ments show that this is approximately the case—deviations of 10° from parallelism being rare, even when, as is often the case, the three canals of one ear are not at right angles to one another.

The methods employed in making these measurements were explained and illustrated.

[*Note by the Author.*—Since presenting to the Society, on January 19th, the Preliminary Note on the Sense of Rotation and the Function of the Semicircular Canals of the Internal Ear, I have seen abstracts of papers on the same subject by Professor Mach and by Dr Breuer. As far as I can judge from these abstracts, while Professor Mach and Dr Breuer refer the action of rotation upon the ampullary nerves to the inertia of the contents of the canals, they do not seem to have noticed the parallelism of the plane of the superior canal of the one ear to that of the posterior canal of the other, nor to have observed that approximate parallelism of these planes is essential, if the semicircular canals are the peripheral organs of the sense of rotation.]

The following Gentlemen were admitted Fellows of the Society:—

R. H. Traquair, M.D., Mus. Science and Art.
Francis Jones, Esq., Lecturer on Chemistry, Manchester.
W. F. Barrett, F.C.S., R. College of Science, Dublin.

Monday, 20th April 1874.

Sir WILLIAM THOMSON, President, in the Chair.

The following Communications were read:—

1. On Last-Place Errors in Vlacq's Table of Logarithms. By Edward Sang, Esq.

Now fifty years ago, while engaged with some heavy calculations connected with engineering work, I became impressed with the advantage of having logarithmic tables much more extensive than those in use. The trouble of the interpolations at the early part of the table, contrasted with the convenience of the small addi-

tional part from 100,000 to 108,000 printed in Hutton, gave rise to the idea of carrying the table onwards even so far as to one million. Although the bulk of such a table appears to be an objection, and the turning of so many leaves a toil, the ease to the habitual computer of finding at once the number of which he is in search is so great as far to outweigh the opposite considerations. Thus, though working only to five places, we prefer to use the extensive seven-place tables rather than to take up Lalande's small volume; and so, while working to seven places, we should gladly avail ourselves of a nine-place million table, the construction of which I proposed to myself, notwithstanding the vast amount of the labour.

The first idea was to interpolate from tables already published, but this was opposed by the feeling of dependency on the accuracy of the previous calculations. On examining the sources of our information on denary logarithms, it became apparent that the original work of Henry Briggs (1620), carried on in the laborious way indicated to him by John Nepair in his "Constructio," is the only foundation; and that the completion of the canon by Adrian Vlacq (1628) was the last of the original labour that has been bestowed on this matter so essential to the progress of exact knowledge.

The more convenient methods of calculation developed by the progress of logistics have come, as it were, too late to be of service. It is indeed surprising that, after the lapse of two hundred and fifty years, we are still relying on the unchecked calculations of Briggs and Vlacq; that among so many generations of scientific men there has not been zeal enough to effect a revision of the canon.

Even on the supposition that Vlacq's logarithms are true in the last place, the attempt to interpolate between them would lead to frequent uncertainty in the seventh place. In order to form an extensive table of seven-place logarithms true in the last figure, we should have to carry our original computations at least five steps farther.

Thus I came to perceive the necessity of making the whole computation anew. From time to time I took up the work to lay it down in alarm at its magnitude, for years of labour only seemed to make a beginning; but about 1849 I happened to obtain a copy

of the great "Table des Diviseurs," by Burckhardt. The facility afforded by this admirable work for finding convenient formulæ of approximation, determined me to persevere in the construction of the large table; and, putting aside all my previous calculations, I arranged a comprehensive scheme for recording each step of the process, so that it might serve as occasion might arise to facilitate subsequent steps, and so that any suspected error might be traced to its source. By this means the progress of the work was effectually secured, because each little addition took its proper place, at however long an interval of time it might happen to be made.

Without going into the details of the procedure, it is enough to mention here that the logarithms of prime numbers up to 3600, and of many others occurring incidentally, have been computed to twenty-eight places with the view of being exact to twenty-five, and that the logarithms of all their products under 10,000 have been tabulated; and, by help of these, tables have been made to fifteen places of the logarithms of all numbers from 100,000 to 320,000, with their first and second differences. These, filling in all twenty-four quarto volumes, are laid on the Society's table.

Henry Briggs computed to fourteen places the logarithms of all numbers up to 20,000, and of numbers from 90,000 to 100,000; so that Vlacq, in shortening them to ten places, was safe from error excepting in one or two rare cases. But when Vlacq set himself to fill in the intermediate 70,000, he sought to lessen the labour by using only twelve places, thus making his last figure insecure in many more cases; and, moreover, the process followed by him wanted the quality of self-verification. On these accounts I suspected the occurrence of last-place errors in Vlacq's part of the table. Seeing that each tenth logarithm of my own computation from 200,000 to 300,000, should agree with Vlacq's from 20,000 to 30,000, the comparison was made, and the result was the discovery of forty-two errors in this single myriad—an exceedingly small number when the nature of the process is considered, but a very large number to have escaped detection for two centuries and a half. At the same rate for each of the remaining six myriads, we may expect a total of nearly three hundred errors.

In 1658, that is thirty years after Vlacq, John Newton published a translation of Gellibrand's "Trigonometria Britannica," in which

he gives an eight-place table of logarithms arranged in the compact manner now usually adopted. In the address to the reader, he speaks contemptuously of Adrian as "Vlaq the Dutchman," "from whose corrupt and imperfect copy," &c.; and in the introduction he describes a mode of computing logarithms which the innocent reader may believe to have been followed by the author of the book, but a collation shows that Vlacq's misprints have been slavishly copied by the indignant Newton.

It was not until 1794 that anything claiming to be a revision of the original table appeared; this was the ten-place table given by Georg Vega in his "Thesaurus Logarithmorum," the arrangement being after the compact manner introduced by Newton. Vega gives a long list of corrections on Vlacq's table, which by that time had become scarce, and it was generally understood that he had at least taken the precaution of adding up Vlacq's differences in order to eliminate the misprints. But on collating the list of errors which I have just discovered in Vlacq, with Vega's table, we are forced, however reluctantly, to the conclusion that Vlacq's identical table had been used by the compositor of Vega's pages. A review of the character of the errors will make this clear; a list of them is subjoined, showing the logarithms true to fifteen places (the first five being omitted), the last group as it should have been in Vlacq, Vlacq's corresponding five, and Vega's last group of three.

Of the forty-two errors shown in Vlacq, forty are last-place errors, such as we are considering; and two, marked with asterisks, are misprints, as is known by the circumstance that the adjoining differences are correct. As was to have been expected, all the final errors are copied by Vega, who never pretended to have made a new computation; of the misprints one, a 9 for a 6, is corrected; but the other, 646 instead of 626, is retained. Not only so, among the final errors there are six belonging to numbers ending in 0; now these logarithms occur in the preceding part of the table, where they are correctly given, and yet these also, of easy detection, are retained by Vega. Thus, again, Vega is only Vlacq in a new and much more convenient form.

The only work claiming to be an original computation of logarithms is that done in the Bureau du Cadastre, at the instance of the French Government. This unpublished work contains to

nineteen places the logarithms of numbers from 1 to 10,000, and to fourteen places of those from 10,000 to 200,000. In the year 1819 the House of Commons, on the motion of Mr Davies Gilbert, presented an address to the Prince Regent, recommending that our Government should join with that of France in the expense of publishing these and the accompanying Trigonometrical Tables; but the negotiations fell through, for reasons that have not been made public. I have not learned that these computations have been used for the verification of those already printed, or that they have served for the production of any seven-place table; and thus, up to the present moment, we have no verification of Vlacq's great work.

The eminent astronomer Lalande, in publishing his little five-place table, was able confidently to assert that it does not contain a single error, and although many thousands of copies have been in use now for seventy years no fault has been detected. Thus the production of a faultless table is quite within the range of possibility; it is a matter of time, of care, of expense; and with our modern appliances the endless reproduction of the plates is easy; so that computers ought to be in possession of tables trustworthy throughout, especially of such tables as are of universal application.

Though not needed for the every-day work of the computer, tables of excessive precision are not the less needed in special departments, and in the preparation of other tables for ordinary use. Their extent and the expense of preparing them, coupled with the smallness of the number of those by whom they are desired, precludes their preparation by private parties, and relegates the matter to the care of public authorities.

In the same way that the "Nautical Almanac," which is far beyond the reach of private enterprise, and yet is needed for the advancement of navigation and astronomy, is undertaken by the Government, it would be right to carry out the idea of Davies Gilbert, and to confer, by the publication of exact tables, a similar boon upon the other branches of science.

It would be fitting that this should be done by the British Government, seeing that the invention and completion of the logarithmic method belong to the Island; and it would be not less fitting that the first public body to move in the matter should be the Royal Society of Edinburgh, from whose place of meeting

we could almost have seen the roof under which John Nepair elaborated his invention, and could fancy to have heard the creaking of the screw with which Andrew Hart imprinted the "Canon Mirificus."

Number.	Log. to 15 Places.	To 10.	Vlacq.	Vega.
20071	90109 36054	90109	90110	110
20280	79506 61298	79507	79506	506
20375	76174 12014	76174	76175	175
20645	48872 10721	48872	48873	873
20822	24421 41256	24421	24422	422
20866	92030 46050	92030	92031	031
21245	67354 25533	67354	67355	355
21749	92932 69231	92933	92932	932
21795	68733 53703	68734	68735	735
21904	34307 89915	34308	34309	309
22016	84165 55417	84166	84167	167
22200	29744 50639	29745	29744	744
22312	85012 57993	85013	85012	012
22877	90721 80887	90722	90721	721
22996	22999 73937	23000	22999	999
23274	10299 83700	10300	10299	299
23492	99921 70919	99922	99923	923
23820	17571 46759	17571	17572	572
24156	50209 36279	50209	50210	210
24580	18785 50435	18786	18785	785
25173	49758 10852	49758	49759	759
25524	87359 50354	87360	87359	359
25586	23955 50655	23956	23955	955
25707	13975 50452	13976	13975	975
26004	01573 67443	01574	01573	573
26407	90654 45820	90654	90655	655
26517	43886 18717	43886	43889*	886
26642	68239 65258	68240	68239	239
26699	49953 49034	49953	49954	954
26717	76904 57995	76905	76904	904
26728	64626 30075	64626	64646*	646
27291	94494 30434	94494	94495	495
27560	92132 35588	92132	92133	133
27586	87318 72159	87319	87318	318
27861	67002 67696	67003	67002	002
27921	09686 32521	09686	09687	687
28486	14699 52392	14700	14699	699
28680	91469 95763	91470	91469	469
29226	93799 55414	93800	93799	799
29446	63077 50861	63078	63077	077
29639	35467 49658	35467	35468	468
29703	03152 31285	03152	03153	153