Algebra in Schools
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ALGEBRA IN SCHOOLS.

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I AM venturing to call the attention of the Association to a difficulty that I have to contend with, as a private tutor, when assisting boys fresh from school either to continue their mathematical studies or to prepare for some of the public examinations. It is of course impossible for me to assert positively that this difficulty is experienced by tutors generally, but the strong conviction that it is felt by others as well as by myself has led to my bringing the matter forward.

In the case of the great majority of schoolboys their mathematical outfit includes some knowledge of arithmetic, algebra, Euclid, and trigonometry. In the case of three of these, the arithmetic, Euclid, and trigonometry, the tutor finds a certain amount of knowledge gained, which he will probably have to strengthen and make more accurate, but which will in some degree serve as a foundation. In algebra, however, the work has usually to be done all over again. The reason for this appears to be that there is an ever-growing divergence between the conception of the nature and objects of algebra that dominates school teaching and the conception that regulates the application of algebra to more advanced mathematics. As a rough attempt to give, in as few words as possible, the character of this divergence, it may be said that at school the end in view is the solution of equations, while later on the student finds his main business is the study of functions.

A teacher who makes it his endeavour to treat the subject in a more satisfactory way has, fortunately, the advantage of the guidance of eminent authorities. Chrystal's *Algebra* and Clifford's *Common Sense of the Exact Sciences* will help him to lay out the best road, and in the published addresses of Chrystal, Henrici, Sylvester, and others there are many directions and suggestions that will be of the greatest service to him. Nevertheless, these high authorities have not as yet had much influence on school books. I do not know of any English book suitable for elementary teaching that is constructed according to their recommendations. A "Chrystal for Beginners" is sadly wanted. The *Indian Text-Book of Algebra* by Rhadakrishna, in the library of the Association, partly supplies this want, and I have myself used it with pupils with satisfactory results, but hitherto all negotiations for an English edition have broken down, and it would be necessary to send to Madras for a copy.

This persistence of school books in the old tracks is doubtless due to the magnitude of the mass to be moved before progress can be made. We sometimes, I think, forget how comparatively modern is the introduction of algebra into Europe. When it first came into England it was called the Cossick art, or the art of Cossick numbers, and Cossick was from "cosa," a thing to be found. For a long period, more than a hundred years, the notion that Professor Chrystal ascribes to his pupils that "higher algebra is the solution of harder and harder equations," was the notion gravely entertained by the best mathematicians. It was through the effort to solve harder and harder equations that all the earlier advances were made, and school teaching in this, as well as in many subjects other than mathematical, has but stood still while the boundaries of knowledge were being enlarged.

I disclaim at the outset any right to speak on mathematical subjects with authority, and the only plea on which I ask for your consideration of some suggestions as to the form which algebraical elementary teaching should take is, that whether right or wrong, they are at least based on practical experience. To save time, and to make my meaning clear, they may perhaps seem to be brought forward in too decided a manner, but I fully recognise that they have no value at all until, and so far as, they are confirmed by others of attainments superior to my own.

Algebra being described as a generalisation of arithmetic, the similarities and differences between the algebraical $f(x)$ and the arithmetical $f(t)$, where t stands for 10, should be fully explained. In the first instance, $f(x)$ should have numerical coefficients only. It would be necessary here to give the student a new view of the symbol 0, as being not merely a negation of magnitude or a starting-point of the positive scale, but as a definite point in a scale extending infinitely on each side of it. Nothing should be defined as that which is less than any assignable quantity, and from the fact that it is indifferent which sign we ascribe to it, the corresponding indifference in the sign of $\frac{1}{0}$ or ∞ should be mentioned as something to be more fully treated later on, and might, I think, with advantage be made the subject of some geometrical illustrations. Clifford's *Common Sense of the Exact Sciences* affords a model for the treatment of the associative, commutative, and index laws, and when once these have been mastered, the pupil may be told that he may deal with the fundamental processes of algebra just as if they were processes in arithmetic, except that there is no carrying.

In this first chapter, however, on addition, subtraction, multiplication, and division as applied to an integral function of x with numerical coefficients, there is room for some things that are almost always overlooked or omitted. The student should be encouraged to work everything with detached coefficients, and he should be trained to verify his results by substituting 1, 2, or 3 for x. A habit of constant verification cannot be too soon encouraged, and the earlier it is acquired the more swiftly and almost automatically it is practised.

In the second chapter we might have $f(x)$ still integral and finite, but with literal coefficients; the theorem often spoken of as the remainder theorem, that if $f(x)$ be divided by $(x-a)$ the remainder is $f(a)$, may then be introduced; and from the corollary that if $f(a)=0$, $f(x)$ is divisible by $x-a$, the subject of factorisation is made plain and easy. There may be some doubt whether or not at this stage to introduce the principle of indeterminate coefficients, though it might easily be done. I believe it would be wise to do so, as it would afford the means of treating more completely the separation of quadratic functions into linear factors. This separation, which is the same thing as the solution of quadratic equations, would necessitate the use of the radical symbol. Following Chrystal, Todhunter, Hall and Knight, and the majority of writers, $\sqrt{a}$ should be considered a quantity having one and not two values, although the algebra of C. Smith and the article by Professor Kelland in the *Encyclopædia Britannica* make $\sqrt{a}$ have two values. Supposing some work done to give the pupil readiness in multiplying and dividing functions with literal coefficients, he might be ready to pass on to chapter three.

The third chapter would require some knowledge of the first and most elementary results in permutations and combinations. It should be noticed that Clifford in the work so often referred to starts from these at the very beginning, while Chrystal defers the subject to a very late period. These first results, being purely arithmetical in character, may be put in anywhere, and I am inclined to think the sooner the better. The student might now proceed from $f(x)$ to $f(xy)$, $f(xyz)$, and so on, noting the number and kinds of terms he may reckon on finding in a function of any of these classes of given degree. The consideration of these expressions and their classification would afford ample material for explaining the terms "dimensions," "homogeneous," and "symmetry." At present, although "homogeneous" is usually defined somewhere in the first three pages of a school algebra, the schoolboy never knows anything about its meaning, as he has not been used to apply it. Symmetry should then be explained and freely illustrated, the symbols Σ and Π used by Chrystal for a product should be introduced. If, for example, xyz, abc were the elements, such symmetrical products as $\Pi(x+y-c)$ and $\Pi(x+y-a-b)$ should be expanded, the result being expressed with the Σ and Π symbols, the pupil being taught strictly to account for the twenty-seven terms of the first or the sixty-four terms of the second. The permutations and combinations that were mentioned just now will be wanted for the consideration of the product $(x+a)(x+b)(x+c)$ . . . etc.; and the binomial theorem for a positive integral index may be then deduced.

If the next chapter deals with fractions, and the pupil has been taught to work Least Common Multiple in arithmetic by separation into factors and not by the division method, and has been shown the special characteristics of the Highest Common Factor process in algebra, I think he may be told that every process in algebra is step by step identical with the corresponding process in arithmetic, with one exception, namely, that it is often better to add algebraic fractions first two together, then a third, and so on, instead of all at once. The subjects of ratio, proportion, and variation are all fractions over again, with some new notation and new names added, several of the symbols being purely alternative, some, as for instance, $a:b:c:d=p:q:r:s$, expressing in a concise form what could have previously been stated only in some lengthy fashion.

I will not trespass on your time longer by carrying the subject on further. It is not the treatment of indices, surds, the complete binomial theorem and logarithms that makes the tutor's task a difficult one. It is because he finds that what ought to have been learnt at the beginning has never been learnt, and that his pupils have got into a clumsy mechanical way of dealing with questions that it is almost impossible to get them out of. No doubt this difficulty has arisen from hurrying over the rudiments so as to arrive at problems in equations, in order that boys may have some plausible answer to their natural question, "What is the good of algebra?" and because it is believed that these problems interest the youthful mind. I sometimes feel a doubt, however, whether boys really enjoy being introduced to such exercises as, "A says to B, how much money have you got?" and B makes a very singular hypothetical reply; or to the fish whose body is half as long again as his head and tail together, while head and tail have given relations of magnitude. I cannot but suspect that they must feel that there is something unpractical in these problems. There must, however, be equations, and there should be some illustrative problems, but the other parts of algebra should not be sacrificed to them. So long as the classification of functions, the remainder theorem, and the analogies and differences between arithmetic and algebra are all duly attended to, the fish, the market-woman with her eggs, the stage-coaches in opposite directions, and other old friends may be occasionally welcome to enliven the scene.

Let me conclude by again earnestly asking you to believe that I am not making these recommendations in any spirit of presumption. While thoroughly convinced that there is something wrong in our school system, I am equally convinced of the great improbability of my having hit on the best remedy, but have still a hope that even crude and imperfect suggestions may point the way to something that will be far better. G. HEPPEL.

MATHEMATICS FOR ASTRONOMY AND NAVIGATION.

I was led to the present subject by observing that every one, almost without exception, is interested in astronomy, yet from lack of learning spherical trigonometry very few of those who have passed through an ordinary school and college course of mathematics can understand a book of physical astronomy, or at least any other than a very elementary and popular exposition of this royal science. The ordinary well-educated man cannot only not predict an eclipse, but he has no idea of the method followed in the prediction. His astronomy is all taken on faith or on authority; he knows only results, nothing of processes. The work of Kepler is as much a mystery to him as a theological dogma. If he possess a telescope, he can use it little better than a toy; he has no power of adding anything to the sum of astronomical knowledge. Now this seems a pity: I think it is not necessary, for my experience tells me that spherical trigonometry is not much harder than plane trigonometry.

I make bold to say that an ordinary well-educated man has less notions of physical astronomy than a lady who has been taught at her boarding school the use of the globes. Take, *e.g.*, this problem from Keith, p. 351, Prob. lxxvi: *The lat. of a place and the day of the month being given, to find all those stars that rise and set acronycally and heliacally.* I confess my own inability to answer it without the book, even were I allowed the globe. By the bye, should not a pair of globes be an article of teaching apparatus in every school? It is only on a globe that the relative form and position of countries can be accurately represented. Two angles, lat. and long.; and two signs, N. and S., and E. and W. (given the centre and the radius), would serve to represent the dimensions of a sphere far better than length, breadth, and thickness; and it is probable that if mankind had earlier known that they were inhabitants of a globe, geometry would have been very different from what it is now. How much would a little knowledge of spherics simplify the doctrine of poles and polars and reciprocation. Is it not a real disgrace how few men or women have the slightest knowledge of how to draw the curved lines on any map; or what is meant by conical, orthogonal, or Mercator's projection; or how to ascertain and record their own position if they happen to be travelling outside the region of maps, clocks, and telegraphs; or how to make a sun-dial; or how to tell the time from the position of sun, moon or star; or how to set a wind-vane; or which way the variation of the compass needle is now changing in England or on the Atlantic? A very clever man who had spent most of his life road-surveying in New Zealand told me that *the sun was always due E. or W. at six o'clock!* How many know what the *Saros* is, though known to the Greek astronomer; or the relation between a sphere and a cylinder of equal height and diameter, discovered by Archimedes; or how eclipses are calculated, though to explain this principle requires no mathematics higher than algebra; or whether eclipses of the sun or moon are most frequent? How many could tell that the planets are to be looked for in the zodiac, but not the asteroids? How many understand anything about precession of the equinoxes, though known before the Christian era; or from what point R. A. is reckoned; or where the first point in Aries is? How many know that a ship's shortest course is along a great circle; or how long. and lat. are discovered by the sextant?

I believe there are very few references to plane trigonometry in Casey's *Spherical Trigonometry*, which I suppose is at least as good as any other hand-book.

I would say experience teaches me never to ask for definitions, unless the pupil has been bidden (which he seldom should be) to repeat a definition, and never to flatly contradict a pupil's answer if you can avoid it, but if it is vague and uncertain, try and get him by questioning and suggesting to improve it into what is clear and sufficient. As to the question of *names* v. *things*, I think the modern preference for things may be carried too far. Mr. Ruskin says truly, I think, that the main thing learnt even at a university is the power of expression. It may be said we must have something to express; but there is not likely to be any lack of that when we have to teach the elements or write even business letters. A pupil may be excellent in the manipulation of algebraic formulæ, but from his non-cultivation of language very deficient in power of expressing even a simple geometrical truth in ordinary language.

To teach spherical trigonometry or spherics, I take it we should have a large wooden globe painted a dull black and marked with a faint equator and meridians. On this arcs of great and small circles should be drawn with a pair of compasses provided with white chalk pencil. To draw a great circle easily I suppose one leg at least should be