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XIX. *On the Theory of the Conduction of Electricity through Gases by Charged Ions.* By J. J. THOMSON, *M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge**.

THE electrical conductivity possessed by gases under certain circumstances—as for example when Röntgen or uranium rays pass through the gas, or when the gas is in a vacuum-tube or in the neighbourhood of a piece of metal heated to redness, or near a flame or an arc or spark-discharge, or to a piece of metal illuminated by ultra-violet light—can be regarded as due to the presence in the gas of charged ions, the motions of these ions in the electric field constituting the current.

To investigate the distribution of the electric force through the gas we have to take into account (1) the production of the ions; this may either take place throughout the gas, or else be confined to particular regions; (2) the recombination of the ions, the positively charged ions combining with the negatively charged ones to form an electrically neutral system; (3) the movement of the ions under the electric forces. We shall suppose in the subsequent investigations that the velocity of an ion is proportional to the electric intensity acting upon it. The velocity acquired by an ion under a given potential gradient has been measured at the Cavendish Laboratory by several observers—in the case of gases exposed to the Röntgen rays by Rutherford and by Zeleny; for gases exposed to uranium radiation or to

* Communicated by the Author.

ultra-violet light by Rutherford; for the ions in flames by McClelland and H. A. Wilson; and for the ions in gases near to incandescent metals or to the arc discharge by McClelland. The velocities in the different cases vary very much; the velocity of an ion in the same gas is much the same whether the conductivity is due to Röntgen rays, uranium rays, or ultra-violet light; it is much smaller when the conductivity is produced by an arc or by incandescent metal. Thus the mean velocity of the positive and negative ions under a volt per centimetre in air exposed to Röntgen rays was found by Rutherford to be about 1·6 cm./sec., while for gas drawn from the neighbourhood of an arc-discharge in carbonic acid the mean velocity of the positive and negative ions was found by McClelland to be only ·0035 cm./sec.

This difference is caused by the ions acting as nuclei about which condensation, whether of the gas around them or of water-vapour present in the gas, takes place. The power of these ions to act as nuclei for the condensation of water-vapour is strikingly shown by C. T. R. Wilson's * experiments on the effects of Röntgen and uranium radiation on the formation of clouds, and also by R. v. Helmholtz's † experiments on the effects produced by ions on a steam jet. If the size of the aggregation which forms round the ion depends on the circumstances under which the ion is liberated and the substances by which it is surrounded, the velocity which the ion acquires under a given potential gradient will also depend on these circumstances, the larger the mass of the aggregation the smaller will be this velocity. A remarkable result of the determination of the velocities acquired by the ions under the electric field is that the velocity acquired by the negative ion under a given potential gradient is greater than (except in a few exceptional cases when it is equal to) the velocity acquired by the positive ion. Greatly as the velocities of the ions produced in different ways differ from each other, yet they all show this peculiarity. The relative velocities of the negative and positive ions differ very much in the different cases of conduction through gases; thus in the case of imperfectly dried hydrogen traversed by the Röntgen rays, Zeleny found that the speed of the negative ions was about 25 per cent. greater than that of the positive, while in the case of conduction through hot flames H. A. Wilson found that the velocity of the negative ion was 17 or 18 times that of the positive. In the case of the discharge through vacuum-tubes, the measurements which I made of

* Wilson, Phil. Trans. A, 1897; Proceedings of Cambridge Phil. Soc. vol. ix. p. 333.

† R. v. Helmholtz, Wied. *Ann.* vol. xxvii. p. 509 (1886).

the ratio of the charge to the mass for the particles constituting the cathode rays and those of W. Wien * for the ions carrying the positive charge indicate that the ratio of the velocity of the negative ion to that of the positive one under the same potential gradient would be very large. This fact is, I think, sufficient to account for most of the differences between the appearances at the positive and negative electrodes in a vacuum-tube. Schuster (Proc. Roy. Soc. vol. xlvii. p. 526, 1890), from observations on the rates at which positively and negatively electrified bodies lost their charges in a vacuum-tube, came to the conclusion that the negative ions diffused more rapidly than the positive; other phenomena connected with the discharge led me later (Phil. Mag. vol. xl. p. 511, 1895) independently to the same result.

We shall now proceed to find equations satisfied by the electric intensity in a gas containing charged ions. To simplify the analysis we shall suppose that the electric force is everywhere parallel to the axis of x, and that if X is the value of the electric intensity at a point, the velocity of the positive ion at that point is $k_1\mathrm{X}$, and that of the negative ion in the opposite direction $k_2\mathrm{X}$; we shall suppose that at this point the number of positive ions per unit volume is n_1, the number of negative ions n_2; let q be the number of positive or negative ions produced at this point in unit volume in unit time: the number of collisions per unit time between the positive and negative ions is proportional to n_1n_2. We shall suppose that in a certain fraction of these collisions recombination between the positive and negative ions takes place, so that a number αn_1n_2 of positive and negative ions disappear in unit time from unit volume in consequence of the recombination of the ions. If e is the charge carried by each ion, the volume density of the electrification is $(n_1-n_2)e$, hence we have

$$\frac{d\mathrm{X}}{dx}=4\pi(n_1-n_2)e, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

if ι is the current through unit area of the gas, and if we neglect any diffusion except that caused by the electric field,

$$k_1n_1e\mathrm{X}+k_2n_2e\mathrm{X}=\iota, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and if things have settled into a steady state, ι is constant throughout the gas; from these equations we have

$$n_1e=\frac{1}{k_1+k_2}\left\{\frac{\iota}{\mathrm{X}}+\frac{k_2}{4\pi}\frac{d\mathrm{X}}{dx}\right\}, \quad . \quad . \quad . \quad (3)$$

$$n_2e=\frac{1}{k_1+k_2}\left\{\frac{\iota}{\mathrm{X}}-\frac{k_1}{4\pi}\frac{d\mathrm{X}}{dx}\right\}. \quad . \quad . \quad . \quad (4)$$

* W. Wien, *Verhandl. der phys. Gesellsch. zu Berlin*, vol. xvi. p. 165.

In a steady state the number of positive ions in unit volume at a given place remains constant, hence

$$\frac{d}{dx}(k_1 n_1 \mathrm{X}) = q - \alpha n_1 n_2, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$-\frac{d}{dx}(k_2 n_2 \mathrm{X}) = q - \alpha n_1 n_2.$$

Substituting in either of these equations the values of $n_1 n_2$ previously found we get, since $d\iota/dx = 0$,

$$\frac{1}{4\pi e}\frac{k_1 k_2}{k_1+k_2}\frac{d}{dx}\left(\mathrm{X}\frac{d\mathrm{X}}{dx}\right) = q - \frac{\alpha}{\mathrm{X}^2 e^2 (k_1+k_2)^2}\left(\iota + \frac{k_2}{4\pi}\mathrm{X}\frac{d\mathrm{X}}{dx}\right)\left(\iota - \frac{k_1}{4\pi}\mathrm{X}\frac{d\mathrm{X}}{dx}\right)$$

If we put

$$\mathrm{X}^2 = 2y,$$

$$\frac{dy}{dx} = p,$$

this equation becomes

$$\frac{1}{4\pi e}\frac{k_1 k_2}{k_1+k_2}\, p\frac{dp}{dy} = q - \frac{\alpha}{2ye^2(k_1+k_2)^2}\left(\iota + \frac{k_2}{4\pi}p\right)\left(\iota - \frac{k_1}{4\pi}p\right).$$

I have not been able to integrate this equation in the general case when q is finite and k_1 not equal to k_2. We can, however, integrate it when q is constant and $k_1 = k_2 = k$. In this case the equation may be written

$$\pi\frac{d}{dy}\left(\frac{k^2}{16\pi^2}p^2 - \iota^2\right) = \frac{\alpha}{8ek}\frac{1}{y}\left(\frac{k^2}{16\pi^2}p^2 - \iota^2\right) + qek,$$

the solution of which is

$$\frac{k^2}{16\pi^2}p^2 - \iota^2 = \frac{qek}{\pi\left(1 - \dfrac{\alpha}{8\pi ek}\right)}y + \mathrm{C}(2y)^{\frac{\alpha}{8\pi ek}},$$

where C is a constant of integration.

If the current through the gas passes between two parallel plates maintained at a constant potential-difference, $d\mathrm{X}/dx = 0$ midway between the plates; at the positive plate $n_1 = 0$, while $n_2 = 0$ at the negative plate; hence if X_0, X_1 be respectively the values of X midway between the plates and at either plate, we have, putting $p = 0$, to get X_0

$$-\iota^2 = \frac{\dfrac{qek}{4\pi}}{1 - \dfrac{\alpha}{8\pi ek}}\mathrm{X}_0^2 + \mathrm{C}\mathrm{X}_0^{\frac{\alpha}{4\pi ek}}.$$

But

$$\iota = 2nkeX_0,$$

since when $dX/dx=0$, $n_1=n_2$.

The measurements of X for gases exposed to Röntgen rays show that unless the current is approaching the maximum value it can attain, X is practically constant for some distance near the middle of the plates; hence in this case we have $d^2X/dx^2=0$ midway between the plates, and therefore by equations (3) and (5) $q=\alpha n^2$; substituting this value of n we have

$$-X_0^2\left(\frac{4k^2e^2q}{\alpha}+\frac{\frac{qek}{2\pi}}{1-\frac{\alpha}{8\pi ek}}\right)=CX_0{}^{\frac{\alpha}{4\pi ek}};$$

or

$$-X_0^2\frac{\frac{4k^2e^2q}{\alpha}}{1-\frac{\alpha}{8\pi ek}}=CX_0{}^{\frac{\alpha}{4\pi ek}}. \qquad (6)$$

At either plate $n_1n_2=0$, so that $\frac{k^2}{16\pi^2}p^2-\iota^2=0$, thus

$$-X_1^2\frac{\frac{qek}{2\pi}}{1-\frac{\alpha}{8\pi ek}}=CX_1{}^{\frac{\alpha}{4\pi ek}}; \qquad (7)$$

hence from (6) and (7)

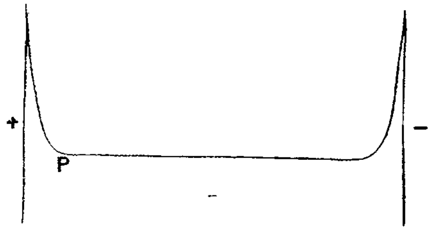
$$\frac{8\pi ke}{\alpha}=\left(\frac{X_0}{X_1}\right)^{\frac{\alpha}{4\pi ek}-2};$$

or if $\frac{8\pi ek}{\alpha}=\beta$,

$$\frac{X_1}{X_0}=\beta^{\frac{1}{2-2/\beta}}. \qquad (8)$$

It follows from this equation that X_1/X_0 is greater than unity, and that the value of this ratio increases from unity to infinity as β increases from zero to infinity. We see that β does not involve either q or ι. So that, to take a particular case, when the gas between two plates is exposed to Rontgen rays, the ratio of the electric intensity at the plates to that midway between them is independent of the intensity of the radiation and of the current through the gas. The curves giving the connexion between the electric intensity and the distance between the plates are found by experiment to be somewhat as represented in fig. 1. The variation in X (fig. 1)

Fig. 1.



occurs only in two layers near the plates and X is approximately constant in the rest of the field. As the current through the gas increases the layers of inconstant X expand until they touch, and then there is no longer a region in which X is constant. We can easily find an inferior limit to the value of λ the thickness of one of these layers when we are given the value of the current. For suppose P (fig. 1) is at the boundary of the layer next the positive electrode, then at P, since X becomes constant, half the current must be carried by the positive and half by the negative ions; if ι is the current and e the charge carried by an ion, then $\iota/2e$ positive ions must cross unit area of a plane through P in unit time, so that at least that number must be produced in unit time in the region between P and the positive plate. Now if λ is the thickness of the layer, $q\lambda$ is the number of positive ions produced in unit time; the number that cross the plane in unit time cannot then be greater than $q\lambda$, and will only be as great as this if no recombination of the ions takes place; hence

$$q\lambda > \frac{\iota}{2e},$$

or

$$\lambda > \frac{\iota}{2qe}.$$

Thus $\iota/2eq$ is an inferior limit to λ. It will not, however, I think be very far from the true value, for we can show that but little recombination will take place in the time taken by the positive ions to traverse a layer of this thickness. For the rate of combination of the positive ions is given by

$$\frac{dn_1}{dt} = -\alpha n_1 n_2.$$

If N_2 is the maximum value of n_2, then $\frac{1}{\alpha N_2}$ measures the time that is taken before recombination diminishes the number of the ions to any very appreciable extent. In this time the positive ions would move a distance δ given by the equation

$$\delta = \frac{kX_1}{\alpha N_2},$$

where X_1 is the value of X at the plate.

$$kN_2X_1e = \iota;$$

thus

$$\delta = \frac{k^2X_1^2e}{\alpha\iota}.$$

Let $X_1 = \gamma X_0$ where $\gamma = \beta^{\frac{1}{2-2/\beta}}$,

$$\delta = \frac{\gamma^2k^2X_0^2e}{\alpha\iota}$$

$$= \frac{\gamma^2\iota}{4qe} = \frac{\gamma^2}{2}\lambda.$$

Thus if γ is tolerably large, the positive ions will traverse a space much greater than λ before recombining.

The greatest current that can pass between the plates is when all the ions are used in carrying the current; if l is the distance between the plates, then lq positive and negative ions are produced in unit time; thus if I is the maximum current which can pass between the plates

$$I = lqe;$$

hence we can write

$$\frac{\lambda}{l} = \frac{\iota}{2I}.$$

The equations

$$\frac{dX}{dx} = 4\pi(n_1 - n_2)e,$$

$$\frac{d}{dx}(k_1n_1X) = q - \alpha n_1n_2,$$

$$\frac{d}{dx}(k_2n_2X) = -(q - \alpha n_1n_2),$$

$$(k_1n_1 + k_2n_2)Xe = \iota,$$

are satisfied by

$$n_1 = n_2 = \{q/\alpha\}^{\frac{1}{2}},$$

$$k_1 n_1 \mathrm{X} e = \frac{k_1}{k_1 + k_2}\iota,$$

$$k_2 n_2 \mathrm{X} e = \frac{k_2}{k_1 + k_2}\iota,$$

$$\mathrm{X} = \left(\frac{\alpha}{q}\right)^{\frac{1}{2}} \frac{\iota}{e(k_1 + k_2)},$$

where ι is the current through the gas. In this case the amounts of the current carried by the positive and negative ions respectively are proportional to the velocities of those ions. When, however, the current passes between two parallel plates, this solution cannot hold right up to the plates, For, consider the condition of things at a point P, between the plates AB and CD, of which AB is the positive and CD the negative plate. Then across unit area at P

$$\frac{k_1}{k_1 + k_2}\frac{\iota}{e}$$

positive ions pass in unit time, and these must come from the region between P and AB ; this region can, however, not furnish more than $q\lambda$, and only as much as this if there are no recombinations ; hence the preceding solution cannot hold when the distance from the positive plate is less than

$$\frac{k_1}{k_1 + k_2}\frac{\iota}{qe};$$

similarly it cannot hold nearer the negative plate than the distance

$$\frac{k_2}{k_1 + k_2}\frac{\iota}{qe}.$$

We shall assume that the solution given above does hold in the parts of the field which are further away from the plates than these distances ; and further, that in the layers in which this solution does not hold, there is no recombination of the ions. Let us now consider the condition of things near the positive plate between

$$x = 0 \quad \text{and} \quad x = \frac{k_1}{k_1 + k_2}\frac{\iota}{qe} = \lambda \text{ say.}$$

Then, since in this region there is no recombination, our equations are

$$\frac{dX}{dx} = 4\pi(n_1 - n_2)e,$$

$$\frac{d}{dx}(k_1 n_1 X) = q,$$

$$\frac{d}{dx}(k_2 n_2 X) = -q.$$

If q is constant, we have

$$k_1 n_1 X = qx,$$

$$k_2 n_2 X = \frac{\iota}{e} - qx,$$

the constant has been determined so as to make $n_1 = 0$ when $x = 0$; substituting these values for n_1, n_2, in the equation giving dX/dx we get

$$X\frac{dX}{dx} = 4\pi e\left\{qx\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{\iota}{ek_2}\right\},$$

or

$$X^2 = 8\pi e\left\{\frac{qx^2}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{\iota x}{ek_2}\right\} + C;$$

the constant may be determined from the condition that when $x = \lambda_1$

$$X^2 = \frac{\alpha}{q}\frac{\iota^2}{e^2(k_1 + k_2)^2},$$

from this we find

$$C = \frac{\alpha}{q}\frac{\iota^2}{e^2(k_1 + k_2)^2}\left\{1 + \frac{4\pi e}{\alpha}\frac{k_1}{k_2}(k_1 + k_2)\right\},$$

Since C is the value of X^2 when $x = 0$, it is the value of X^2 at the positive plate; if we call X_1 the value of X at the positive plate and X_0 the value of X between the layers we have

$$X_1 = X_0\left\{1 + \frac{4\pi e}{\alpha}\frac{k_1}{k_2}(k_1 + k_2)\right\}^{\frac{1}{2}},$$

thus X_1 is always greater than X_0, the value of X between the layers.

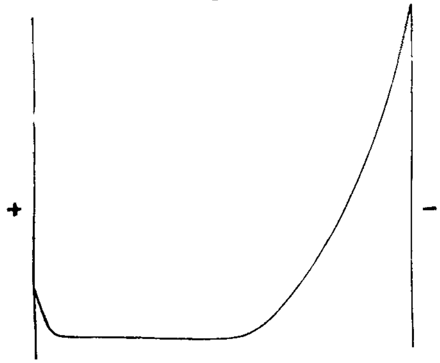
If X_2 is the value of X at the negative plate we have

$$X_2 = X_0\left\{1 + \frac{4\pi e}{\alpha}\frac{k_2}{k_1}(k_1 + k_2)\right\}^{\frac{1}{2}}.$$

Thus, if k_2, the velocity of the negative ion, is very large compared with k_1, the velocity of the positive, the value of X at the negative plate is large compared with its value at the positive.

The curve representing the electric intensity between the plates is shown in fig. 2. In this case k_2/k_1 is a large quantity.

Fig. 2.



The fall of potential across the layer whose thickness is λ_1 is equal to

$$\int_0^{\lambda_1} \mathrm{X}dx,$$

and this is equal to

$$\frac{1}{2}\mathrm{X}_1\lambda_1 + \frac{1}{2}\frac{\mathrm{X}_0\lambda_1}{\sqrt{\beta}}\log(\sqrt{\beta} + \sqrt{1+\beta}),$$

where

$$\beta = \frac{4\pi e}{\alpha}\frac{k_1}{k_2}(k_1+k_2),$$

and

$$\mathrm{X}_1 = \mathrm{X}_0(1+\beta)^{\frac{1}{2}},$$

Hence, if β is large, the fall of potential across the layer whose thickness is λ_1 next the positive plate, is approximately

$$\frac{1}{2}\mathrm{X}_1\lambda_1 = \frac{1}{2}\mathrm{X}_0(1+\beta)^{\frac{1}{2}}\frac{k_1}{k_1+k_2}\frac{\iota}{qe},$$

Similarly, if

$$\beta_1 = \frac{4\pi e}{\alpha}\frac{k_2}{k_1}(k_1+k_2),$$

and if β_1 is large the fall of potential across the layer whose thickness is λ_2 is approximately

$$\frac{1}{2}\mathrm{X}_2\lambda_2 = \frac{1}{2}\mathrm{X}_0(1+\beta_1)^{\frac{1}{2}}\frac{k_2}{k_1+k_2}\frac{\iota}{qe}.$$

The distance between the plates in which the electric intensity is constant and equal to X_0 is $l-(\lambda_1+\lambda_2)$, where l is the distance between the plates. Since $\lambda_1+\lambda_2=\iota/qe$ the fall of potential in this distance is equal to

$$X_0\left(l-\frac{\iota}{qe}\right);$$

hence if V is the potential-difference between the plates

$$V=X_0\left\{\tfrac{1}{2}(1+\beta)^{\frac{1}{2}}\frac{k_1}{k_1+k_2}\frac{\iota}{qe}+\tfrac{1}{2}(1+\beta_1)^{\frac{1}{2}}\frac{k_2}{k_1+k_2}\frac{\iota}{qe}+l-\frac{\iota}{qe}\right\},$$

and

$$X_0=\left(\frac{\alpha}{q}\right)^{\frac{1}{2}}\frac{\iota}{e(k_1+k_2)},$$

so that

$$V=\left(\frac{\alpha}{q}\right)^{\frac{1}{2}}\frac{\iota}{e(k_1+k_2)}\left\{\tfrac{1}{2}(1+\beta)^{\frac{1}{2}}\frac{k_1}{k_1+k_2}\frac{\iota}{qe}+\tfrac{1}{2}(1+\beta_1)^{\frac{1}{2}}\frac{k_2}{k_1+k_2}\frac{\iota}{qe}-\frac{\iota}{qe}+l\right\}.$$

This gives the relation between the current and the potential-difference between the plates. It is of the form

$$V=A\iota^2+B\iota.$$

In a paper by Mr. Rutherford and myself in the Phil. Mag. for Oct. 1896, a relation between V and ι was given on the assumption that the electric intensity was constant between the plates; in this investigation I have tried to allow for the variation in the electric intensity. The above investigation ceases to be an approximation to the truth when the two layers touch each other. In this case the current has its limiting value lqe, and there is no loss of ions by recombination; we may therefore neglect the recombination and proceed as follows.

Equations (5) become in this case

$$\frac{d}{dx}(k_1n_1X)=q,$$

$$\frac{d}{dx}(k_2n_2X)=-q.$$

If q is constant, the solutions of these equations are

$$k_1n_1X=qx, \qquad (9)$$

$$k_2n_2X=q(l-x), \qquad (10)$$

where x is the distance measured from the positive plate and l the distance between the plates, for these solutions satisfy the differential equations and the boundary conditions $n_1=0$ when $x=0$, and $n_2=0$ when $x=l$. From the equation

$$\frac{dX}{dx}=4\pi(n_1-n_2)e,$$

we have

$$X\frac{dX}{dx}=4\pi qe\left\{x\left(\frac{1}{k_1}+\frac{1}{k_2}\right)-\frac{l}{k_2}\right\},$$

or

$$X^2=8\pi qe\left\{\frac{x^2}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)-\frac{lx}{k_2}\right\}+C, \quad . \quad . \quad (11)$$

where C is the constant of integration. When X has its minimum value we see from equation (1) that $n_1=n_2$; hence from (9) and (10) at such a point we have

$$\frac{k_1}{k_2}=\frac{x}{l-x}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

hence if we determine the point Q where X is a minimum, this equation will give us the ratio of the velocities of the positive and negative ions.

We see that a positive ion starting from the positive plate, and a negative ion starting from the negative plate, reach this point simultaneously.

If X_0 is the minimum value of X, and ξ the distance of a point between the plates from Q, we may write equation (11) in the form

$$X^2=X_0^2+4\pi qe\xi^2\left(\frac{1}{k_1}+\frac{1}{k_2}\right);$$

if I is the maximum current, this may be written

$$X^2=X_0^2+4\pi I\xi^2\frac{1}{l}\left(\frac{1}{k_1}+\frac{1}{k_2}\right). \quad . \quad . \quad . \quad (13)$$

We see from this equation that if we measure the values of X at two points and I, the maximum current, we can deduce the value of

$$\frac{1}{k_1}+\frac{1}{k_2},$$

and since from (12) we know the value of k_1/k_2, we can deduce the values of k_1 and k_2.

If the positive ion moves more slowly under a given potential gradient than the negative ion, then we see from (12) that Q is nearer to the positive than to the negative plate. Hence it follows from (13) that the electric force at the negative plate is greater than that at the positive.

A very convenient method of determining the velocities of the ions, and one which can be employed in nearly every case of conduction through gases, is to produce the ions in one region and measure the electric intensity at two points in a region where there is no production of ions, but to which ions of one sign only can penetrate under the action of the electric field. Thus let A, B represent two parallel plates immersed in a gas, and let us suppose that in the layer between A and the plane LM we produce a supply of ions, whether by Röntgen rays, incandescent metals, ultra-violet light, or by other means, and suppose that the gas between LM and B is screened off from the action of the ionizer. Then if A and B are connected to the poles of a battery a current will pass through the gas, and this current in the region between LM and B will be carried by ions of one sign. These will be positive if A is the positive pole, negative if it is the negative pole. Let us find the distribution of electric intensity in the region between LM and B. Let us suppose A is the positive plate, then all the ions in this region are positive and we have, using the same notation as before,

$$\frac{dX}{dx} = 4\pi n_1 e,$$

$$k_1 n_1 X e = \iota,$$

where ι is the current through unit area; from these equations we have

$$X\frac{dX}{dx} = \frac{4\pi\iota}{k_1},$$

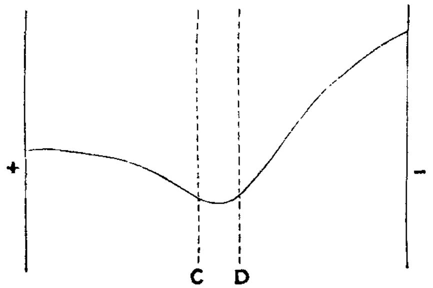
or

$$X^2 = \frac{4\pi\iota}{k_1}x + C.$$

Hence if we measure the values of X at two points in the region between LM and B, and also the value of ι, we can, from this equation, deduce the value of k_1, and hence the velocity of the positive ion in a known electric field. To determine the velocity of the negative ion we have only to perform a similar experiment with the plate A negative.

When the ionization is confined to a layer CD between the plates A and B, the distribution of electric intensity is represented by fig. 3, where A is the positive and B the negative plate, and the velocity of the negative ion is supposed to be much greater than that of the positive.

Fig. 3.



The investigation of the distribution of electric intensity given on p. 262 shows that when the velocity of the negative ion is much greater than that of the positive, the distribution of the intensity has many features in common with that associated with the passage of electricity through a vacuum-tube, especially the great increase in electric intensity close to the negative electrode. Thus this feature of the discharge through vacuum-tubes can be explained by the greater velocity of the negative ion than of the positive, a property which seems to hold in all cases of discharge of electricity through gases. And as the most important of the differences between the phenomena at the two poles of a vacuum-tube are direct consequences of the electric intensity at the cathode far exceeding that at the anode, I think the most striking features of the discharge through vacuum-tubes are consequences of the difference in velocity between the positive and negative ions. In the case discussed on p. 261 we assumed q constant, *i. e.* that the ionization along the path of the discharge was constant ; in the case of the discharge through vacuum-tubes, where the ionization is due primarily to the electric field itself, it is unlikely that the ionization will be constant when the field is so variable. We can derive information as to the distribution of the ionization by a study of the very valuable curves giving the distribution of electric intensity in a vacuum-tube which we owe to the researches of Graham (Wied. *Ann.* lxiv. p. 49, 1898).

From the equations

$$\frac{dX}{dx} = 4\pi(n_1 - n_2)e,$$

$$\frac{d}{dx}(k_1 n_1 \mathrm{X}) = q - \alpha n_1 n_2,$$

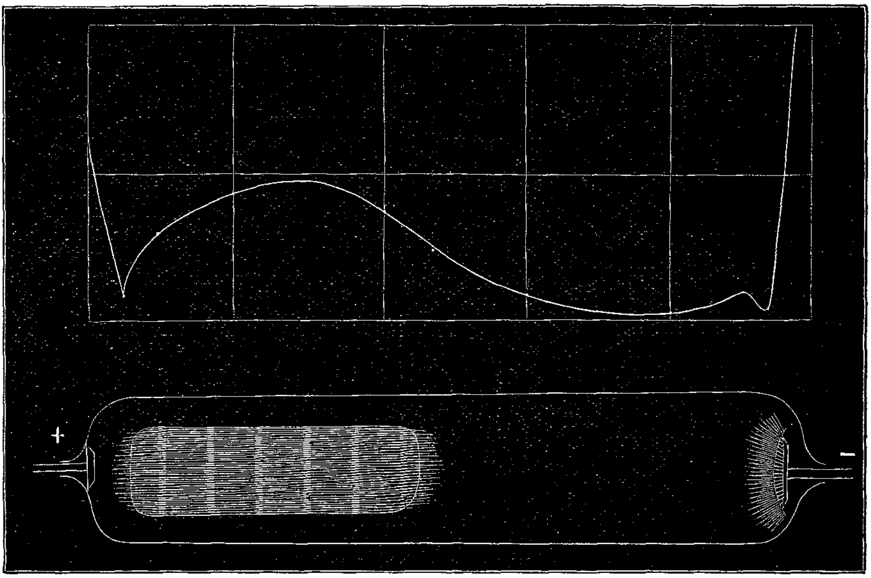
$$-\frac{d}{dx}(k_2 n_2 \mathrm{X}) = q - \alpha n_1 n_2,$$

we get if k_1 and k_2 are independent of x,

$$\frac{d^2\mathrm{X}^2}{dx^2} = 8\pi e(q - \alpha n_1 n_2)\left(\frac{1}{k_1} + \frac{1}{k_2}\right).$$

Thus $q-\alpha n_1 n_2$ is of the same sign as $d^2\mathrm{X}^2/dx^2$. Thus when $q-\alpha n_1 n_2$ is positive, that is when the ionization exceeds the recombination, the curve for X^2 will be convex to the axis of x, and this curve will be concave to the axis of x when the recombination exceeds the ionization. Places of sharp curvature will be regions either of great ionization or recombination. Fig. 4 is a curve for X^2 calculated from Graham's

Fig. 4.



results. It will be seen that there are two places of specially sharp curvature with the curvature in the direction denoting ionization, one, the most powerful one, just outside the negative dark space, the other near the anode, while in the positive light the curvature indicates recombination. It would seem as if the positive ions formed at the centre of ionization near the anode, in travelling towards the cathode, met with the negative ions coming from the centre of ionization near the cathode, that these positive ions combine with the negative until their number is exhausted, and on combining give out light, the region of recombination constituting the positive light. In the dark space between the positive light and the negative glow these positive ions from the centre of ionization near the anode are exhausted, so that there are none of them left for the negative ions coming from the centre near the cathode to combine with.

The nick in the curve denoting the centre of ionization near the cathode is present in all the curves given by Graham; the centre near the anode is not nearly so persistent. In several of the curves given by Graham there is no nick near the anode, though the one near the cathode is well marked, and in these tubes there is no well-developed positive light. The distribution of potential which accompanies the luminous discharge requires a definite distribution of electrification in the tube, this requires ionization and a movement of the ions in the tube before the luminous discharge takes place. There must, therefore, be a kind of quasi-discharge to prepare the way for the luminous one. Warburg (Wied. *Ann.* lxii. p. 385) has, in some cases, detected a dark discharge before the luminous one passes. It seems probable that such a discharge is not limited to the cases in which it has already been detected, but is an invariable preliminary to the luminous discharge.

Besides the "nicks" or places of specially sharp curvature fig. 4 shows that there is a small curvature in the direction indicating an excess of ionization over recombination all through the considerable space that intervenes between the positive light and the negative glow; as this region is one far away from places of great electric intensity it seems probable that in producing ionization the electric intensity at any point is helped by other agencies. The case of the cathode rays shows that the motion of charged ions tends to ionize the surrounding gas. E. Wiedemann, too, has shown that the discharge generates a peculiar radiation, called by him "Entladungstrahlen"; it is possible that these may possess the power of ionizing a gas through which they pass.