

Mathematical Association

88. Geometrical Proofs: (1) $\sqrt{1+\sin A}+\sqrt{1-\sin A}=2\cos\frac{A}{2}$ ($A < 90^\circ$). (2) $\sqrt{\frac{1-\cos A}{2}}=\sin\frac{A}{2}$ ($A < 90^\circ$). (3) $\frac{-1+\sqrt{1+\tan^2 A}}{\tan A}=\tan\frac{A}{2}$ ($A < 90^\circ$)

Author(s): J. V. Thomas

Source: *The Mathematical Gazette*, Vol. 1, No. 24 (Dec., 1900), p. 412

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3604472>

Accessed: 23-10-2015 10:30 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

required for the Sixth Book have been collected together in the Tenth and Eleventh Sections, so that they are clearly marked off from that which must be read to understand the Sixth Book ; but the beginner will find them more and more useful as he goes deeper into the study of Geometry. M. J. M. HILL.

MATHEMATICAL NOTES.

88. [K. 20. d.] Geometrical proofs :

$$(1) \sqrt{1 + \sin A} + \sqrt{1 - \sin A} = 2 \cos \frac{A}{2} \quad (A < 90^\circ).$$

$$(2) \sqrt{\frac{1 - \cos A}{2}} = \sin \frac{A}{2} \quad (A < 90^\circ).$$

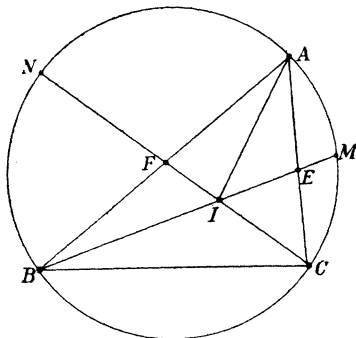
$$(3) \frac{-1 + \sqrt{1 + \tan^2 A}}{\tan A} = \tan \frac{A}{2} \quad (A < 90^\circ).$$

Let P be any point on a semicircle APB , radius unity, centre O , diameter AB . Let $\hat{P}OB$ be A . Draw PN perpendicular to AB .

$$\begin{aligned} (1) \sqrt{1 + \sin A} + \sqrt{1 - \sin A} &= \sqrt{1 + PN} + \sqrt{1 - PN} \\ &= \sqrt{1 + \sqrt{AN \cdot NB}} + \sqrt{1 - \sqrt{AN \cdot NB}} \\ &= \sqrt{\frac{AN + NB + 2\sqrt{AN \cdot NB}}{2}} + \sqrt{\frac{AN + NB - 2\sqrt{AN \cdot NB}}{2}} \\ &= 2\sqrt{\frac{AN}{2}} = 2\sqrt{\frac{AN}{AB}} = 2 \cdot \frac{AN}{AP} = 2 \cos \frac{A}{2}. \\ (2) \sqrt{\frac{1 - \cos A}{2}} &= \sqrt{\frac{1 - ON}{2}} = \sqrt{\frac{NB}{AB}} = \frac{NP}{PB} = \sin \frac{A}{2}. \\ (3) \frac{-1 + \sqrt{1 + \tan^2 A}}{\tan A} &= \frac{-1 + \sqrt{1 + \frac{PN^2}{ON^2}}}{\frac{PN}{ON}} \\ &= \frac{-ON + \sqrt{ON^2 + PN^2}}{PN} = \frac{-ON + 1}{PN} = \frac{NB}{PN} = \tan \frac{A}{2}. \end{aligned}$$

J. V. THOMAS.

89. [K. 1. b.] The equality of internal bisectors.



Let ABC be a triangle, and let the internal bisectors of the angles at B, C meet the opposite sides in E, F and the circumcircle in M, N respectively.

Given $BE = CF$, to prove $\hat{B} = \hat{C}$.

Since $BE \cdot BM = AB \cdot BC$,
 $CF \cdot CN = AC \cdot BC$;
 $\therefore BM : CN = AB : AC$
 and $CN \cdot AB = BM \cdot AC = 0$,
 or $AN(AC + BC) - AM(BC + AB) = 0$,
 or
 $BC(AN - AM) + AC \cdot AN - AB \cdot AM = 0$.