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“Weights of Structures estimated Graphically.”

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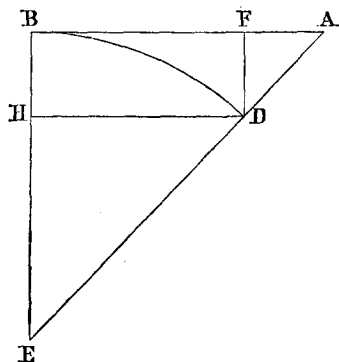
As the graphic method of ascertaining the strains produced on structures of different kinds, by the application of weight, is now deservedly attracting considerable attention, it has occurred to the Author that it might be interesting, and not devoid of practical utility, to endeavour to ascertain to what extent a similar system can be employed in estimating the weights of the structures themselves, in such cases as framed girders, roofs, &c.

The weight of such a structure adapted to carry a distributed fixed load, in addition to its own weight, may be considered as represented by the expression $\frac{WQ}{1-Q}$, where W is the weight carried, and WQ the weight of structure required to carry W alone, or $\frac{Wa}{W-a}$, where $a = WQ$.

These premises having been stated, the Author will show how these formulas, as applied to different types of structure, may be represented by straight lines, and thus the result be obtained simply by scale measurement.

Let BD (Fig. 1) be the arc of a circle, of which BE is the radius, HD the sine, BA the tangent, and EA the secant.

FIG. 1.



Draw FD parallel to BH , intersecting BA in the point F .

Then, by similar triangles, $HE : ED :: FD : DA$, and therefore $DA = \frac{ED \times FD}{HE}$; but $HE = BE - BH$; $ED = BE$, and $FD = BH$.

$$\text{Therefore} \quad DA = \frac{BE \times BH}{BE - BH}.$$

That is to say, DA may be considered as representing the weight of a girder of unknown span carrying the weight BE and its own weight in addition, and BH represents the proportion of the total weight DA due simply to the weight BE ; for if $BE = W$, and $BH = a$, the formula becomes $DA = \frac{W a}{W - a}$ as before. Again, if

$BE = 1$, $BH = \frac{a}{W}$ or Q , and therefore $DA = \frac{Q}{1 - Q}$, or the former value divided by W , that is, the proportion of the total weight of the structure necessary to support a unit of load and its own weight in addition; consequently if the load be again represented by W , the total weight of the structure becomes $\frac{W Q}{1 - Q}$, as before;

which, in fact, is the system adopted in the use of tabular sines, &c., where radius = 1, and where, in consequence, a multiplier W has to be used when radius = W .

The conclusion arrived at is, that if the weight W carried by the structure = radius, and the weight of structure necessary to carry W alone = versed sine to radius W ; then the weight of structure required to carry W and its own weight in addition,

$$= (\secant \text{ to radius } W) - (\text{radius } W), \text{ or } W (\text{tabular sec.} - 1).$$

FIG. 2.

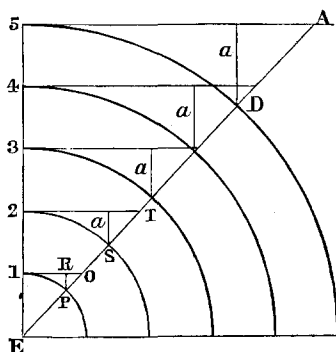


Fig. 2, showing five concentric circles, whose radii respectively

represent the weights 1, 2, 3, 4, 5, further elucidates the subject. Let D A be the weight of a girder of known span, capable of carrying the weight 5, and its own weight in addition,

$$D A = \frac{5 \times a}{5 - a} = 5 \text{ times } \frac{1 \times R P}{1 - R P},$$

but

$$R P = \frac{a}{5} = Q.$$

Therefore

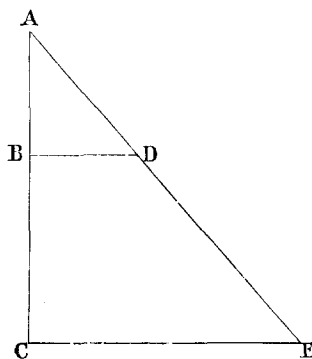
$$D A = 5 \times \frac{Q}{1 - Q}.$$

Also P O represents the weight of a girder of the same span and general proportions as that the weight of which is represented by D A, but carrying 1 instead of 5; S T represents the weight of such a girder carrying 2, and so on.

If, now, a graphic method be found of ascertaining the value of a or Q for any other span of the same description of structure, all the elements will exist necessary for expressing graphically the weight of any other structure of the same type, with any load and span.

In the case of solid beams carrying both a fixed and a moving load, when the same factor of safety is allowed to each, and the proportion of depth to span remains constant, Q varies as the span; or, in other words, the weight of beam required to carry the whole external load alone, varies as the span when the load remains constant. It will be convenient, and sufficiently accurate for the present purpose, to proceed on this assumption in every case.

FIG. 3.

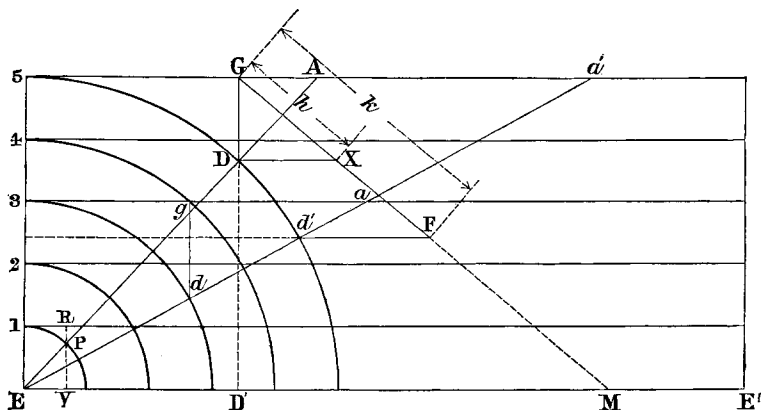


Let $AB = a$ (Fig. 3) for a girder of which the span $= h$; and let $AC = a$ for a girder of the same description, of which the

span = k ; then $h : k :: AB : AC$, and therefore $AC = \frac{AB \times k}{h}$, obtained graphically thus. Place any convenient drawing scale in the position shown by the line ADE , with zero at A , making AD = the span h , and AE = the span k to that scale. Join BD , and parallel to it draw EC , intersecting AC in the point C . It is evident that the required proportion $AD : AE :: AB : AC$ exists, whatever scale is used for measuring the distances AD and AE , and it would therefore hold good if those lines were drawn to the same scale as AB and AC , that is, $h : k :: AB : AC$, and therefore $AC = \frac{AB \times k}{h}$, the required length.

The application of this principle to Fig. 2 is shown in Fig. 4. Let DA = the weight of a given girder of the span h , carrying 5

FIG. 4.



and let it be required to find the weight of a girder of the same description of the span k , carrying 3. From the point D draw the line DX parallel to the tangents, and, with any convenient scale, set off the distance $GX = h$, intersecting DX in X , also making $GF = k$; from the point F draw Fd' parallel to the tangents, intersecting the circle with radius 5 in d' . Through the point d' draw the radiating line $Ed'a'$, then $d'a'$ is the weight of the girder with the span k carrying 5, and da is the weight of the girder with the span k carrying 3. The procedure would be similar if k were the span of the given girder. If there were no data, the weight of a girder of the span k , of the strength required to carry 3 alone, would have to be estimated in the usual way.

This would be represented by gd , and da would give the total weight of the girder as before.

It is now to be observed that in the formulas at the commencement of this Paper the limiting span is reached when $Q = 1$, or when $a = W$, that is, the span being proportional to a , or versed sine, it reaches its limit when the versed sine becomes equal to the radius, or, in other words, when (secant - radius) the total weight of the girder is infinite, and the result is immediately obtained in the diagram by producing GF to meet EE' in the point M , for if GD be also produced to meet EE' in D' , $GD : GX :: GD' : GM$; but GX is the span when $GD = a$, therefore GM is the span when $GD' = a$, but $GD' = 5 = W$; therefore GM is the span when $a = W$, or the limiting span. But when $GD' = a$, $RV = Q$; and $RV = 1$, therefore GM is the span when $Q = 1$, or the limiting span.

When the external load is proportional to the span, as in the case of most bridges, and of roofs having principals the same distance apart in each instance,

$$W = (\text{weight per lineal foot on given structure}) \times (\text{span of proposed structure in feet});$$

and the results obtained under these conditions are the most accurate: but it must be observed that in the case of roofs, unless an adequate weight be adopted in each instance to represent the weight of snow and the equivalent of the wind-pressure, and added to the weight of covering, the resulting limiting span will be too low, and the weight of principal obtained by the diagram will be too high, as it will be obtained by applying the (sec. - rad.) of the circle whose radius = (weight of covering + weight of snow + equivalent of wind-pressure), to the circle whose radius = weight of covering only, which is the same thing as using too large a scale, causing the radiating lines to form a less angle than they should with the horizontal line EE' (Fig. 4). Also, when a greater or less load per lineal foot of principal is carried, owing to the principals being farther apart or closer than in the case of the given structure, the proportion of wind-pressure, to the total external load will not remain the same, as that portion of the covering consisting of cross-girders may be considered to vary per lineal foot of principal as the square of the distance between the principals; and the remainder of the covering, including the weight of snow and the equivalent of the wind-pressure, to vary per lineal foot simply as the distance between the principals, though, strictly speaking, the wind-pressure and dead load should

not be combined. This combination, however, is useful and fairly accurate when an actual equivalent in respect of the resultant weight of structure is adopted in the given roof instead of what may be termed an average vertical component per lineal foot of principal; but this is merely accidental, and entirely dependent on practical considerations, and the Author believes that its tendency is to slightly under-estimate the weights of smaller roofs than the sample one. The necessary allowance for wind-pressure must therefore, in the Author's opinion, to some extent rest on experience.

Of course, having by these means estimated the weight of any structure, the cubical content in feet can be obtained by dividing the weight by the weight per cubic foot; and in the case of solid beams the width is given by dividing the cubical content by (span \times depth).

It will be seen that a diagram drawn even to a small scale, on the system hitherto considered, to be of practical utility in estimating the weights of large girders heavily loaded, would be of inconveniently large dimensions, and to obviate this objection the Author has designed a "unit-diagram," the radius, which represents unity, being divided into one hundred equal parts, each of which therefore represents 0.01. These divisions are ruled the whole length of the diagram, similarly to the five divisions in Fig. 4.

From what has been already said, and by reference to Figs. 2 and 4, it will be clear that as the total weight of a structure divided by the external load represents the proportion of that weight necessary to support a unit of load, and its own weight in addition, this diagram can be used in approximately estimating the weights of structures of any span and load. This is effected by dividing the weight of the sample structure by its load, finding the position of a radiating line equal in length to the quotient (measured by the diagram scale) between the quadrant and the top of the diagram, as at D A (Fig. 4), noting the horizontal line intersecting the circumference at the same point, to which line the scale of spans is to be applied with the reading representing the span in contact therewith, and with the scale in such a position that zero may fall on the line at the top of the diagram; then observing upon which line the reading representing the span of the proposed structure falls, and, from the point in which this line intersects the circumference, measuring another radiating line as at $d' a'$, the length of which, by the diagram scale, after being multiplied by the proposed load, will give the weight of the proposed struc-

ture. The remark already made as to the procedure to be adopted when no data are at hand applies to the unit diagram, but the weight represented by $g d$ (Fig. 4) must be divided by the external load before being made use of.

The Paper is accompanied by the diagrams from which the woodcuts in the text have been prepared; also by two unit-diagrams with suitable scales, each with millimetre divisions, but one having a complete quadrant yielding the limiting spans, and with radius = 1 decimetre, each millimetre division therefore representing 0.01, so that the measurements are obtained to two places of decimals; and the other having only a portion of the quadrant, but drawn to ten times the scale, each millimetre therefore representing 0.001, thus enabling the measurements to be read to three places of decimals, but confining its applications to structures of shorter span.
