised. It is true that the greater the altitude of the moon above the horizon, and the nearer to its centre the apparent path of the star, so much the smaller will be the difference of parallax caused by a sphere and an oblate spheroid‡ : and consequently the more easily might the

knowledgeed talents of the persons employed in those surveys ; and prevent us from placing too great reliance on a method which produces results, differing so widely from each other. The learned Boscovich might well exclaim that "the greater the number of degrees that are measured, the more uncertain does the figure of the earth seem to be." B.

\* *De Expeditione Litteraria, Lib. v. § 230, et seq.*

† It does not appear that the altitude of the moon should be limited to the quantity mentioned by the author in the text, since the variations, alluded to, will be perceptible at the height of 20, and even 30 degrees. Thus, in the case mentioned in § 9, it appears that the variation in the parallax of the moon at the height of 10° is 8'',5 : but, if the moon were at the height of 20° above the horizon, the variation would be 8'',1 ; and at 30° it would be 7'',4. B.

‡ The smaller the chord of the lunar disc (or the smaller the ver-sine of the arc) the more perceptible will be the difference in the duration of the occultation in such cases ; as will appear from the following table ; which shows the duration of an occultation of the star on the supposition that the place of the star, when it passed the vertical circle, would be 60', 30', 15', 10', and 8'',5 within the moon's disc, provided the earth were a perfect sphere : together with the corresponding durations provided the earth were an oblate spheroid, having the axis compressed =  $\frac{1}{305}$ . Latitude of the place 60° : height of the moon 10°.

Earth = Sphere.		Earth = $\frac{1}{305}$ .		Difference of Duration.
* within D's disc.	Duration.	* within D's disc.	Duration.	
60	20. 7	51'',5	18.41	1.26
30	14.20	21,5	12.10	2.10
15	10.11	6,5	6.43	3.28
10	8.19	1,5	3.14	5. 5
8,5	7.41	0	Appulse.	7.41

In

the errors of the elements made use of in the calculation be confounded with that difference. But these are precisely the occasions most favourable for the observations with which these particular occultations may be compared; and by which (as I have said) the errors of the tables are corrected, and the geographical longitude determined.

§ 16. It is true that the conditions here required, for ascertaining the flattening of the poles, cannot be very frequently obtained: but, if we look out for them with the diligence that the importance of the object demands, we shall perhaps meet with them more frequently than we imagine. For, I may venture to assert that there does not pass a month without the occurrence of an occultation of some star whose position is well known: and there is no occultation that will not afford, to some part or other of the earth's surface, the conditions required\*. If only once out of twenty times they should occur, in a place where there is an astronomical observatory, the question as to the compression, and also as to its quantity, would be very soon determined.

§ 17. For the solution of such questions, for which immense sacrifices have hitherto been made (for instance, in the measurement of the degrees of the meridian), it surely is not requiring too much that the trifling expense should be incurred of enabling astronomers to travel to places more favourably situated for making observations of such occultations. This would be an undertaking worthy of a sovereign who wishes to distinguish himself as a patron of science. We should, by such means, gradually arrive at a knowledge of the relative length of the terrestrial radii, in a great number of places: and, it is most probable that we might

In the APPENDIX I have given other tables, showing the differences that would arise from varying the latitude of the place, the height of the moon, and the quantity of the earth's compression: whereby the reader may be better able to judge of the maximum of difference which would arise under the most favourable circumstances. B.

\* Since this was written, the positions of most of the zodiacal stars have been determined with a degree of accuracy sufficient for the purposes detailed in the memoir. The labours of *Cagnoli* himself, of *Piazzi*, *Harding*, *Zach*, and *Bessell* have contributed much to this end: so that we may now safely assert that scarcely a *night* passes "without the occurrence of an occultation of some star whose position is well known." Nearly forty years ago *Messier* made the following remark, at the close of a numerous list of observed occultations: "We see by this collection of occultations how many new ones I have observed, in the first quarter of the moon, on the dark limb; which are distinguished with the greatest precision. They are frequent, and much preferable to the observations of Jupiter's satellites, or lunar eclipses, for determining the longitude. It were much to be wished that the conductors of our Ephemerides should announce, for the first part of each lunation, the immersions of stars of even the 7th, 8th and 9th magnitude, which are as readily observed as those of the 1st, 2nd and 3rd magnitude." *Connaissance des Temps*, Année viii. page 319. B.

thereby be enabled finally to deduce the true and exact figure of the earth.

§ 18. Although it appears, from the different measurements of the degrees of the meridian, that the figure of the earth is not regular, still it is possible that the irregularities do not belong so much to the figure, or the radii, as to the nature of the upper strata, the different density of which may occasion the concealed error in the perpendicularity of the instruments: an error which (as I have elsewhere shown) may be quite sufficient to reconcile all the disagreements between the measurements hitherto taken\*. Consequently the variations in the parallax in different latitudes might very well proceed with as much regularity as appears to exist in the variation of gravity, and in the length of the pendulum.

§ 19. It is in the power of every principal Academy materially to assist in such a discovery, by two methods. First, in regard to times *past*, to collect together, from all quarters, the observations of occultations stated to have been made in a given interval; for instance, in the last ten years: and to employ some calculator to select and compute all those which are proper for showing the variation of curvature at different places. Secondly, with respect to the *future*, to insert in the Ephemerides accurate notices of those places or districts where it would be most important that any occultation should be observed (particularly of the principal stars), in order that it might serve to apprise, and excite the attention of such astronomers as might be favourably situated themselves, or contiguous to more advantageous situations. It appears to me that such notices would be more important and useful than those of the phases of solar eclipses, about which the calculators of Ephemerides are in the habit of taking so much trouble.

#### END OF THE MEMOIR.

[The Appendix will be given in our next.]

\* Independent of the deviations arising from the causes here alluded to, the plumb-line has been sometimes known to be attracted towards the sides of the *glass* vessel, containing the weight, as powerfully as gold-leaf towards an electrical tube. The remark appears to have been made by M. Flaugergues. "Astronomers ought to avoid using a *glass* vessel for the water in which the weight of the plumb-line is suspended; for, I have observed twice, in one year, a singular deviation in the plumb-line occasioned by the attraction of the ball of the plumb-line towards the side of the vessel in which it was suspended. This ball was drawn towards the side with as much rapidity as gold-leaf is attracted by an electrified tube: and I was obliged (in order to destroy the effect of this spontaneous electricity, so as to enable me to take equal altitudes) to put a coating of sealing-wax upon the ball. But, since I have substituted a *metal* vessel, this singular phenomenon has not again occurred." *Connaissance des Temps, Année xiii.* page 413. Although such a powerful impulse as this may not often have occurred, yet it is possible that slight deviations from the perpendicular may frequently have arisen from the cause here alluded to; and which may account for some anomalies which have been remarked in the observations made in the course of the surveys. B.