Note on the Voltage Ratios of an Inverted Rotary Converter

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L. *Note on the Voltage Ratios of an Inverted Rotary Converter.* *By* W. C. CLINTON, *B.Sc., A.M.Inst.E.E.; Demonstrator in the Pender Electrical Laboratory, University College, London* *.

IF a separately excited rotary converter be driven at a constant speed with the rings and brushes on open circuit, the ratios of the maximum alternating E.M.F. between the rings to the E.M.F. over the brushes is given by a well-known expression on the assumption of a sinusoid distribution of the air-gap magnetic flux over the surface of the armature.

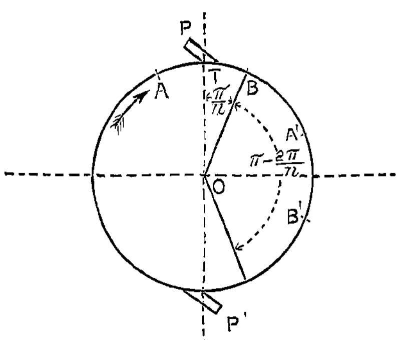
Putting E_a and E_c for the alternating and continuous E.M.F.'s respectively, and n for the number of rings, then

$$\frac{E_a}{E_c} = \sin\frac{\pi}{n}$$

where $\frac{2\pi}{n}$ is the angular width in degrees of phase between the contacts for two successive rings.

If, however, the converter be driven by current supplied to either side, the other side being on open circuit, then a correction on the above value becomes necessary owing to ohmic drop in the armature conductors.

Let direct current be supplied to the commutator, the rings being on open circuit. Let the resistance of a single path



through the armature from brush to brush be R, and let the current flowing in that path be C. Then the ohmic drop from brush to brush is CR.

* Read March 24, 1905.

Let AB be points of connexion of a pair of rings to the winding.

The P.D. between the rings due to drop is zero at AB, increases uniformly till A is under the brush at P, and then remains constant until B comes under the brush P′. It decreases to zero again with A 180 phase degrees from its first position.

Let $\frac{2\pi}{n}$ = angular distance in phase degrees from A to B, and let θ = phase displacement of the centre T of the section AB from the line OP.

As θ changes from O to $\frac{\pi}{n}$ the P.D. between A and B due to ohmic drop increases from O to $\frac{2CR}{n}$ as AB is the $\frac{2}{n}$th part of a single path between the brushes.

The value of this P.D. for an intermediate value of θ may be written as

$$\frac{2CR}{n} \cdot \frac{\theta}{\frac{\pi}{n}} = \frac{2CR\theta}{\pi}.$$

For further increase of θ up to $\pi - \frac{\pi}{n}$ this drop is constant and equal to $\frac{2CR}{n}$.

From $\pi - \frac{\pi}{n}$ to π the ohmic drop diminishes to zero again, in the same manner as it rose in the section 0 to $\frac{\pi}{n}$

The other half of the cycle is similar with the polarity of the rings reversed.

Let V = voltage on brushes, then the total instantaneous voltage on the rings is given by :—

$$(V - CR) \sin\frac{\pi}{n} \sin\theta + \frac{2CR\theta}{\pi} \qquad \text{when } \theta \text{ is between O and } \frac{\pi}{n} \quad . \quad . \quad (1)$$

$$(V - CR) \sin\frac{\pi}{n} \sin\theta + \frac{2CR}{n} \qquad ,, \qquad ,, \qquad \frac{\pi}{n} \text{ and } \pi - \frac{\pi}{n} \quad . \quad (2)$$

$$(V - CR) \sin\frac{\pi}{n} \sin\theta + \frac{2CR(\pi - \theta)}{\pi} \; ,, \qquad ,, \qquad \pi - \frac{\pi}{n} \text{ and } \pi \quad . \quad (3)$$

The R.M.S. value of the voltage on the rings can be obtained by proceeding in the usual way with the above expressions.

$$\int_0^{\pi/n} \left\{ (\mathrm{V}-\mathrm{CR}) \sin\frac{\pi}{n} \sin\theta + \frac{2\mathrm{CR}\theta}{\pi} \right\}^2 d\theta$$

$$= \left\{ (\mathrm{V}-\mathrm{CR})^2 \sin^2\frac{\pi}{n}\left(\frac{\pi}{2n} - \frac{\sin\frac{2\pi}{n}}{4}\right) + \frac{4\mathrm{CR}(\mathrm{V}-\mathrm{CR})\sin\frac{\pi}{n}}{\pi}\left(\sin\frac{\pi}{n} - \frac{\pi}{n}\cos\frac{\pi}{n}\right) + \frac{4\mathrm{C}^2\mathrm{R}^2\pi^2}{n^3} \right\}. \quad (4)$$

$$\int_{\pi n}^{\pi-\pi/n} \left\{ (\mathrm{V}-\mathrm{CR}) \sin\frac{\pi}{n} \sin\theta + \frac{2\mathrm{CR}}{n} \right\}^2 d\theta$$

$$= (\mathrm{V}-\mathrm{CR})^2 \sin^2\frac{\pi}{n} \left\{ \frac{\pi}{2} - \frac{\pi}{n} + \frac{\sin\frac{2\pi}{n}}{2} \right\} + \frac{8\mathrm{CR}(\mathrm{V}-\mathrm{CR})\sin\frac{\pi}{n}\cos\frac{\pi}{n}}{n} + \frac{4\mathrm{C}^2\mathrm{R}^2}{n^2}\left(\pi - \frac{2\pi}{n}\right). \quad (5)$$

The sum of the squares for the period 0 to π is equal to twice expression (4) + expression (5).

The value of this is

$$(\mathrm{V}-\mathrm{CR})^2 \sin^2\frac{\pi}{n} \cdot \frac{\pi}{2} + \frac{4\mathrm{CR}(\mathrm{V}-\mathrm{CR})}{n} \sin\frac{\pi}{n} \left\{ 2\sin\frac{\pi}{n} - \frac{2\pi}{n}\cos\frac{\pi}{n} + 2\cos\frac{\pi}{n} \right\} + \frac{8\mathrm{C}^2\mathrm{R}^2\pi^2 + 4\mathrm{C}^2\mathrm{R}^2(n-2)\pi}{n^3}.$$

Dividing through by π, taking the square root, and dividing this again by V we get as the value of the ratio of the alternating P.D. to the direct current P.D.

$$\frac{1}{\mathrm{V}}\sqrt{\frac{(\mathrm{V}-\mathrm{CR})^2\sin^2\frac{\pi}{n}}{2} + \frac{8\mathrm{CR}(\mathrm{V}-\mathrm{CR})\sin\frac{\pi}{n}}{n\pi}\left\{\sin\frac{\pi}{n} + \cos\frac{\pi}{n} - \frac{\pi}{n}\cos\frac{\pi}{n}\right\} + \frac{4\mathrm{C}^2\mathrm{R}^2}{n^3}(2\pi + n - 2)}$$

Let $\mathrm{CR} = \frac{\mathrm{V}}{m}$. Then the expression for the ratio becomes :—

$$\sqrt{\frac{(m-1)^2\sin^2\frac{\pi}{n}}{2m^2} + \frac{(m-1)\sin\frac{\pi}{n}}{m^2 n\pi}\left\{\sin\frac{\pi}{n} + \cos\frac{\pi}{n} - \frac{\pi}{n}\cos\frac{\pi}{n}\right\} + \frac{4}{m^2 n^3}(2\pi + n - 2)}.$$

The corrected ratios of conversion are compared with the uncorrected values for different values of m in the following table :—

No. of Phases.	Corrected Ratios (as percentages).						Uncorrected Ratio as percentages.
	$m=10.$	$m=20.$	$m=50.$	$m=100.$	$m=200.$	$m=500.$	
Single phase ...	67·13	68·32	69·61	70·14	70·42	70·58	70·72
Three phase ...	56·70	58·71	60·18	60·69	60·95	61·10	61·24
Four phase ...	46·04	47·87	49·11	49·55	49·75	49·905	50·00
Six phase	32·46	33·82	34·73	35·03	35·19	35·29	35·35
Twelve phase...	15·60	17·50	17·97	18·13	18·21	18·26	18·30

It is of interest to construct a table showing the differences between the above ratios and the corresponding uncorrected ratio, as a percentage of the latter :—

No. of Phases.	Percentage difference from uncorrected ratio.					
	$m=10.$	$m=20.$	$m=50.$	$m=100.$	$m=200.$	$m=500.$
Single phase	5·076	3·39	1·57	·82	·42	·20
Three phase	7·413	4·13	1·73	·90	·47	·23
Four phase	7·920	4·26	1·78	·90	·50	·19
Six phase	8·176	4·33	1·75	·83	·45	·17
Twelve phase ...	14·75	4·37	1·80	·93	·49	·22

This table brings out the fact that the percentage correction due to voltage drop on the unloaded machine is practically independent of the number of phases, if the value of m is of the order usually found in good design.