88. Geometrical Proofs: (1) $\sqrt{1+\text{sin}\ A}+\sqrt{1-\text{sin}\ A}=2\text{cos}\frac{A}{2}$ (A < 90°). (2) $\sqrt{\frac{1-\text{cos}\ A}{2}}=\text{sin}\frac{A}{2}$ (A < 90°). (3) $\frac{-1+\sqrt{1+\text{tan}^{2}A}}{\text{tan}\ A}=\text{tan}\frac{A}{2}$ (A < 90°)

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required for the Sixth Book have been collected together in the Tenth and Eleventh Sections, so that they are clearly marked off from that which must be read to understand the Sixth Book; but the beginner will find them more and more useful as he goes deeper into the study of Geometry. M. J. M. HILL.

MATHEMATICAL NOTES.

88. [K. 20. d.] *Geometrical proofs:*

$$(1)\ \sqrt{1+\sin A}+\sqrt{1-\sin A}=2\cos\frac{A}{2}\quad (A<90^\circ).$$

$$(2)\ \sqrt{\frac{1-\cos A}{2}}=\sin\frac{A}{2}\quad (A<90^\circ).$$

$$(3)\ \frac{-1+\sqrt{1+\tan^2 A}}{\tan A}=\tan\frac{A}{2}\quad (A<90^\circ).$$

Let P be any point on a semicircle APB, radius unity, centre O, diameter AB. Let $P\hat{O}B$ be A. Draw PN perpendicular to AB.

$$(1)\ \sqrt{1+\sin A}+\sqrt{1-\sin A}=\sqrt{1+PN}+\sqrt{1-PN}$$

$$=\sqrt{1+\sqrt{AN\,.\,NB}}+\sqrt{1-\sqrt{AN\,.\,NB}}$$

$$=\sqrt{\frac{AN+NB+2\sqrt{AN\,.\,NB}}{2}}+\sqrt{\frac{AN+NB-2\sqrt{AN\,.\,NB}}{2}}$$

$$=2\sqrt{\frac{AN}{2}}=2\sqrt{\frac{AN}{AB}}=2\,.\,\frac{AN}{AP}=2\cos\frac{A}{2}.$$

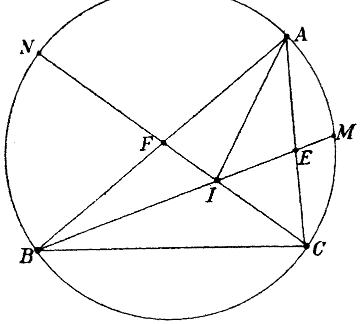
$$(2)\ \sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{1-ON}{2}}=\sqrt{\frac{NB}{AB}}=\frac{NP}{PB}=\sin\frac{A}{2}.$$

$$(3)\ \frac{-1+\sqrt{1+\tan^2 A}}{\tan A}=\frac{-1+\sqrt{1+\dfrac{PN^2}{ON^2}}}{\dfrac{PN}{ON}}$$

$$=\frac{-ON+\sqrt{ON^2+PN^2}}{PN}=\frac{-ON+1}{PN}=\frac{NB}{PN}=\tan\frac{A}{2}.$$

J. V. THOMAS.

89. [K. 1. b.] *The equality of internal bisectors.*



Let ABC be a triangle, and let the internal bisectors of the angles at B, C meet the opposite sides in E, F and the circumcircle in M, N respectively.

Given $BE=CF$, to prove $\hat{B}=\hat{C}$.

Since $BE\,.\,BM=AB\,.\,BC$,
$CF\,.\,CN=AC\,.\,BC$;

$\therefore\ BM : CN=AB : AC.$

and $CN\,.\,AB-BM\,.\,AC=0$,

or $AN(AC+BC)-AM(BC+AB)=0$,

or

$BC(AN-AM)+AC\,.\,AN-AB\,.\,AM=0.$