XXXIII. On the Hindú Quadrature of the Circle, and the infinite Series of the proportion of the circumference to the diameter exhibited in the four S'ástras, the Tantra Sangraham, Yucti Bháshá, Carana Padhati, and Sadratnamála

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XXXIII. *On the Hindú Quadrature of the Circle, and the infinite Series of the proportion of the circumference to the diameter exhibited in the four S'ástras, the Tantra Sangraham, Yucti Bháshá, Carana Padhati, and Sadratnamála.* By CHARLES M. WHISH, *Esq., of the Hon. East-India Company's Civil Service on the Madras Establishment.*

(*Communicated by the* MADRAS LITERARY SOCIETY *and* AUXILIARY ROYAL ASIATIC SOCIETY.)

Read the 15th of December 1832.

A′RYAB'HATTA, who flourished in the beginning of the thirty-seventh century of the *Cáli Yuga*,* of which four thousand nine hundred and twenty years have passed, has in his work, the *Aryab'hatiyam*, in which he mentions the period of his birth, exhibited the proportion of the diameter to the circumference of the circle as 20000 to 62832, in the following verse :

> *Chaturadhicam ṣatamaṣhṭaguṇandwáshashṭistathá sahasráṇám*
> *Ayutadwaya vishcambhasyásannó vritta pariṇáhah.*†

Which is thus translated :

" The product of one hundred increased by four and multiplied by eight, added to " sixty and two thousands, is the circumference of a circle whose diameter is twice ten " thousand."

The author of the *Lilávati*, who lived six centuries after A′RYAB'HATTA, states the proportion as 7 to 22, which, he adds, is sufficiently exact for common purposes. As a more correct or precise circumference, he proposes that the diameter be multiplied by 3927, and the product divided by 1250 ; the quotient will be a very precise circumference. This proportion is the same with that of A′RYAB'HATTA, which is less correct than that of

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* Or the sixth century of the Christian era.

† This verse is in the variety of the *Aryavrittam* measure, called *Vipula*.

METIUS, *viz.* 113 to 355, though in all three the circumference is rated at more than the truth. The *slóca* in the *Lilávati* is as follows:

> *Vyásé bhanandagnihaté vibhacté*
> *Chabánasúryaih paridhissussúcshmah*
> *Dwávimṣatinighnévihritétha sailaih*
> *Stúlóthavásyádvyavahárayógyah.*"*

A'RYAB'HATTA has not left behind him any record of the means by which he obtained so accurate a proportion. His work however shews so much geometrical knowledge, that there can be little doubt that he effected his approximation in the same manner with ARCHIMEDES, by a numeral calculation of the perimeters of inscribed and circumscribed polygons with the circle; for in the polygon of 768 sides to a circle whose radius is *one*, the perimeter of the inscribed polygon is 628316, and of the circumscribed 628326. The circumference of the circle will therefore be greater than 628316, but less than 628326, or nearly half their sum, namely, 628321, to which the above approximation is nearly similar.

In the *Súrya Sid'dhánta*, the radius of a circle is stated at 3438, in terms of minutes of the circumference, being about one-quarter of a minute too great; and in the following *slóca*, it appears that a very coarse rule is laid down by the author.

> *Yojanánám ṣatányashṭau bhúkarnó dwigunánitu*
> *Jadvargató daṣagunát padam bhúparidhirbhavét.†*

" Square the *yojanas* of the earth's diameter 1600, and multiply the square by 10, " and extract the root of the product: the root is the equatorial circumference of the " earth."

In such a matter, the strictest accuracy need not be required; but for a general rule, this would err considerably from the truth, being less correct than the proportion of 7 to 22; and yet we find the same in the *Cáma-dógdhrí*, a commentary on the *Súrya Sid'dhánta*, laid down in the following terms:

> *Vritta vargát daṣa hritat padamvyásóbhavédiha*
> *Evam parícshitóvyásah vrittastu viparítatah.*

" The root of the square of the circumference divided by ten, is the diameter; to find the circumference of a given diameter, reverse the rule."

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* This verse is in the *Upajátivrittam* measure.
† This verse is in the *Anushtubvrittam* measure.

In the *śástra* named *Tántra Sangraha,* we find the following lines :

> *Ishṭa vyásé haté nágavédavahnyabdhichéndubhih*
> *Tithyaṣwivibudhairbhacté susúcshmah paridhirbhavét*
> *Paridhérvyatyayáchaivamsusúcshmam vyasamánayét*
> *Tithyaṣwivibudhairnighnát chachatancéndunétratah*
> *Súcshmó vyásóhi védagni critóbhréndu hritóthava*
> *Tríṣaghna chacra liptábhyó vyásórtthéshwagnibhirhritéh.*

Which are thus translated :

" Multiply any given diameter by 104348, and divide the product by 33215; the " quotient is a very correct circumference. To find the diameter, if the circumference " be given, reverse the above numbers. If then you multiply the astronomical circum- " ference by 33215, and divide the product by 104348, you will have the diameter *in* " *terms of minutes of the periphery ; or it will be sufficiently exact if you multiply* " *the circumference by* 113, *and divide by* 355, *for a diameter.*"

The astronomer's circle, *anantapura* or " the universe," and *chachatan-cendunetram,* is, according to alphabetical notation and symbols respectively, 21600, that is, the terms of the minutes in the circumference of twelve signs, containing each thirty degrees. In the first proportion of 33215 to 104348, if the diameter of a circle be 1, the circumference will be 3.141592653921, &c., which is an excellent approximation, being correct to the ninth place of decimals, the tenth being too large ; the second proportion, 113 to 355, that of Metius, is a very useful one, being as 1 to 3.1415929, &c., correct to the sixth place of decimals, the seventh being rather more than the truth.

In another work, named *Carana-Padhati,* and in one called *Sadratnamála,* are the two following verses ; the first is from the former work :

> *Anúnanutnánananunnanutyaih*
> *Samáhataṣchacra kálávibhacṭah*
> *Chandámṣuchandrádhama cumbhipálaih*
> *Vyásastadardhantribhamaurvicásyát.*

" If the circumference of a circle in minutes be multiplied by 10000000000, and the " product be divided by 31415926536, the quotient will be the diameter of the circle " *in terms of the minutes of the circumference,* and its half will be the radius."

In the latter work is the subjoined verse :

> *Evanchátra parárdha vistritimahá vrittasya nahocsharaih*
> *Syádbhadrámbudhi sidha janma gaṇitasradáṣmayadbhápúgih.*

" If you proceed thus (as laid down in the former verse), and measure the circum-
" ference of a great circle by 100000000000000000 parts, the circumference will be
" equal to 314159265358979324 of *such parts.*"

The approximations to the true value of the circumference with a given
diameter, exhibited in these three works, are so wonderfully correct, that
European mathematicians, who seek for such proportion in the doctrine of
fluxions, or in the more tedious continual bisection of an arc, will wonder by
what means the *Hindú* has been able to extend the proportion to so great
a length. Some quotations which I shall make from these three books, will
shew that a system of fluxions peculiar to their authors alone among *Hindús,*
has been followed by them in establishing their quadratures of the circle;
and a few more verses, which I shall hereafter treat of and explain, will
prove, that by the same mode also, the sines, cosines, &c. are found with
the greatest accuracy.

I proceed to quote extracts from the *Tántra Sangraham.* The first, of the
measure called *A'nushtubvrittam,* is from the chapter upon sines, &c.

Vyásárdham prathamannítwatatóványátguṇannayét
Sambandhanniyamanchaivam vynéyóvyása vrittayóh.

" Having found the radius, you may construct the sines ; but you must first know the
" proportion between the diameter and the circumference."

The next is of the *Gítivrittam* measure :

Vyásé varidhinihité rúpa hrité vyásaságarábhihate
Trisarádhi vishama sanchyá bhactamriṇamswam prithacramát curyát
Yatsanchyayátra haraṇé critérnivrittá hritistujámitayá
Tasyá úrdhwa gatáyássamasanchyá taddalamgunóntésyát.
Jadvarggó rúpayutó háró vyásábdhighátacah prágwat.
Tabhyámáptam swamriṇé critédhané ṣódhananchacaraṇíyám
Súcshmah paridhissasyát bahucritwóharaṇatóti súcshmascha.

" Multiply the diameter by 4, and from it subtract and add alternately the quotients
" obtained by dividing four times the diameter by the odd numbers 3, 5, 7, 9, 11, &c.,
" do thus to the extent required ; and having fixed a limit, take half the even number
" next less than the last odd divisor for a multiplier, and its square *plus* one for a divisor.
" Multiply four times the diameter by the multiplier, and divide the product by the
" divisor, and add it or subtract it, according to the sign of the last quote in the series,
" from the sum of the series · thus the circumference of the given diameter will be
" obtained very correctly."

If we proceed according to the rule, we have an infinite series of the following form :

$$C = 4\,d \; - \; \frac{4\,d}{3} + \frac{4\,d}{5} - \frac{4\,d}{7} + \frac{4\,d}{9} - \frac{4\,d}{11} + \&c. \; \pm \; \frac{4\,d \times \frac{1}{2}p}{p^2 + 1}.$$

where C=circumference, d diameter, and p the last odd divisor diminished by unity. When $d=1$ the series becomes

$$C = 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \pm \&c. \frac{\frac{1}{2}\,p}{p^2 + 1} \right)$$

Then follows a verse of the *Gitivrittam* measure, explaining more fully the correction by which this series is brought to greater perfection.

Asmat sucshmatarónyó vilichyatécaschanápi samscárah
Anté samasanchyá dala varyassaicó gunassá éva punah
Yuga gunitó rúpayuttassamasancyá dala ható bhavéddhárah
Triṣarádi vishama sanchyá haranát paramé tadéva vácáryam.

" I now shew how the correction may be made more complete than in the former " rule: take the even figure next greater than the last odd divisor in the series $4 \times$ " $\left(1 - \frac{1}{3} + \frac{1}{5} - \&c. \right)$ that may have been fixed upon, and square its half, and increase " it by *unity ;* this is to be a multiplier : this multiplier multiply by 4, and the product " increased by unity multiply by half the original even figure ; this last product will be " a divisor : add to the result of the series the quotient of four times the diameter " multiplied by the new multiplier, and divided by the new divisor ; the sum will be a " more correct circumference."

The series by means of this correction becomes :

$$C = 4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \&c. \; \pm \; \frac{\frac{p^2}{4} + 1}{\left(p^2 + 4 + 1 \right)\frac{p}{2}} \right)$$

p being here the last odd divisor increased by unity.

The author being aware how slowly the series converges, found it necessary to correct the last quote, which is done very correctly by the rule he has exhibited.

Next follows a verse of the *Anushtubvrittam* measure.

Vyásavargádravihatat padamsyát prathamam phalam
Jatastattat phaláchchápiyávadiṣchántribhirharét
Rupádyayugmasancyábhirllabdéshwéshuyathácramam
Vishamánám yutétyacté yugmayógé vritirbhavét.

" Multiply any given diameter squared by 12, and extract the root of the product ;
" this is the first quote : divide this first by 3 for the second quote ; this second, and each
" so obtained quote, divide by 3 continually, place them in order, and divide them in
" succession by the odd numbers 1, 3, 5, 7, 9, 11, &c. Then add together the first,
" third, fifth, &c., quotes ; and add together also the second, fourth, sixth, &c. quotes ;
" then subtract the sum of the even from that of the odd, and the remainder will be the
" circumference of the circle, whose diameter was given."

The series laid down in the verse is thus expressed algebraically :

$$C = \sqrt{12} \times \left(1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \frac{1}{9.3^4} - \frac{1}{11.3^5} + \&c. \right)$$

The next verse is of the *Gítivrittam* measure :

Samapancháhatayóyá rupádyayujás chaturghna múlayutáh.
Tabhisshódaṣa guṇitádvyását prithagá hṛitétu vishamayaté.
Sama phala yágé tyacté syátishṭa vyása sambhavah paridhih.

" Divide the diameter multiplied by 16 severally by the fifth power of the odd
" numbers 1, 3, 5, 7, 9, &c., adding to each fifth power four times its root ; of the
" quotes thus obtained subtract the sum of the second, fourth, sixth, &c. from that of
" the first, third, fifth, and seventh, the remainder will be the circumference of the circle
" whose diameter was taken."

This is an infinite series of the following form :

$$C = \frac{16\,d}{1^5 + 4.1} - \frac{16\,d}{3^5 + 4.3} + \frac{16\,d}{5^5 + 4.5} - \frac{16\,d}{7^5 + 4.7} + \frac{16\,d}{9^5 + 4.9} - \&c.$$

Or, if the diameter be *one :*

$$C = 16 \times \left(\frac{1}{1^5 + 4.1} - \frac{1}{3^5 + 4.3} + \frac{1}{5^5 + 4.5} - \frac{1}{7^5 + 4.7} + \&c. \right)$$

The next verse of the *Gítivrittam* measure is as follows :

Vyásádwaridhi nihatát pṛitagáptantryáyugvimúld ghanaih
Trighna vyásé swamṛiṇam cramasah kritwápi paridhivánéyah.

" Divide four times the given diameter by the cubes of the odd numbers 3, 5, 7, 9,
" 11, 13, &c., subtracting from each its respective root ; the quotes thus obtained
" alternately add to, and subtract from three times the diameter ; this is the circumfe-
" rence of the circle whose diameter was given."

This is an infinite series (given in the *Carana Padhati* in nearly the same
words), as follows :

$$C = 3\,d + \frac{4\,d}{3^3 - 3} - \frac{4\,d}{5^3 - 5} + \frac{4\,d}{7^3 - 7} - \frac{4\,d}{9^3 - 9} + \&c.$$

Which, if d=1, will be:

$$C = 3 + 4 \times \left(\frac{1}{2.3.4} - \frac{1}{4.5.6} + \frac{1}{6.7.8} - \frac{1}{8.9.10} + \&c. \right)$$

The author then proceeds in the same measure:

Dwyádiyujám vacritayó vyécá hárádwinighna vishcambhe
Dhanamrinamanténtyórdhwa gataujacritidwi sahitáharódwighní.

" Multiply the diameter by 4, and divide the product severally by the squares of the
" numbers 2, 4, 6, 8, 10, &c., subtracting 1 from each square : the quotients alternately
" add to, and subtract from, twice the diameter : rectify the ṣum obtained, by taking the
" next odd number less than the last even figure squared, squaring it, adding 2 to the
" square, doubling this sum, and with this thus obtained as a divisor, dividing four times
" the diameter : this quotient add or subtract, as is required, from the sum formerly
" obtained, for a very correct circumference."

The series thus obtained is :

$$C = 2d + \frac{4d}{2^2-1} - \frac{4d}{4^2-1} + \frac{4d}{6^2-1} - \frac{4d}{8^2-1} + \&c. \frac{\pm}{} \frac{4d}{\overline{p-1}^2 + 2 \times 2}$$

Where C=circumference, d=diameter, and p=the last even number
squared in the series, which, if the diameter be 1, becomes,

$$C = 2 + 4 \times \left(\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \&c. \frac{\pm}{} \frac{1}{\overline{p-1}^2 + 2 \times 2} \right)$$

This series, *viz.* $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \&c.$ can easily be proved to be
equal to the arc of $90° - \frac{1}{2}$, therefore 2+4 into the series will equal the
whole circumference, the diameter being *one*, as in our author's series.

Two other series are then exhibited in lines of the *Gíti* measure.

Dwyádéschaturádérva chaturadhicánánniréca vargassyuh
Háráh cunjara gunitá vishcambhaswamati calpitóbhájyah
Phalayutirádyé vrittam bhájyadalam phalavihínamányatra.

" Take the squares of the terms, diminished by 1, of the two arithmetical progressions
" whose first terms are respectively 2 and 4, and ratio of progression 4, for divisors : in
" the former series, divide 8 times the diameter by the divisors severally, and the sum of
" the quotients is the circumference of the circle ; in the latter, subtract from 4 times
" the diameter the sum of the quotients of 8 times the diameter divided by the divisors
" severally, and the result is the circumference in the second case."

The two series thus explained are found to be of the following forms :

$$C = \frac{8\,d}{2^2-1}+\frac{8\,d}{6^2-1}+\frac{8\,d}{10^2-1}+\frac{8\,d}{14^2-1}+\frac{8\,d}{18^2-1}+ \&c.$$

$$C = 4\,d - \left(\frac{8\,d}{4^2-1}+\frac{8\,d}{8^2-1}+\frac{8\,d}{12^2-1}+\frac{8\,d}{16^2-1}+ \&c.\right)$$

Which, if $d=1$, become respectively :

$$C = 8 \times \left(\frac{1}{1\cdot3}+\frac{1}{5\cdot7}+\frac{1}{9\cdot11}+\frac{1}{13\cdot15}+\frac{1}{17\cdot19}+ \&c.\right)$$

$$C = 4 - 8 \times \left(\frac{1}{3\cdot5}+\frac{1}{7\cdot9}+\frac{1}{11\cdot13}+\frac{1}{15\cdot17}+\frac{1}{19\cdot21}+ \&c.\right)$$

Now, let the former series within the brackets be a, and the latter b; then it can be easily proved that $a+b$, or the series $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\frac{1}{9\cdot11}+\&c.=\frac{1}{2}$; and it has been mentioned above that the series $\frac{1}{1\cdot3}-\frac{1}{3\cdot5}+\frac{1}{5\cdot7}-\frac{1}{7\cdot9}+\&c.=$ (when the diameter is one), the arc of $90°-\frac{1}{2}=a-b$; having therefore the values of $a-b$ and $a+b$, it can be easily proved that $a=$ the arc of $45°$, and $b=\frac{1}{2}-$ arc of $45°$; therefore, in the first of these quadratures, $8a=$ circumference, and in the second, $4-8b=$ circumference also, as taught by the author.

The author, after laying down the above series, proceeds to shew in numbers a proportion of the circumference to the diameter in the verse which has been originally quoted and translated, and then finishes his chapter by rules for finding the sines and cosines, of which mention will be made hereafter.

In the *Carana Padhati*, the sixth chapter commences thus :

> *Vyásácchaturghnát bahuṣah prithac sthát*
> *Tripanja saptadyayugáhritáni*
> *Vyasé chatuoghné cramaṣastwriṇam swam*
> *Curyáttadásyát paridhissusúcshmah.*

" Divide the given diameter multiplied by 4 severally and continually by the odd
" numbers 3, 5, 7, 9, 11, &c., and the quotients thus obtained, alternately subtract
" from, and add to, the diameter multiplied by 4. The result is the precise circum-
" ference."

This series is the same with the first laid down by the author of the *Tantra Sangraha*, namely,

$$C = 4 \times \left(1 - \frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+ \&c.\right)$$

The next verse in the chapter is of the measure called *Gitivrittam.*

Vyásádwana samgunitát prithagáptantryádyay ugvimúlaghanaih
Irigunita vyásé swamrinam Cramaṣah critwápi paridhiránéyah.

" Divide the given diameter multiplied by 4 severally and continually by the cubes of
" the odd numbers 3, 5, 7, 9, 11, &c., subtracting from each cube its respective
" root; the sums thus obtained alternately add to, and subtract from, 3 times the
" diameter: thus you will obtain the circumference of the circle whose diameter was
" given."

This series is also the same with one of the former book, namely,

$$C = 3 + 4 \times \left(\frac{1}{2.3.4} - \frac{1}{4.5.6} + \frac{1}{6.7.8} - \frac{1}{8.9.10} + \&c. \right)$$

The fourth verse of the chapter is of the measure called *Indra Vajra-vrittam.*

Vargairyujámvadwigunairnnirécaih
Vargícritairvarjitayugma vargaih
Vyásancha shadghnam vibhajét phalam swam
Vyásé trinighné paridhistadásyat.

" Add to three times the diameter the sum of the quotes obtained by dividing six
" times the diameter by the square of twice the square minus one of the even numbers
" 2, 4, 6, 8, 10, &c., subtracting from each the square of its even figure respectively.
" The sum is the circumference."

This is an infinite series thus expressed algebraically :

$$C = 3d + \frac{6d}{(2.2^2-1)^2-2^2} + \frac{6d}{(2.4^2-1)^2-4^2} + \frac{6d}{(2.6^2-1)^2-6^2} + \&c.$$

Which, if the diameter = 1, becomes

$$C = 3 + 6 \times \left(\frac{1}{1.3.3.5} + \frac{1}{3.5.7.9} + \frac{1}{5.7.11.13} + \frac{1}{7.9.15.17} + \frac{1}{9.11.19.21} + \&c. \right)$$

The author proceeds with the verse originally quoted, for determining the
diameter and radius in terms of minutes of the circumference, and then
teaches how, by certain series, the sines, cosines, &c. are to be constructed.
He next exhibits a stanza for finding an arc of the circumference of a
circle by means of the sum obtained by multiplying the sine of the arc into
its radius and dividing the product by its cosine. This sum, it will be
readily observed, is equivalent to a tangent of that arc, for cosine is to
radius as sine is to tangent; but the tangent individually does not appear in
the mathematics of the *Hindús.*

The lines are as follow :

Vyásárdhénahatádabhíshṭa guṇatah Cótyaptamádyam phalam
Jyávargéna vinighnamádimaphalam tattat phalanchá harél
Crityá cóti guṇasya tatratuphaléshwécatripanchá dibhi
Bhuctéshwójayutéstyajét samayutim jívádhanussishtyaté.

" Multiply the given sine by radius, and divide the product by cosine for the first
" quote ; multiply this quote by the square of the sine, and divide the product by the
" square of the cosine for the second quote : multiply and divide this last quote, and so
" continually each obtained quote, by the square of the sine and the square of the cosine
" respectively, and the quotes obtained by this means divide in succession by the odd
" numbers 1, 3, 5, 7, 9, 11, &c.; then the sum of the 2d, 4th, 6th, 8th, &c. quotes
" being subtracted from the sum of the 1st, 3d, 5th, 7th, 9th, &c. quotes, the remainder
" will be the arc of the sine which was taken."

Note.—*If the cosine be less than the sine of the given angle, change the names of*
the two, and proceed as laid down in the rule.

The infinite series thus beautifully and concisely expressed in the *Sanscrit*
slóca, is the following :

$$a = \frac{r \times s}{cos.} - \frac{r \times s^3}{3\,cos.^3} + \frac{r \times s^5}{5\,cos.^5} - \frac{r \times s^7}{7\,cos.^7} + \frac{r \times s^9}{9\,cos.^9} - \&c.$$

Where a=arc ; r=radius ; s=sine, and cos=cosine.

In this series, the first quote equals the tangent of the arc $=t$; the second
quote equals tangent cubed, divided by thrice radius squared ; the third
equals tangent to the fifth power divided by 5 into radius to the fourth power ;
or, algebraically, thus :

$$a = t - \frac{t^3}{3\,r^2} + \frac{t^5}{5\,r^4} - \frac{t^7}{7\,r^6} + \frac{t^9}{9\,r^8} - \&c.$$

Which is easily proved to be true by a process in fluxions, which demon-
strates the fact that the fluxion of the tangent of an arc *is to* the fluxion of the
arc itself *as* the square of the secant is to the square of the radius ; in which
case, the fluxion of the arc is proved to be equal to $\frac{r^2\,\dot{t}}{r^2 + t^2}$ and if $r^2\,\dot{t}$ be

divided by $r^2 + t^2$, the quotient will be $\dot{t} - \frac{t^2\,\dot{t}}{r^2} + \frac{t^4\,\dot{t}}{r^4} - \frac{t^6\,\dot{t}}{r^6} + \frac{t^8\,\dot{t}}{r^8} - \&c. = \dot{a}$; and

the fluents of each being taken, it becomes $t - \frac{t^3}{3\,r^2} + \frac{t^5}{5\,r^4} - \frac{t^7}{7\,r^6} + \frac{t^9}{9\,r^8} - \&c. = a =$

the arc itself, as is laid down in the *Carana Padhati.*

It is also seen that from this verse the matter is derived of the former series ; for if (in the first series) the arc of 45° be taken, then the tangent will be equal to radius; and if radius $= 1$, the above series becomes $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} +$ &c. $= a$; and four times this sum will be the semi-circumference, or when the diameter is 1, the whole circumference; therefore $4 \times \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \right.$ &c.,$\left. \right)$ the circumference, as shewn in the *Tantra Sangraha,* and in the *Carana Padhati.*

I proceed now to quote some verses from the *Sadratnamála.* The first which I shall extract is from the chapter on sines, and is of the measure called *Sálini-vrittam.*

> *Vargádwyásasyárcanighnát padamyat*
> *Tatryamṣó yastécha tattannavámṣáh*
> *Dwigna vyécaicadwi purvaujayugmah*
> *Chinnányaicya dwyantaram vrittanáhah.*

" Square the diameter and multiply the product by 12, and extract the root of this
" product ; the root obtained will be the *modulus* of odd quotes, which if you divide
" by 3, the quotient will be the *modulus* of even quotes. Divide each modulus
" continually by 9, and the quotient thus obtained from the former, divide by double
" the numbers 1, 3, 5, 7, 9, &c. minus 1 respectively, and the quotient obtained from
" the latter, by double the number 2, 4, 6, 8, 10, &c. minus 1 respectively, add up the
" new obtained quotes, and subtract the sum of those gotten from the even from the
" sum of those gotten from the odd modulus, the remainder is the circumference of the
" circle."

The next verse of the *Sarddúla-viccridita* measure is this :

> *Vyasárcaghnacriteh padegnibhiratónítécha tattat phaláh*
> *Cháthaicyadyayugá hriteshu paridhirbhédóyugójaicyayóh.*
> *Evanchátra parárdha vistriti mahávrittasya náhácsharaíh*
> *Syátbhadrámbudhi sidha janma ganitasradhásmayatbhúpagih.*

" Square the diameter and multiply the product by 12, and extract the root of this
" product; this root divide continually by 3, and the quotients thus obtained by 1, 3,
" 5, 7, 9, 11, &c., and subtract the sum of the 2d, 4th, 6th, 8th of the last obtained
" quotes from the sum of the 1st, 3d, 5th, 7th, 9th, &c. If you do thus, and measure
" the diameter of a great circle by 100000000000000000 equal parts, the circumference
" will be equal to 314159265358979324 of such parts."

The rule laid down in this verse is exactly similar to that communicated by Dr. Halley to the Royal Society of London ; and is founded upon the

principles laid down in another *slóca* of the *Vasantitilaca-vrittam* measure in
the same chapter, as follows :

> *Cotíhrita triguna báhu vadhé chatasmá*
> *Tattat phalácchcha bhujavarga hatáttu cotyáh*
> *Crityá críteshucha dharágni sarádi bhacte*
> *Shwojaicyatastyajatuyugmayutim dhanustat.*

" The radius into the sine divided by the cosine is the first quote : this multiplied by
" the square of the sine, and divided by the square of the cosine, is the second quote ;
" this second, and those obtained continually in the same way, multiply and divide by
" the square of the sine and the square of the cosine respectively : divide the quotes in
" order by 1, 3, 5, 7, 9, 11, &c. respectively, and the difference of the sum of the 1st,
" 3d, 5th, &c., and of the 2d, 4th, 6th, &c., will be the arc whose sine was taken."

This, of which the meaning is the same with that in the *Carana Padhati*, is
expressed thus :

$$a = \frac{r \times s}{cos.} - \frac{r \times s^3}{3\,cos.^3} + \frac{r \times s^5}{5\,cos.^5} - \frac{r \times s^7}{7\,cos.^7} + \&c.$$

Now if the arc of 30° be taken, the first quote = the tangent of that arc
= $\sqrt{\frac{1}{3}}$; to multiply this by the square of the sine, and divide it by the
square of the cosine, it is sufficient to divide it simply by 3 (because the
square of the sine of 30° is one-third of the square of the cosine of that
angle) : divide then each quote in succession by the odd numbers 1, 3, 5, 7,
9, 11, &c., respectively ; the arc, thus obtained, multiplied by 6, will equal
the semi-circumference when the radius = 1, or the whole circumference if
the diameter be 1 ; but $6 \times \sqrt{\frac{1}{3}} = 2 \times \sqrt{3} = \sqrt{12}$; therefore $\sqrt{12} \times$
$\left(1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \frac{1}{9.3^4} - \frac{1}{11.3^5} + \&c. \right)$ equals the whole circumference when
the diameter is *one*, according to the rule laid down in the *Sadratnamálah*
in the verse " *Vyasárcaghnacriteh*," &c., and in the *Tantra Sangraha* in
the verse " *Vyasa Vargadravihatat*," &c. before quoted. The rule also for
finding the sum of any arc from the tangent is thus expressed in the same
work in four *slócas* of the *Anushtub* measure.

> *Vrittánayana márgínábreshta jyáná dhanurnayét*
> *Ishta jyá trijyayórghátát cótyáptam prathamam phalam*
> *Jyá vargom gunacam critwá cotí vargancha háracam*
> *Prathamádi phalébhyóthá néyáphalalatirmuhuh.*
> *Ecatryádyája sanchyábhirbhactéshwétéshwanucramát*
> *Ojánám yutitastyacte yugma yógé dhanurbhavét*
> *Dóh cótyóralpaméveshtam calpaníyamihóditam*
> *Muhuh critépi labd'hínánnanyathávasitirbhavet.*

" According to the genius of the rules which we have laid down for the circumference
" of the circle, the arc of any given sine may be found thus : take a given sine multiplied
" by radius, and divided by the cosine to the given sine for the first quote ; let the square
" of the sine be a constant multiplier, and the square of the cosine a constant divisor,
" the quotes obtained by multiplying and dividing (by these) the first quote, must be
" placed in order one under the other, and divided in succession by the odd numbers 1, 3,
" 5, 7, 9, 11, &c. ; then subtract the sum of the even from that of the odd quotes,
" and the remainder will be the arc whose sine was taken. If the sine of the given arc
" be greater than its cosine, call *that* the cosine, and the cosine to it the sine."

Having thus submitted to the inspection of the curious eight different
infinite series, extracted from *Bráhmanical* works for the quadrature of the
circle, it will be proper to explain by what steps the *Hindú* mathematician
has been led to these forms, which have only been made known to Euro-
peans through the method of fluxions, the invention of the illustrious
NEWTON. Let us first, however, know the age of these works ; and as far
as can be determined, the authors. First, then, it is a fact which I have
ascertained beyond a doubt, that the invention of infinite series of these
forms has originated in Malabar, and is not, even to this day, known to
the eastward of the range of *Gháts* which divides that country, called in
the earliest times *Céralam*, from the countries of *Madura, Coimbatore,
Mysòre*, and those in succession, to that northward of these provinces.

The author of the *Sadratnamálah* is SANCARA VARMA, the younger brother
of the present *Rájá* of *Cadattanáda* near *Tellicherry*, a very intelligent man
and acute mathematician. This work, which is a complete system of
Hindú astronomy, is comprehended in two hundred and eleven verses of
different measures, and abounds with fluxional forms and series, to be
found in no work of foreign or other Indian countries.

The author of the *Carana Padhati*, whose grandson is now alive in his
seventieth year, was PATHUMANA SÓMA YAJI, a *Nambútiri Bráhmana* of
Tirusívapura (Trichúr) in Malabar. In the first verse of the commentary
on his work, he is thus mentioned :

> *Nutuna griha sama sutarachitayah carana padhatervidusha*
> *Bhasham vilichati caschit balanam bhodhanarthamalpadhiyam.*

" The *Carana Padhati*, a path to arithmetic, composed by NUTUNAGRIHA SÓMA
" SUTA, I now proceed to explain in the common tongue, to adapt it to the genius of
" young scholars."

Nutunagriha Sóma Súta bears the meaning in *Sanscrita* of the Malabar terms " Puthamana Sóma Yaji," the first word being the name of his house.

The last verse of his work contains its date in numerical letters in terms of days of the *Caliyuga ;* a mode of dating which, in the country, is to be traced back to upwards of one thousand seven hundred years ago, as may be seen in the inscriptions taken from the pagoda of *Tiruvunnúr* near Calicut. The verse is as follows :

Iti sivapura nam agramajah Capiyajwa
Cimapi Carana padhatyahwayam tantra rupam
Vyadhita ganita metutsamyagalocyasantah
Cathitamihavidantassuntu santosha pantah.

The words *Ganita Metutsamyac,* written in numbers, amount to 1765653 in terms of days of the present age; which is found to agree with A.D. 1733, being 86 years and 271 days before the 31st of December 1819 ; Sóma Yaji was not, however, inventor of the system by which he formed his infinite series.

The author of the *Tantra Sangraha,* who was educated in the college of *Bráhmanas* at Trichúr (which college still exists, and which is a place of reputed sanctity, and certainly of respectable learning), has, among his works on arithmetic and astronomy, laid the foundation for a complete system of fluxions, and opened a mine of wealth in mathematics to those students who can boast of being his followers, of which they are most eminently jealous. The copy of the work, which I have obtained with some difficulty, by frequent intercourse with this interesting society, bears in itself marks of antiquity, the commentary in the vulgar tongue being written in a language which is not now generally current in Malabar, and the forms of many letters differing materially from those of the present day.
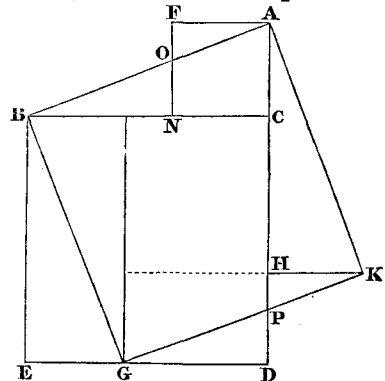
The author of the work is Talaculattúra Nambútiri, of *Cérala*, or Malabar ; the first epithet being the name of his house, the latter the title of his rank : he flourished in the forty-sixth century of the present age, or *Caliyuga.*

The testimonies as to the author, and the period in which he lived, are the following, *viz.* The general consent of the learned in Malabar ; the date which is shewn in the commencement of the work itself, namely, the year 4600 of the *Caliyuga ;* the mention made of him in the first chapter of a work named *Driccaranam* by his commentator, the author of the *Yucti-Bháshá,* Cellalura Nambutiri, in which, while relating the history of the progress of astronomy, from the improvements of A'ryab'hatta up

to his own period, he thus observes: "Much difference having occurred
"in astronomical calculations, in the year of the *Caliyuga* 4532 a *Bráh-*
"*man* of high rank who lived on the coast of the western ocean, having
"examined the heavens for twelve years, established what is laid down
"in the *Tantra-Sangraha.*" This is the evidence of the author of the
Yucti-Bháshá, the commentary on the *Tantra-Sangraha,* concerning the
author of the latter work: the date of the *Driccaranam* is mentioned in
the latter part of the work, *viz.* the 783d of the Malabar era; and in the
summary account of the periods of astronomy, it is written 4708 of the
Caliyuga, both of which coincide with the year 1608 of the Christian era.

A farther account of the *Yucti-Bháshá,* the demonstrations of the rules
for the quadrature of the circle by infinite series, with the series for the
sines, cosines, and their demonstrations, will be given in a separate paper:
I shall therefore conclude this, by submitting a simple and curious proof of
the 47th proposition of Euclid, extracted from
the *Yucti-Bháshá.* In the accompanying figure,
let ABC be a triangle, having the angle at C
a right angle: on AC describe the square
ACFN, and on BC describe the square
BCDE: on ED take EG equal to AC, and
on AD take. AH equal to BC, draw HK
perpendicular to AD and equal to AC, join
BG, GK, and AK.

The mathematician will easily prove that the three spaces AFO, PGD,
and BEG, parts of the squares SACFN and BCDE not included in the
figure ABGK, are equal to, and identical with, the spaces KHP, OBN, and
AHK, not occupied by any parts of the above two squares; *id est,* that the
sum of the two squares ACFN and BCDE equals the figure ABGK; but
this, from its construction, is a square, and is drawn upon the hypothenuse
ABC. This is probably the form by which PYTHAGORAS discovered the
celebrated problem, which EUCLID afterwards so beautifully illustrated in
the 47th proposition of the first book of his 'Elements.'