

Galvanometers

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By placing the eye very nearly in the plane of the paper and looking along the lines, the curvatures can be seen to follow very closely those sketched on fig. 2. I would specially draw attention to the partial curvature of such lines as *c d e f* and *j k l m n*.

LV. *Galvanometers.* By Prof. W. E. AYRTON, *F.R.S.*,
T. MATHER, and W. E. SUMPNER, *D.Sc.**

IN order that a number of students may be able to work at the same time in the Physical Laboratories of the City and Guilds of London Central Institution we have endeavoured, as far as possible, to arrange the apparatus so that each of the many experiments should be complete in itself. This has led to the necessity of our constructing, and having constructed for us, a large number of galvanometers of various types, and from the specimens that are on the table it will be seen that several of the galvanometers contain points of novelty. A long series of comparative tests has been carried out with all the more important types that are in our laboratories, and we have therefore thought that a record of the results obtained by the students, and by ourselves, will be of value in affording data to guide others in the selection of instruments most suitable for the objects in view, as well as in suggesting future improvements in the manufacture of galvanometers.

I. *Astatic or Non-Astatic.*

One of the first questions that arises is whether it is desirable to employ the astatic type in the construction of sensitive galvanometers. Prof. A. Gray, in his treatise on 'The Theory and Practice of Absolute Measurements in Electricity and Magnetism,' maintains that it is not, for he says (page 311):—"Sensibility is sometimes obtained by the use of astatic galvanometers, but these are rarely necessary and are more troublesome to use than ordinary non-astatic instruments."

This opinion carries special weight as it may be very probably taken as expressing Sir William Thomson's view on this subject, seeing that the usual method adopted by Sir

* Read January 17, 1890.

William for measuring small currents is to employ a galvanometer with a single set of magnetic needles, and to weaken the magnetic controlling field as far as requisite by means of adjustable magnets. Theoretically there is no limit to the sensibility obtainable in this way; but with any strength of the resultant controlling field due to the earth and the adjustable magnets an astatic combination of magnetic needles must give a more sensitive arrangement than a non-astatic one. For let F be the resultant strength of this controlling field, let M be the magnetic moment of a needle within the coil whose magnetic constant is G , and let C_1 be the current flowing, then, if α be the deflexion, we have for small deflexions

$$\alpha = \frac{C_1 G}{F}.$$

Next, let a needle of moment m be attached to the suspended system outside the coil, so as to form an approximately astatic system with the needle inside, then, even if we neglect the action of the coil on the needle outside, an action wh. h of course assists the action of the coil on a needle inside, we have, if C_2 be the current now flowing,

$$\alpha = \frac{C_2 G M}{F(M-m)}.$$

Hence, if the deflexions be the same in the two cases,

$$C_2 = C_1 \frac{M-m}{M}.$$

Now M may be made large, and $M-m$ small, hence C_2 may be a small fraction of C_1 . Consequently for any adjustment of the controlling magnet, the current required to produce a given deflexion can be much diminished by using an astatic combination instead of a non-astatic one.

But, perhaps, the most important advantage of an astatic over a non-astatic instrument is its much greater freedom from outside magnetic disturbance. In order that C_1 , the current with the non-astatic galvanometer, may be small for a given value of the deflexion α , the resultant controlling field F must be small; whereas with the astatic combination there is no necessity to make F small in order that C_2 may be small,

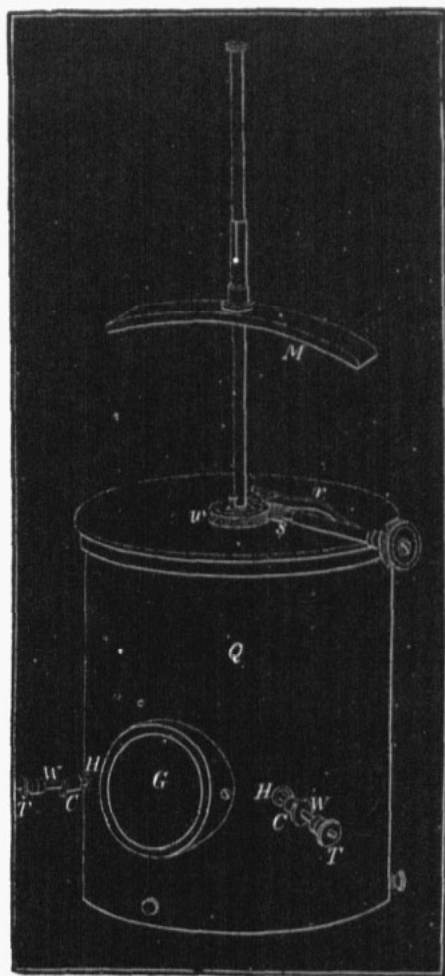
since $\frac{M}{M-m}$ may be made large. Now the disturbance that an extraneous magnetic body can bring about depends on the strength of the magnetic field it sets up compared with that of the field F produced by the earth and the adjusting magnet combined. Hence the moving about of a magnetic body in the neighbourhood of the galvanometer will produce more disturbance on a non-astatic galvanometer than on an astatic galvanometer of the same sensibility. The preceding are very important considerations; hence, so far from agreeing with Prof. Gray that it is more troublesome to work with astatic galvanometers than with non-astatic ones, the preceding considerations, which we find fully borne out by experience, have led us to regularly employ the astatic principle with sensitive galvanometers.

And generally we may conclude that when it is desired to utilize a magnetic control, for example, with electrometers, &c., it is better, when a small controlling moment is required, to obtain this by the use of a weak magnet on the suspended system than by attempting to simply weaken the controlling field. Indeed, we may mention that even in the case of electrometers, where there is no question of the action of currents on magnets, and where (when a magnetic control has been employed in place of a bifilar or torsional control) it has been usual to attach a single little magnet to the aluminium electrometer-needle, to be acted on by a stationary outside magnet, we have found it more convenient to attach an *astatic* combination of magnets to the moving needle, and to direct the system by a fairly strong permanent magnet, which acts of course differentially on the suspended astatic system of magnets, since with such an arrangement great freedom from outside magnetic disturbance is secured, combined with the power of employing a wide range of sensibility.

It is worth noticing that in galvanometers, where the distance of the controlling magnet from the needles is varied by simply raising the magnet M , as in the case of the galvanometer seen in fig. 1, it is more convenient to place the galvanometer so that the plane of the coils is in the magnetic meridian of the laboratory when the controlling magnet is removed; or, in other words, to place the galvanometer so that

the spot of light comes to the zero of the scale for no current passing through the instrument and with no controlling magnet. For in that case the sensibility of the instrument can be subsequently varied by simply raising or lowering the

Fig. 1.



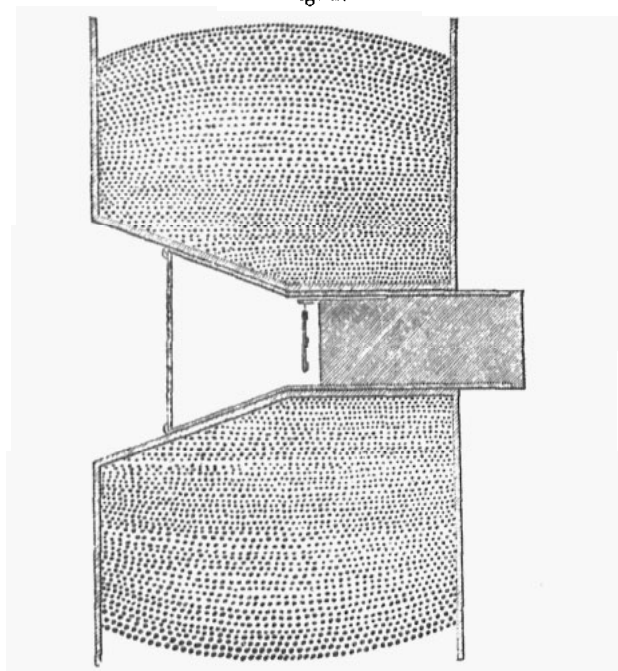
controlling magnet *parallel to itself*; whereas, if the galvanometer be fixed in any other position, it is necessary, when raising or lowering the controlling magnet, to give it a care-

ful screwing motion to prevent the spot of light going off the scale.

II. *Position of Mirror.*

According to the ordinary method of constructing a Thomson's reflecting-galvanometer, a tubular space at the centre of the coil is left unwound, and the mirror, with the magnets at the back of it, hung at the centre of this unwound space (see fig. 2, which illustrates a well-known form of this type of galvanometer). To allow the ray of light to pass out

Fig. 2.



of the coil after reflexion at the mirror, when the mirror is deflected, the end of the tubular space has to be made trumpet-shaped, which leads to still more space in the neighbourhood of the needle being left unwound. And when the galvanometer is an astatic one, a somewhat similar tubular space is left unwound on each side of the lower coil to allow room for the diamond-shaped aluminium vane to turn.

In this way the most valuable part of each coil, viz. that

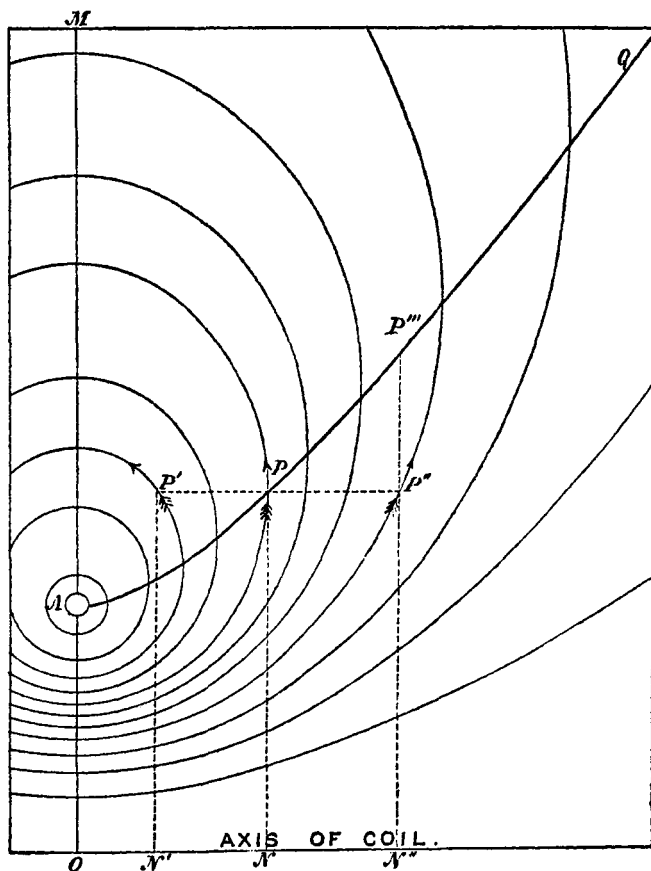
close to the magnetic needles, is left unwound. This disadvantageous mode of construction so impressed itself on Mr. Mudford, while a student at the City Guilds' Institute, that he suggested that the needles alone should be placed within the coil, and that the mirror (which is, of course, employed to reflect a ray of light, and not, as its position *inside* the coil would seem to infer, to be acted on electromagnetically) should be placed outside the coil, or between the upper and lower sets of coils when two pair of coils are employed. A specimen of a galvanometer constructed in this way and containing certain other improvements, which will be referred to later on, was submitted some years ago by Mr. Mudford to Sir William Thomson, who expressed approval of the devices employed, and, since that time, many galvanometers differing in other details, but all embodying the principle of not wasting valuable space in the coils, have been constructed for the Central Institution.

It is interesting to notice that the common method of placing the mirror inside the coil is an irrational survival of an old custom kept up, like the two buttons at the back of a coat, when its use is almost forgotten. Gauss and Weber put their mirror inside the coil because the mirror was made of polished steel and was also the magnetic needle. Sir William Thomson put the mirror inside the coil in his "speaking galvanometers," for receiving messages on submarine cables, because he desired to render the space near the mirror as air-tight as possible in order to obtain great damping. And the ordinary maker of astatic galvanometers puts the mirror inside the coil, because he has seen it there in other galvanometers, and it has never occurred to him to put it anywhere else.

If the ray of light has not to go into the coil no hole need be left and the coil may be wound practically to the centre. It is necessary, however, that the space left unwound inside the coil should be somewhat larger than is required simply to allow the needle to turn through a small angle, because if the windings were carried to the axis of the coil in the immediate neighbourhood of the needle the effect on the latter of the current passing through the innermost coils would be opposed to that of the rest of the coil. To find the shape and volume of the least space that should be left unwound we proceeded as follows :—

Let OA (fig. 3) be the radius of a convolution of wire whose plane is perpendicular to the paper, the section of the convolution being represented by the small circle at A , then, from

Fig. 3.



Maxwell's 'Elect. and Mag.' vol. ii. plate xviii., the lines of force due to a current passing round this convolution are as indicated. Therefore, if a magnet whose half length is NP be placed at a distance ON from the plane of the convolution such that NP is a tangent to the line of force at P , no torque will be exerted on the magnet by a current passing round the convolution, since the direction of the force at P is along the magnet NP . But if the magnet be placed nearer to the convolution, as at $N'P'$, or farther from it, as at $N''P''$, there will be a torque exerted on it due to this convolution of radius OA ; the two torques, however, will be in opposite directions.

Now as the radii of the greater number of convolutions contained in the coil are greater than the half length of the magnet, it is clear that the torque exerted by the convolution OA when the magnet is to the right of NP is in the same direction as that exerted by the coil as a whole, and that the torque exerted on the magnet by this convolution when the magnet is between OA and NP is opposed to the torque exerted by the rest of the coil. In other words, if NP be the position of the half-magnet in the coil, a convolution of smaller radius than OA must not be wound in the plane that passes through OAM at right angles to the paper. And if a line APQ be drawn so as to cut all the lines of force at points where the tangents are perpendicular to the axis of the coil, OA gives the smallest radius that a coil must have in the plane passing through OAM at right angles to the paper, whether the magnet be at N and its half length be NP, or the magnet be at N'' and its half length be N''P'''.

For our purpose, however, we have not to consider the problem of magnetic needles of different lengths at different distances from a fixed plane, but the converse problem of a magnetic needle of a fixed length in a fixed position, which is at different distances from different planes, the critical radius of convolution in each of which we wish to determine.

If NP were the fixed half length of the magnet, then, as already seen, OA would be the critical radius of a convolution at a distance NO from N, and the critical radius of a convolution for another distance can be obtained by imagining the figure 3 reduced in the proportion of NP to N''P''', in which

case a critical radius $\frac{NP}{N''P'''} \times OA$ will be obtained at a distance

distance $\frac{NP}{N''P'''} ON''$ from N. Or, lastly, if we take OA as the

half length and position of our magnetic needle, the critical radius of convolution at every distance from O will be obtained by taking every such ordinate as N''P''' and multiplying OA by the ratio of OA to N''P''', and the distance from

O for which this is the critical radius is equal to $\frac{OA}{N''P'''} \times ON''$

A curve giving the locus of such critical radii of convolution is shown in fig. 4 for a needle of length 2, shown therefore in

the figure about twelve times its full size, and we learn from it that at a distance from the centre of the coil equal, say, to 0.4 of the half length of the needle, the smallest convolution should have a radius about 0.75 of the half length of the needle, and that the wire must not be wound close to the axis until the distance from the centre along the axis is about 0.72 of the half length of the needle.

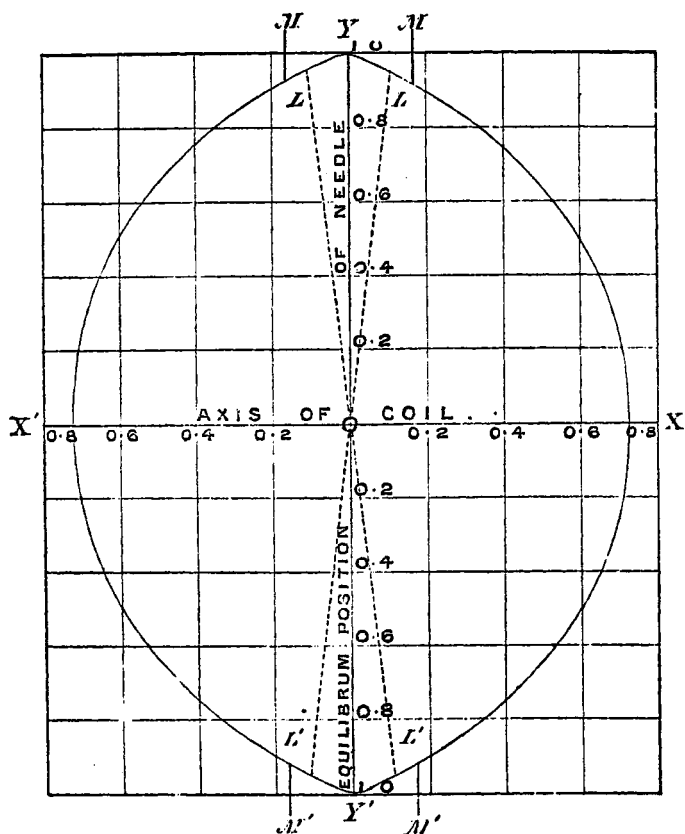
With an ordinary reflecting-galvanometer the needle requires to have a free angular space for turning of about 15° , represented by the space between the two lines LOL'. In this region the radii of the smallest convolutions must be a little greater than theory allows, otherwise the needle would touch the coils, and generally a sufficiently near approximation can be made to the cavity which theoretically ought to be left unwound by making it an oblate spheroid with a polar axis about 0.72 of its equatorial diameter, the latter being of course slightly larger than the length of the needle.

Since winding wire inside the surface whose section is given in fig. 4 would oppose the effect of the wire wound outside this surface if the current flowed in the same sense throughout the whole coil, it is possible, by causing the current to circulate in opposite directions in the two portions, to wind all the space and to cause all the convolutions to help one another. This we have not yet tried, but in view of the fact that the space which has been left unwound in the galvanometers constructed according to Mr. Mudford's suggestion is very near the needle, and therefore very valuable, it seems important to try and utilize it in the way just suggested. In that case the surface we have theoretically determined would be the surface separating the coil into two parts, wound respectively in opposite directions.

As the coils of a reflecting-galvanometer have to be supported by a central framework, there is necessarily a certain distance left between the coils when they are in position. This space we find when drawn to scale is that contained between the two vertical lines MM', and which therefore allows more than sufficient room for the needle to turn. Hence if the plan of opposite winding that we have suggested be tried, the face of the coils may be made quite plane and no portion

whatever of the coils left unwound, even to allow the needle space to move.

Fig. 4.



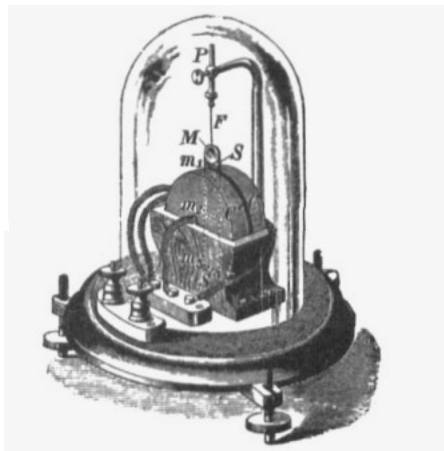
Shape of cavity to be left unwound inside the coil when the length of the magnetic needle is 2.

III. *Damping the Oscillations.*

Since no hole is left in the lower coil for the aluminium damper, as is the case with Messrs. Elliott's reflecting-galvanometers, a different mode of damping is adopted. The plan we find best is to attach the mirror *M* (fig. 5) to the middle of a long vertical strip of mica, *S*, across the upper and lower ends and the middle of which the magnets m_1 , m_3 , and m_2 are fixed. Such an arrangement is also seen in position in the galvanometer shown in figure 6 later on. With this arrangement, which requires very little space, we find that when the controlling magnet is adjusted so

that the periodic time of vibration is 7 seconds or more, the spot of light does not visibly pass through the zero

Fig. 5.



or the instrument is dead-beat, and even when the period is much shorter than this, the spot of light does not pass through the zero more than twice*. With galvanometers in which less damping is desired, a portion of the strip of mica is cut away, so as to make it narrower, and in extreme cases, when for example no damping is required, as in the case of a ballistic galvanometer, the mica strip is replaced by a vertical piece of wire.

With a ballistic galvanometer, especially when intended for teaching-purposes, it is desirable to have some easy means of adjusting the amount of damping. In the case of the galvanometer used by our students for experimenting on the variation produced in the swing of the needle of a ballistic galvanometer by variations in the amount of damping, this variable damping is effected by enclosing the mirror in a cell whose glass ends can be simultaneously made to approach or recede. This motion of the ends of the cell is effected by

* Quickness in the needle coming to rest, which is produced by using a powerful controlling field, must not, of course, be confounded with considerable damping, since, other things being the same, the decrement is the greater the weaker the controlling field and the slower the motion of the needle.

turning a milled head outside the instrument which works a right-and-left-handed screw inside.

In order to correct the throw of a ballistic galvanometer for the damping of the oscillations of the needle, it is well-known that it is necessary to multiply the sine of half the angle of throw by the expression

$$V = \frac{1}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \sum \frac{\lambda}{\pi} \tan^{-1} \frac{\pi}{\lambda},$$

where λ is the logarithmic decrement of the oscillations. The value of this quantity when λ is so small that its square may be neglected is

$$V = 1 + \frac{1}{2}\lambda.$$

It seemed, however, desirable to work out the value of V more fully and also to express it in terms of the numerical ratio of each amplitude to its successor, since in this way the need of finding the Napierian logarithm is avoided. This we did at first by direct calculation, but while forming a table of values connecting V and the decrement we observed that the matter could be treated much more simply as follows:—

By taking logarithms of the complete expression for V we have

$$\log V = -\frac{1}{2} \log (1 + x^2) + x \tan^{-1} \frac{1}{x},$$

where

$$x = \frac{\lambda}{\pi}.$$

Differentiate both sides of this equation once, twice, and thrice with respect to x and substitute for x the value 0. We then obtain successively,

$$\frac{dV}{dx} = \frac{\pi}{2}; \quad \frac{d^2V}{dx^2} = \frac{\pi^2}{4} - 3; \quad \frac{d^3V}{dx^3} = \frac{1}{8}\pi^3 - \frac{9}{2}\pi.$$

By Maclaurin's theorem we then have

$$V = 1 + \frac{\pi}{2}x + \left(\frac{\pi^2}{4} - 3\right)\frac{x^2}{2} + \left(\frac{1}{8}\pi^3 - \frac{9}{2}\pi\right)\frac{x^3}{6} + \&c.;$$

or substituting for x , and working out numerically, we obtain

$$V = 1 + 0.5\lambda - 0.027\lambda^2 - 0.054\lambda^3,$$

which shows that until λ approaches unity the approximate value for V generally used is sufficiently accurate. Now if the ratio of the amplitude of any swing to that of its successor is $(1+y)$, we have

$$\lambda = \log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \&c.,$$

$$\lambda^2 = y^2 - y^3,$$

$$\lambda^3 = y^3.$$

Neglecting powers higher than the third, we obtain by substitution

$$V = 1 + 0.5y - 0.277y^2 + 0.130y^3,$$

from which we can at once calculate the value of the complete correcting factor when we have observed the decrement.

It is usual to determine the sensibility of a ballistic galvanometer either by the employment of an earth-inductor, or by discharging through the galvanometer a condenser charged to a known P.D. The first method, however, necessitates an *exact* knowledge of the horizontal or vertical component of the magnetic intensity at the spot, while the second requires an *exact* knowledge of the value of the capacity of the condenser and of the P.D. employed. But now that it is possible to obtain an ammeter calibrated with a high degree of accuracy, the simplest method of determining the sensibility of a ballistic galvanometer is to first calibrate it absolutely as a galvanometer for measuring currents by direct comparison with the ammeter. Its absolute calibration as a ballistic galvanometer can then be determined from the fact that

The swing per micro-coulomb in scale-divisions

$$= \text{the deflexion per micro-ampere} \times \frac{2\pi}{T},$$

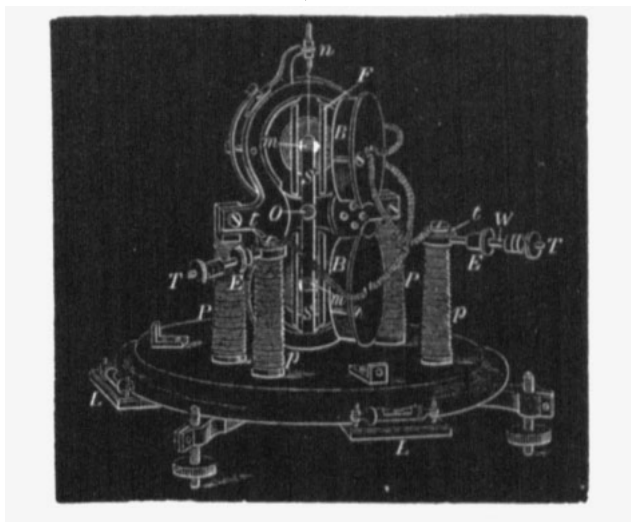
where T is the periodic time of oscillation of the needle in seconds.

IV. *Supporting the Coils.*

In olden days Messrs. Elliott supported the coils in their galvanometers by simply employing three long screws to squeeze each coil between a brass plate and the framework of the instrument to which the plate was screwed. The disadvantage, however, of such an arrangement was made painfully

clear in the use of a galvanometer constructed by this firm for one of the authors in 1868. For on transporting this galvanometer across India, soon after it was made, the brass screws expanded more than the coils, and the coils consequently slipped down, shearing away the needles, the mirror, and the aluminium vane. It was therefore suggested to Messrs. Elliott that the coils should be supported in boxes B, preferably hinged, as shown in fig. 6, so that when the coil-boxes are opened, as

Fig. 6.



in this figure, the suspended system S could be got at and even removed without detaching the connecting wires from the coils. The galvanometer shown in fig. 6 is, when in use, covered up by the brass case supporting the controlling magnet, illustrated in fig. 1.

Hinged boxes containing the coils of a reflecting-galvanometer appear to have been employed even earlier than this by Mr. Pepper, at the Polytechnic, but that form of construction was, apparently, in 1868 unknown to, or at any rate unused by, instrument makers.

In the case of galvanometers intended to be highly insulated, these hinged boxes may with great advantage be made of ebonite, and to prevent the boxes becoming electrified and

acting on the suspended needle electrostatically, one terminal of the galvanometer should be joined to the outside brass case.

In the galvanometer shown in fig. 6, the boxes are kept closed by a screw which is screwed into the framework of the instrument, but in the galvanometer shown in fig. 7 this screw is replaced by a spring button, which is a more convenient arrangement.

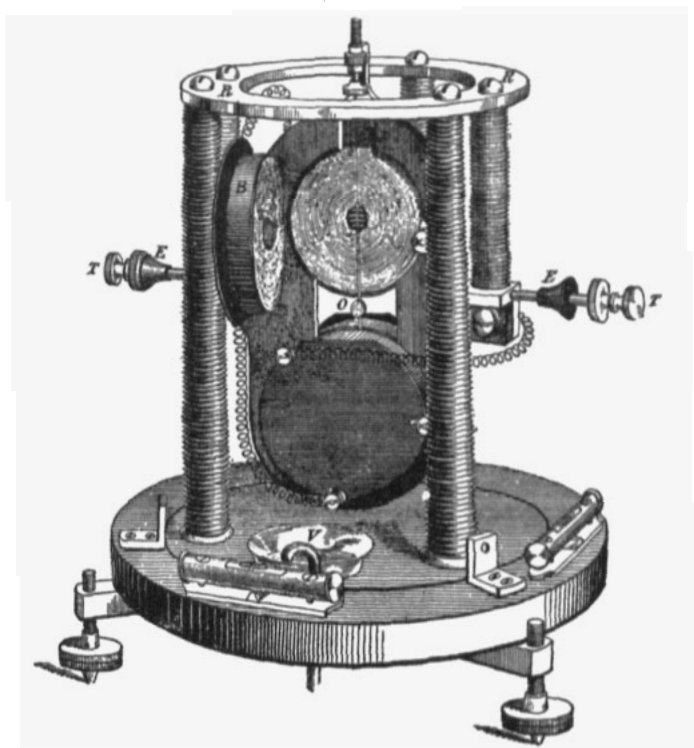
V. Insulation of Coils and Terminals.

In the ordinary method of constructing reflecting-galvanometers the wires are brought under the base to terminals fastened on to it. But such an arrangement is most unsatisfactory, since, no matter how thoroughly the upper part of the ebonite base may be cleaned and dried, leakage is almost sure to take place between the two wires, between the terminals, and between both the wires and the terminals to the ground along the inaccessible under surface of the ebonite base. In fact, when one of the authors was engaged, on behalf of the Indian Government, in 1872 testing telegraph insulators at Messrs. Siemens's works at Charlton, it was the common practice every morning to light a gas-burner near the galvanometer to dry it, and to diminish the leakage to a workable amount. And it appears to the authors that the form of expensive astatic reflecting-galvanometer, that is still made by some firms, with the wires underneath the base, is but a survival of the antiquated type of leaky electrostatical apparatus, to use which required that the whole air of the room should be first dried. Fig. 6 shows the obvious way of overcoming this difficulty, and which consists in supporting the framework and the coils of the apparatus as well as the terminals on paraffined corrugated ebonite pillars *p*, which are maintained clean and dry by their being kept entirely inside the brass cover of the instrument *Q* (fig. 1). The terminals *T* at the ends of the wires *W* project through holes *H* in the brass case, these holes being kept closed by tightly-fitting ebonite collars *C*, fig. 1, *E*, fig. 6, when the galvanometer is not in use, or when it is being employed for some test where a small amount of leakage is not important.

Still higher insulation is obtained with the galvanometer

shown in fig. 7, since in this instrument the coils are supported from two corrugated ebonite rods which hang from

Fig. 7.



Very high resistance galvanometer with brass case removed.

a brass ring *R*, carried on the top of three corrugated ebonite pillars fixed to the slate base-plate. This instrument, the four coils of which have a joint resistance of nearly 400,000 ohms, was constructed by Messrs. Nalder Bros.; but the device, by means of which the shortest path along which surface-leakage can take place from the coils, or from the terminals, to the base of the instrument is between 30 and 40 inches of ebonite artificially dried by sulphuric acid, is due to Messrs. Eidsforth and Mudford. To prevent any possible overflowing of the sulphuric-acid vessel, a syphon passes through the base on the Tantalus cup principle, and which empties the liquid when it rises to the level of the bend of the syphon.

VI. *Proportionality of Deflexion and Current.*

Some authorities are of opinion that it is of comparatively little consequence what is the law connecting deflexion and current, since for very accurate work any galvanometer must be carefully calibrated experimentally, and the exact law connecting deflexion and current thus ascertained. But, on the contrary, although the necessity of calibration for accurate work is perfectly true, experience has shown us that for ordinary practical work much time is saved if the readings on the scale are approximately, even if not quite accurately, proportional to the currents producing them. This latter result can, of course, be obtained either by fitting the lengths of the divisions on the scale to the peculiarities of the galvanometer, or by starting with a uniformly divided scale and constructing the galvanometer in such a way that equal additions to the current produce equal angular deflexions.

In certain cases, when, for example, the galvanometer is mainly employed for measuring currents all having about one definite value, as for example in the case of a voltmeter used on an electric-lighting circuit, it is obviously desirable to have the scale widely extended at the part where it is most used, and to obtain this result it is better to have the divisions crowded together elsewhere. But, since the power of accurately subdividing the spaces on a scale by eye is much increased if all the spaces be of equal length, it is preferable, in the case of galvanometers for general use, and when all parts of the scale are equally valuable, to judiciously construct the galvanometer in such a way that the whole scale may be divided into equal distances which are directly proportional to equal increments of current, than to give the coils and needle a shape arrived at in a haphazard fashion, and then attempt to experimentally subdivide the scale to suit the vagaries of the galvanometer.

In a former paper communicated by Prof. Perry and one of the authors to this Society, it was pointed out that while a galvanometer may be a "proportional" instrument if the zero for no current be that corresponding with the needles being in a symmetrical position to the coils, the same instrument may be very far from proportional if the zero for no current

be taken at one end of the scale and the spot of light deflected right across the scale for increasing currents, since in this latter case a variation in the law occurs as the plane of the needles passes through the symmetrical position. This want of proportionality is exhibited by the following figures :—

Ordinary Reflecting-Galvanometer.

Deflexion from one end of the Scale.	Relative Strength of Current.	Error.	Percentage Error.
0	0	0	
60	62.0	+2.0	3.4
120	121.0	+1.0	0.8
180	181.5	+1.5	0.9
240	242.5	+2.5	1.0
300	303.0	+3.0	1.0
360	363.5	+3.5	1.0
420	423.5	+3.5	0.8
480	483.0	+3.0	0.7
540	542.0	+2.0	0.4
600	600.0	0	

This defect is even more marked in some parts of the scale with the ordinary d'Arsonval galvanometer when the zero for no current is at one end of the scale. This instrument, as usually made by Messrs. Carpentier, has a tubular iron core supported from a standard which carries the top suspension of the coil, and the stationary permanent magnets are not provided with pole-pieces. In consequence of the core and the suspension not being supported independently, there is great difficulty in centering the coil relatively to both the core and the stationary magnets ; further, neither the core nor the coil can be readily removed. And partly because of this difficulty of centering the coil, partly because its centre of gravity is generally not in the line of suspensions, and partly because the stationary magnets have no properly shaped pole-pieces, there is (as may be seen from the following sample of the results of many experiments we have made on d'Arsonval galvanometers) an error in different parts of the scale far less regular in its value than in the case of the ordinary reflecting-galvanometer with stationary coil.

Ordinary d'Arsonval Galvanometer.

Deflexion from one end of Scale.	Relative Strength of Current.	Error.	Percentage Error.
0	0	0	
60	59	-1	-1.7
120	118.5	-1.5	-1.3
180	180	0	0
240	240.6	+0.6	+0.2
300	300	0	0
360	359.8	-0.2	-0.06
420	416.5	-3.5	-0.8
480	473.5	-6.5	-1.4
540	537	-3	-0.6
600	600	0	

We find that it is possible, however, to obtain with a d'Arsonval galvanometer a far more exact proportionality between the deflexion and the current than is given in the preceding table, by:—

Supporting the coil independently of the iron core, so that fairly accurate centering becomes possible. This result has been to a certain extent accomplished in the small type of d'Arsonval galvanometer constructed by Messrs. Nalder.

Allowing the coil to hang freely from the top suspension so that the centre of gravity of the coil is always in the axis of rotation. Electric continuity at the bottom of the coil is then maintained by a spiral of fine wire offering little torsional rigidity.

Fitting iron pole-pieces to the stationary magnets, and shaping these pole-pieces so that the rate of cutting lines of force by the moving coil is practically constant.

The following gives the results of tests made on a d'Arsonval galvanometer constructed in this way, each divisional length on the scale being 1.05 millim., so that the total deflexion of 600 divisions represented a motion of the spot of light over about 63 centimetres, or about 25 inches. The deflexion could be read easily to 0.2 of a division, and it will be observed that the greatest want of proportionality is only 0.15 per cent., which occurs with a deflexion of 480 scale-divisions.

Improved d'Arsonval Galvanometer.

Deflexion from one end of Scale.	Relative Strength of Current.	Error.	Maximum Percentage Error.
0	0	0	0.15
60	60	0	
120	119.9	-0.1	
180	179.8	-0.2	
240	239.9	-0.1	
300	299.8	-0.2	
360	360.2	+0.2	
420	420.5	+0.5	
480	480.7	+0.7	
540	540.5	+0.5	
600	600	0	

To improve the magnetic circuit in d'Arsonval galvanometers some English makers replace the original tubular core by a solid iron core; and Messrs. Jolin of Bristol form the core of a series of short horizontal magnets laid vertically one above the other. A comparison that we have made of the sensibilities of the instruments constructed in these different ways has not shown that the English instruments are superior in this particular respect to the French. This may be, however, due to the fact that the horseshoe permanent magnets employed by the English manufacturers are inferior to those used by Messrs. Carpentier.

The ordinary methods adopted for attaching the suspension wires of the d'Arsonval coil to the supports seem to have been designed with total disregard of the fact that these wires convey the current into and out of the coil. This is particularly serious when a d'Arsonval galvanometer is used as a voltmeter, especially when it is employed to measure a small fraction of a volt, and when therefore the total resistance in the circuit must be small.

In fact we have found that before any reliance can be placed on the indications of a d'Arsonval galvanometer employed in this way, it is necessary to solder all the joints. And instead of trusting to contact through the supports to which the torsional suspension-wires are attached, we have found it desirable to solder wires coming from the terminals of the instrument directly to the hooks to which the suspension-wires are attached. Perhaps the best plan is to solder

wires at one of their ends to the terminals of the instrument, and at their other ends to an extension of the torsional suspension-wires.

In devising very low-resistance d'Arsonval galvanometers careful consideration must be given to the suspension-wires which lead the current into and out of the moving coil, since being usually made of German silver they may easily become slightly heated by the relatively large current employed with a very low resistance-galvanometer, and in consequence their elasticity temporarily diminished. This would of course have the effect of making the instrument more sensitive for large currents than for small; but as the sensibility for large currents would, until a limit was reached, increase with the time the current was kept flowing through the galvanometer, such an instrument could not be used for accurate measurement. One method of overcoming this difficulty is to make the suspension of very thin phosphor-bronze strip, such as is employed in the construction of delicate "Ayrton and Perry magnifying-springs." For a thin strip has, for its cross-section, a very small torsional rigidity and a very large radiating surface, and therefore is the very thing to employ when we want a conductor with small torsional rigidity, and which will be very slightly raised in temperature by the passage of a current through it.

D'Arsonval galvanometers, when used as voltmeters, are subject to the error that affects all electromagnetic voltmeters, arising from a variation of the resistance of the coil due to a variation in the temperature of the room; if, however, both the coil and the suspension-wires are made of platinum-silver, a very curious compensating effect is brought about, since the percentage increase of resistance with temperature of platinum-silver is almost exactly equal to the percentage diminution in its torsional rigidity. Hence when a rise of temperature diminishes the current flowing through the instrument for a given P.D. maintained at its terminals and so diminishes the deflecting couple, a proportional diminution in the torsional rigidity of the suspending wires, and therefore in the controlling couple, is produced at the same time. *It is thus possible to construct an electromagnetic voltmeter of one metal only, which has no temperature error.*

VII. *Sensibilities of Different Types of Galvanometers.*

Apart from differences in detail (as, for example, in the size shape, or number of the coils, &c.), galvanometers may be divided into four distinct classes, viz.:—

1. Galvanometers in which the moving system consists of one or more magnetic needles turning about an axis at right angles to the magnetic axes of the needles.
2. Galvanometers in which the needles have a motion of translation along their own magnetic axes, the magnetic needles being drawn into, or pushed out of, coils acting like solenoids.
3. Galvanometers in which the magnetic system is stationary, and the moving system consists of a coil turning about an axis on its own plane.
4. Instruments based on the heating of a conductor by the passage of a current through it.

Type 1 includes tangent-galvanometers, Thomson's reflecting-galvanometers—in fact, so many well-known instruments that further description is unnecessary. In addition to galvanometers with straight magnetic needles, it may be noticed that type 1 also includes instruments containing a bell-shaped horseshoe-magnet; since the bell-shaped magnet is only a device for producing the equivalent of a very powerful and permanent short flat magnet.

We may here mention that the instrument shown in fig. 5 is spoken of in this paper as a galvanometer with "one pair of coils," while those illustrated in figs. 6 and 7 are referred to as having each "two pairs of coils."

Type 2 has been employed by Lord Rayleigh, the Profs. Gray, Mr. Rosenthal, by ourselves, and others. Figs. 8 and 9 illustrate an instrument of this type, constructed by Messrs. Edelmann of Munich, and called a Rosenthal Micro-galvanometer, since this instrument has the peculiarity that the coils into which the ends of the magnet are sucked are extremely small, having an *outside* diameter of only 11 mm. and a thickness of 2 mm. Fig. 9 shows the suspended magnet of the Rosenthal galvanometer as constructed by the makers, the arrangement being non-astatic; but we have also made and used with this galvanometer an astatic combination, constructed as in fig. 10.

Fig. 8.

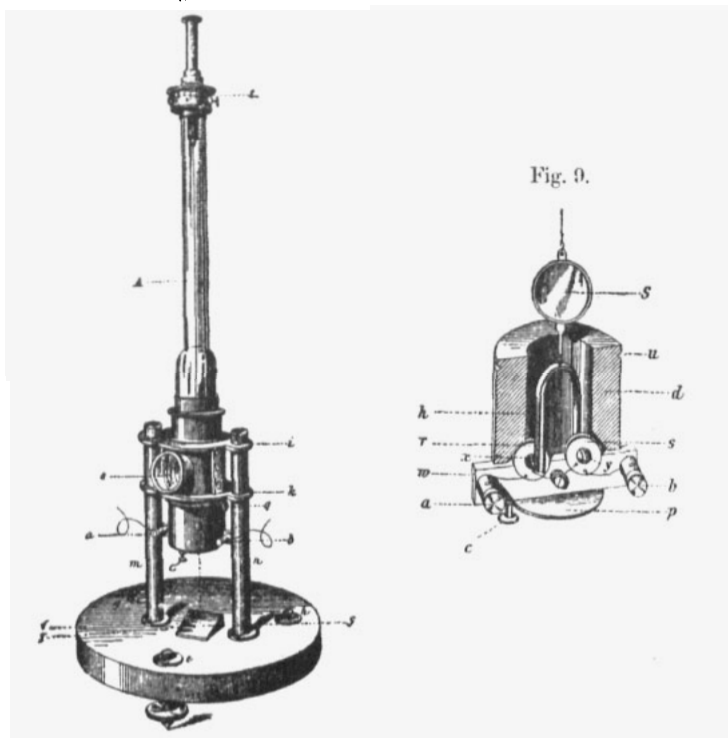
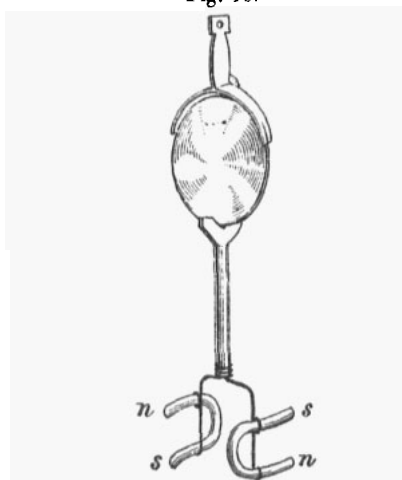


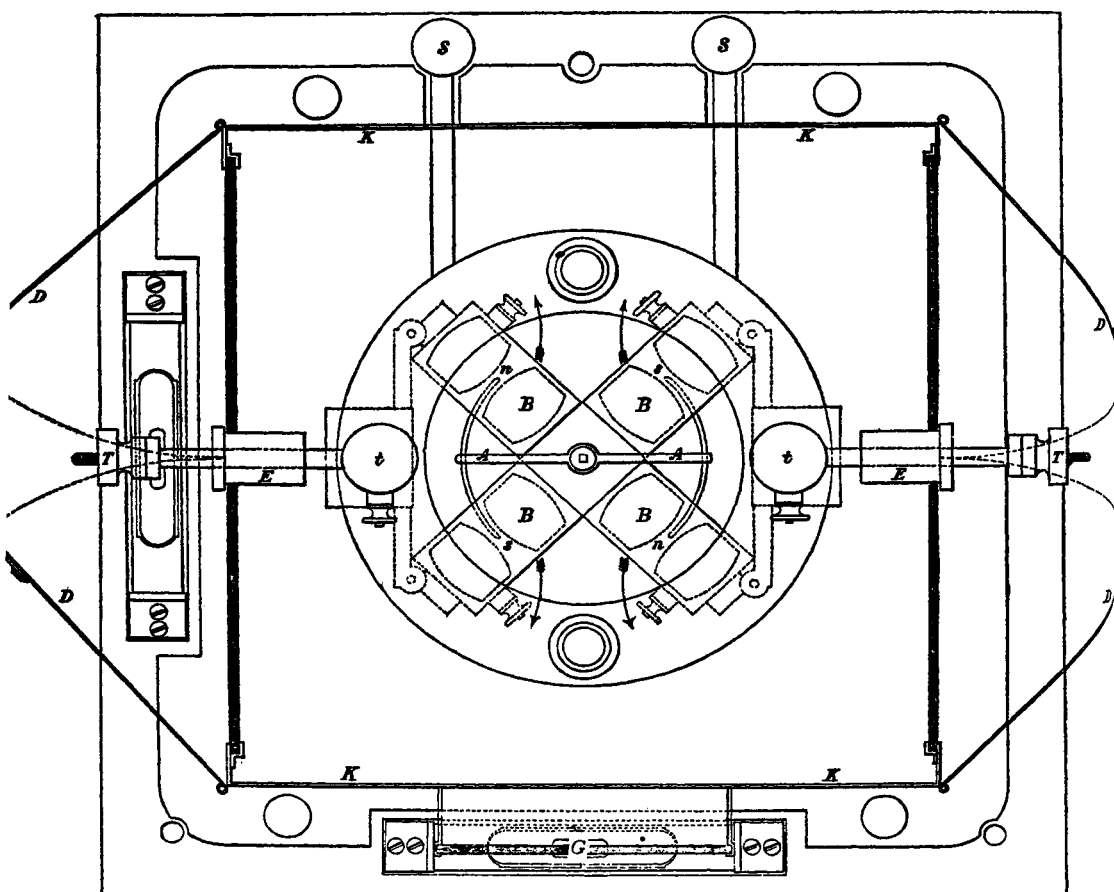
Fig. 10.



Mirror and astatic pair of needles for the Rosenthal Micro-galvanometer.

Figs. 11 and 12 show, in plan and elevation, a ballistic galvanometer constructed for the Central Institution by Messrs. White of Glasgow, from drawings made by us in consultation with Prof. T. Gray. The needles *ns, ns*, which are arranged so as to produce an astatic combination, are each at one end sucked into a coil and at the other end pushed out of a coil, the coils being contained in four small, rectangular, hinged boxes, *B, B*, which are kept closed by spring catches. The vertical aluminium wire, to which is screwed the aluminium bridge *AA* (fig. 11), supporting the needles, carries on one side

Fig. 11.

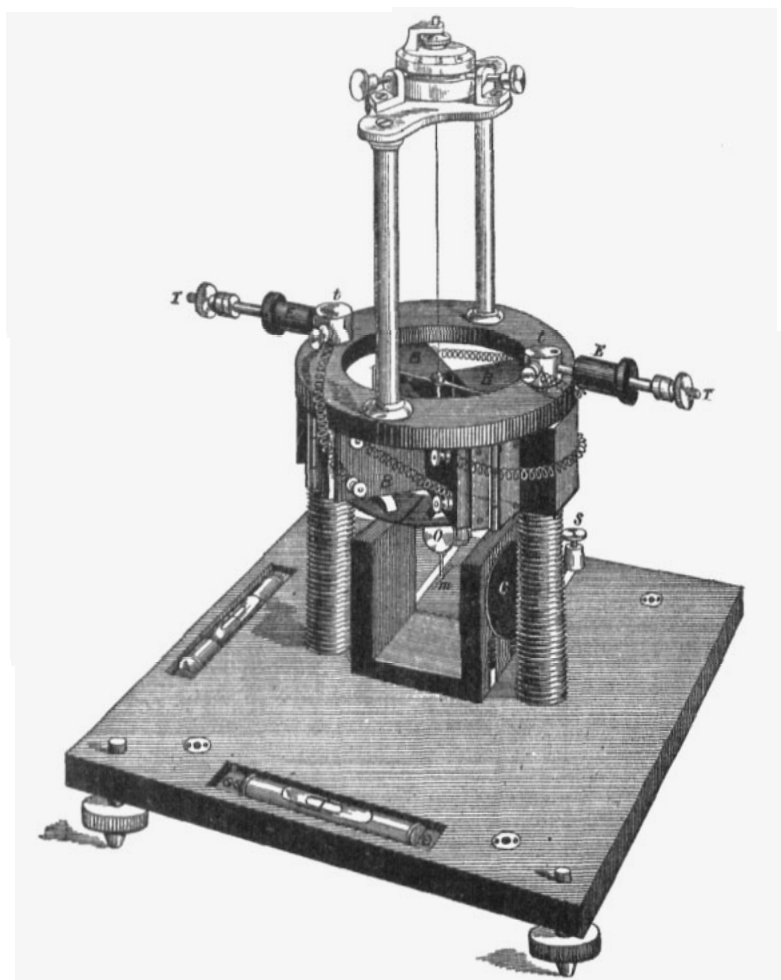


Plan of Ballistic Galvanometer.

of its lower end a concave mirror *O* (fig. 12) to produce a spot of light on a scale, and on its other side a truly

plane mirror, to be viewed with a telescope, if desired. At its lower end it carries a small magnetic needle, *m*,

Fig. 12.



Perspective elevation of Ballistic Galvanometer with brass case removed

to be acted on by a pair of coils, *C*, joined up in series and having their ends led to the terminal screws, *s*, *s*. These coils are quite independent of the main coils of the instrument, and are employed for damping the oscil-

lations of the suspended system. With ballistic galvanometers previously made for us we have not had any such damping-coil attached ; but since to all our galvanometers which are used ballistically it has been found convenient to provide a damping-coil, operated with an auxiliary small cell and roughly-made reversing-key, for the purpose of expediting measurements, we think that such a damping-coil should be attached, by the makers, to every ballistic galvanometer.

The brass case, K, has hinged brass doors D, D, D, D at its sides, to protect the glass windows ; and both doors and windows have holes in them to allow the terminals T, with their ebonite collars E, to be screwed into the brass terminal blocks, *t, t*. At the front of the instrument there is a circular plate-glass window G, to allow the light to pass through, and protected by a brass cap when not in use.

Type 3 includes the well-known d'Arsonval galvanometers, with the modification described to this Society some two years ago for obtaining "invariable sensibility," by replacing the ordinary torsional suspension with one having extremely small torsional rigidity and inserting a number of small magnets in the moving coil, which, being attracted by the large stationary magnet, produce the control.

Type 4 we do not propose touching on in this paper, as that type of instrument formed the subject of communications previously made to this Society by Prof. Perry and by one of the authors on "Hot-Wire Voltmeters" and "Twisted Strip Voltmeters."

It is important that the results of the numerous tests on many specimens of each of the first three types of galvanometer that have been carried out in the laboratories of the Central Institution, as well as the tests that have been made elsewhere, and the results of which we have collected together, should be reduced to the same standard ; so that the relative merits of the different types of galvanometer, as well as of the different specimens of each type, should be able to be usefully compared. A satisfactory standard of comparison, however, cannot easily be arrived at ; and we are not aware that the question of comparing galvanometers of totally different types has previously received much attention. The "figure of merit" of a galvanometer is sometimes defined as "the

amount of current which will produce one division or degree of deflexion”*. But although the figure of merit, so defined, tells us the sensibility of a particular galvanometer with a particular adjustment of the controlling magnet, and with a particular distance of the scale, it gives us no means of comparing this galvanometer with other forms, since this figure of merit can be varied by altering the position of the controlling magnet &c.—in fact, is not a constant of the particular galvanometer. Something much more definite than this is therefore required for our purpose.

A little consideration shows that the sensibility of a galvanometer in actual practice is made up of many factors, some of which depend on the skill of the experimenter himself. In view of the sensibility of any particular galvanometer being a complex result depending on many factors, it is desirable to record the results of experiments on galvanometers in such a form that the influence of each factor can be separately estimated. We have therefore endeavoured to distinguish, as far as possible, the three chief causes that affect the sensibility of reflecting-galvanometers, viz. the arrangement and winding of the coils, the construction of the magnetic system, and the optical method of magnifying the angular deflexion produced. With the same view the question of the susceptibility of various galvanometers to outside magnetic disturbances has been considered separately, since, first, the electrical qualities of the instrument are not altered by such disturbances, and, secondly, their influence is dependent, not only on the kind of instrument, but on the place of test. In the Table that is given later on recording the results of a large number of experiments, we have classified galvanometers in four divisions of decreasing susceptibility to outside magnetic disturbance, comprising :—

Non-astatic instruments working in a weak field having strongly magnetized needles.

Non-astatic instruments working in a strong field and having somewhat weakly magnetized needles.

Astatic galvanometers, working in a fairly strong field.

Galvanometers of the d’Arsonval type, working in an extremely strong field.

* ‘A Handbook of Electrical Testing,’ by H. R. Kempe.

At the end of the Table are given the constants of a collection of non-reflecting instruments. These instruments, being all of a rougher type, have been simply grouped together at the end of the list; consequently their position on the list has no reference to their sensitiveness or unsensitiveness to outside magnetic disturbance.

The optical sensitiveness of an instrument is dependent on many conditions that have no necessary connexion with the electromagnetic qualities of the instrument; for example, the goodness of the mirror, the brightness of the light, the distance of the scale, &c. are important factors. We have therefore given the number of scale-divisions per micro-ampere or per micro-coulomb, assuming the scale to be placed at a distance from the mirror equal to 2000 scale-divisions. As the scale-distance most generally used is a metre, and as the scale-divisions are frequently in half-millimetres, the conditions assumed are often fulfilled in practice. The sensibilities for a steady current, and for a discharge, are thus directly expressed in angular measure, irrespective of optical magnification.

An important and difficult question to decide is whether the factor of merit of a galvanometer should be stated for a constant periodic time of oscillation of the suspended system, or for a constant controlling-moment for unit angle-deflexion, or for a constant controlling-field irrespective of the magnetic moment of the suspended system. The first condition is the most simple to realize experimentally with different instruments, and the results so obtained are the most convenient for use in subsequent calculations; this condition is, further, the one most easily understood. We therefore give, in columns 3 and 4, the values of D and S , which respectively denote the deflexion per micro-ampere and the swing per micro-coulomb, each in scale-divisions, when the period is 10 seconds and the scale-distance is 2000 scale-divisions. The corresponding values at any other period T are of course $DT^2/100$ and $ST/10$. It would, however, be unfair to rest satisfied with the results given in these tables for the purpose of comparing different galvanometers. For a galvanometer having a large mirror, and intended to be used at a considerable distance from the scale, would appear to be an inferior instrument, because on

the one hand the large moment of inertia of the mirror would necessitate the controlling-moment being large to reduce the period to 10 seconds, while, on the other hand, the magnification arising from the much greater distance that separated, in practice, the scale from this particular galvanometer would have no weight in the values given in the columns for D and S. We have therefore given in column 2 the values of M, which are the millimetres deflexion per micro-ampere when the period is 10 seconds and the scale placed as in the ordinary use of the instrument. We have further, in the following way, reduced the numbers to their corresponding values under the condition of *constant controlling-moment*, the condition of constant controlling-field being rejected, because it is inapplicable to instruments of the d'Arsonval type.

The periodic time T equals

$$2\pi\sqrt{\frac{\text{Moment of inertia} \times \sin \theta}{\text{Controlling-moment for a deflexion } \theta}};$$

therefore, in order to obtain the comparative deflexions that would be obtained with the same controlling-moment per unit angle, we must find the deflexions that would be produced with periodic times proportional to \sqrt{I} , where I is the moment of inertia of the suspended system, the values of which for the various galvanometers are given approximately in C.G.S. units in column 6. Hence, to obtain the values in columns 7 and 8, which give numbers respectively proportional to the steady deflexion per micro-ampere and the first swing per micro-coulomb for the same controlling-moments for all galvanometers, we must multiply D by I and S by \sqrt{I} . Now the stability of the zero of any galvanometer depends on the controlling-moment per unit angle of deflexion; hence columns 7 and 8 give numbers proportional to the angular deflexion per micro-ampere and the angular swing per micro-coulomb for the same amount of stability of the zero for all the instruments. The periodic times, however, will be very different for the different numbers either in column 7 or for the different numbers in column 8, the periodic time corresponding with any particular number being equal to $10\sqrt{I}$ seconds, where I is the moment of

inertia given in column 6 for the particular instrument in question. For example, the numbers 3200 and 283, given in columns 7 and 8 for Profs. T. and A. Gray's galvanometer, correspond with a periodic time of $10\sqrt{50}$, or about 70 seconds: while the numbers 542 and 3400, given in the same columns for the 360,000-ohm galvanometer at the Central Institution, correspond with a periodic time of only $10\sqrt{0.01}$, or 1 second. The numbers, then, in columns 7 and 8 give a fair comparison between the various galvanometers when the time required to take an observation is unimportant.

The influence of resistance has next to be considered. If two galvanometers differ only in the gauges of wire with which they are wound, and the convolutions are similarly distributed, then the sensibilities should be proportional to the number of turns. If, again, the thickness of the insulating covering bears a constant proportion to that of the wire, then the resistance of the coils would be proportional to the square of the number of turns; so that the values of $D/r^{\frac{1}{2}}$, $S/r^{\frac{1}{2}}$ would be constants for the same pattern of instrument, where D and S are the number of scale-divisions per micro-ampere and micro-coulomb respectively, and r is the resistance of the galvanometer. The proportion in question, however, varies not only for coils of different resistances, but also in the same coil, for several different gauges are generally used in winding the various portions. To get some idea of the relation between resistance and sensibility, we have made tests on galvanometers of the same pattern but of different resistances, the same suspended system of magnets being used successively in each instrument, the controlling magnet being adjusted to produce the same periodic time in each experiment, and, when the galvanometers were reflecting ones, the scale being placed at the same distance from each of the instruments. The following were the results obtained with three astatic reflecting-galvanometers having coils of exactly the same size in each case, but of very different resistance.

Resistance of Galvanometer, in ohms.	D. .	$D/r^{\frac{1}{2}}$.	$D/r^{\frac{2}{3}}$.
26	349	68.5	94.4
2457	2160	43.5	95.4
6410	3360	42	101

From this it would appear that, if the only thing altered in a galvanometer be the gauge of wire with which it is wound, the sensibility is more nearly proportional to the resistance raised to the power $\frac{2}{3}$ than to the resistance raised to the power $\frac{1}{2}$.

Another set of experiments was now made with four non-astatic galvanometers, the coils of each of which were made in the same mould, totally different, however, in shape from that employed in constructing the coils of the three reflecting-galvanometers given in the last list. When tested with the same suspended system, the controlling magnet being adjusted to produce the same periodic time in each case, the following results were obtained :—

Resistance of Galvanometer, in ohms.	M, Fraction of one degree per micro-ampere.	$M/r^{\frac{1}{2}}$.	$M/r^{\frac{2}{3}}$.
0.87	0.0149	0.01597	0.0157
4.30	0.0333	0.01604	0.0185
64.23	0.1120	0.01397	0.0246
399.25	0.2700	0.01352	0.033

In this case the sensibility appears to be more nearly proportional to $r^{\frac{1}{2}}$ than to $r^{\frac{2}{3}}$. We have, therefore, in the large table given two columns, one (No. 9) for $D/r^{\frac{1}{2}}$ and one (No. 10) for $D/r^{\frac{2}{3}}$, which represent (according as $r^{\frac{1}{2}}$ or $r^{\frac{2}{3}}$ may be regarded as the more accurate divisor to employ) the deflexion per micro-ampere on a scale whose distance is 2000 scale-divisions for a periodic time of 10 seconds that would be obtained if all the different forms of galvanometers were wound with such a gauge of wire that the resistance of

each instrument was 1 ohm. Column 11 gives the swing per micro-coulomb on the same hypothesis, but in this case we have only used $r^{\frac{2}{3}}$ as the divisor.

And whereas columns 9, 10, and 11 give the relative values for the same periodic time, columns 12 and 13 give the deflexion per micro-ampere and the swing per micro-coulomb on a scale whose distance is 2000 scale-divisions for *the same controlling-moment* that would be obtained if all the galvanometers had 1 ohm resistance—the periodic time in any case being equal to $10 \sqrt{I}$ seconds, I being the moment of inertia of the suspended system given in column 6.

Another question to be considered is the factor of merit in connexion with the volume of the coils. As this is important as regards the cost of high-resistance galvanometers, a separate column (5) is therefore given, showing the approximate volume in cubic centimetres occupied by the wire in each instrument, and other columns (14, 15, 16, 17) give the values obtained by dividing this quantity into the various factors of merit. The results of comparison in these latter columns are very interesting. It appears that galvanometers of the Rosenthal or Gray and the d'Arsonval types are by far the most sensitive in proportion to the volume of coil; and we believe that the best way to make a very sensitive galvanometer with a movable magnetic system is to employ several small coils, instead of one or two large ones*, and that the magnets should be horse-shoe-shaped, with the line joining the poles vertical, as in Prof. Gray's instrument, but modified so as to give smaller moment of inertia. Such a magnetic system can be made very delicately astatic, and the weakening of the magnets with time ~~does not~~ greatly influence the astaticism. We also conclude from our investigations that the most sensitive galvanometer of all would be one of the d'Arsonval type suitably modified. The poles of the magnet should be very close together, the coil should be very long and narrow, and no stationary iron core should be used inside the coil as in the ordinary d'Arsonval galvanometer. The sensitiveness could be further increased by employing electro-magnets instead of permanent ones to pro-

* Mr. C. V. Boys, F.R.S., in his Cantor Lectures on "Instruments for Measuring Radiant Heat" (April 1889), has also shown that galvanometers with small coils may be made very sensitive (pp. 22, 23).

duce the deflecting field, the current flowing round the electromagnet being kept constant by means of an auxiliary reflecting "set-up ammeter" producing an image on the same scale as is used for the electromagnetic d'Arsonval galvanometer. With a "set-up ammeter" the suspended system is supported by means of an almost torsionless phosphor-bronze strip, requiring many twists to be given to it to bring the spot of light on to the scale when the current, that is to be kept constant, is flowing through it. A motion, then, of the spot of light over 100 divisions of the scale corresponds perhaps to a variation of only $\frac{1}{20}$ in this current; and consequently, by means of such an instrument and a suitable adjustable resistance, a current can be kept constant to a very small fraction per cent.

It might be objected to our conclusions that, in the papers read by Prof. Threlfall before this Society, and published in the 'Philosophical Magazine' for December 1889, he comes to the conclusion that coils used in what we may shortly call the Mudford way (figs. 5, 6, and 7) give greater sensibility than when used in the Gray way (figs. 8-12). But on studying Prof. Threlfall's papers we find that, from various numerical errors, he makes his coils when used as a Gray galvanometer one ninth as sensitive as they really were, and when used as a Mudford galvanometer seventeen times as sensitive as they were; so that Prof. Threlfall's ratio of the sensibility of the Gray galvanometer to the sensibility of the same coils when used as a Mudford galvanometer requires to be multiplied by the very large number 153*.

In order to correct Prof. Threlfall's calculations, the following alterations must be made:—

Phil. Mag., Dec. 1889: Page 465, line 11, *for* "The current was therefore 1.26×10^{-7} amperes," *read* "The current was therefore 1.43×10^{-8} amperes."

Same page, line 25, *for* " 2.5×10^{-8} amperes" *read* " 2.8×10^{-9} amperes."

Page 469, line 1: "The galvanometer having been brought to a state of sensitiveness of 5 scale-divisions for 10^{-11} amperes, the measurement of the resistance of the sample of sulphur

* These errors were also noticed by Prof. A. Gray, and referred to in an article in the Phil. Mag. for February 1890.

		1.	2.	3.	4.
Type.	Description.	r.	M.	D.	S.
Thomson. One pair of coils.	Elliott's Tripod (F.T.C.) †	0.6	3.8	7.6	4.76*
	Tripod	50	102	204	128*
	Elliott's Tripod. " Specially light {	3860	7550	15,400	9,700*
	needles †	6000	15,750	32,000	20,000*
Thomson. Two pairs of coils.	Elliott's Glass Cylinder (F.T.C.) †	4600	1220	2440	1520*
	Latimer Clark & Muirhead (F.T.C.) † ..	20,000	8700	10,700	6720*
	Elliott's Stock pattern †	7000	4300	7110	4450*
	" Large Coil pattern †	12,300	7680	15,200	9550*
	"	30,000	10710	21,800	13,700*
	" Special form †	30,000	20,000	40,000	25,000*
	Arvton & Perry ballistic (Japan)	20,000	1070	1480	900
Mudford's Thomson. One pair of coils.	Langley's Helomast	20	125	250	156*
	Figure 3 (C.I.)	26	347	349	220*
	"	2,457	1375	2160	1360*
	"	6,410	2720	3360	2110*
	"	2086	880	1720	1080*
	Beil Magnet Ballistic †	7800	720	1350	850*
	"	10,000	640	1200	755*
Mudford's Thomson. Two pairs of coils.	Figure 5 (C.I.)	5330	2280	1440*
	Figure 6 (F.T.C.)	700	1720	2970	1860*
	(C.I.)	686	1660	1600	1010*
	Variable damping Large Coils (C.I.)	9744	4200	4280	2640
	Figure 7 (C.I.)	360,000	50,000	54,200	34,000*
	"	350,000	15,700	31,400	19,700*
	"
Rayleigh, Gray, or Rosenthal.	Figure 8, Non-astatic (C.I.)	44	22.9	25.4	16
	Figure 9, Astatic (C.I.)	44	25	27.7	17.4
	Non-reflecting (C.I.)	5.2	9.06	5.7*
	Gray's (Proc. Roy. Soc. 1884, p. 287) ...	30,000	32	64	40*
	Figures 11 & 12 (C.I.)	4090	54	63	39.2
	Threlfall's (Phil. Mag. Dec. 1889, p. 460)	15680	2.90	3.74	2.35
	"
d'Arsonval.	Jolin's High-resistance (C.I.)	750	137	260	163*
	Carpentier's ordinary	208	286	375	236*
	Jolin's Low resistance (C.I.)	3.8	39	42	26.4*
	Invariable sensibility, Large type (C.I.) ..	21	190	181	114*
	Nalder's type fitted { Pole pieces in (C.I.)	274	169	106*
	with Invariable- " out "	274	242	152*
	sensibility device.
Flat Coil or Multiplier type.	Carpentier milliamperemeter (C.I.)	183	73	45.7*
	Single Needle Post-office pattern (C.I.) ..	800	190	119*
	Coils 6 centim. long outside {	400	303	190*
	" 2.7 " broad " {	350	146	91.7*
	" 2.2 " high " {	64	125	78.7*
	" 2.2 " high " {	4.3	14.8	9.3*
	Winding 0.6 " thick. " {	4.3	35.2	22.1*
	"87	6.8	4.27*
	"87	16.7	10.5*
	Detector nearly astatic †	2000	490	308*
	" partially astatic †	2000	42	26.4*

5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.
V.	I.	DI.	SI ^{1/2} .	D/r ^{1/2} .	D/r ² .	S/r ^{3/2} .	DI/r ^{3/2} .	SI ^{1/2} /r ^{3/2} .	D/r ^{3/2} V.	S/r ^{3/2} V.	DI/r ^{3/2} V.	SI ^{1/2} /r ^{3/2} V.
.....	10	9.4	5.9
125	0.01	81.6	25.6	28.7	42.8	26.9	17.1	5.4
125	0.01	154	970	248	566	356	5.66	35.6	4.53	2.85	0.045	0.23
125	0.01	320	2000	413	986	616	9.86	61.6	7.89	4.93	0.079	0.46
.....	83	83	52
.....	76	203	128
30	0.013	92	507	85	205	129	2.68	14.7	6.8	4.3	0.089	0.049
240	0.013	198	1099	137	350	220	4.53	25.2	1.46	0.91	0.013	0.013
240	0.013	284	1560	126	353	220	4.6	25.3	1.47	0.91	0.013	0.013
240	0.013	231	648	405	2.67	1.60
.....	16.5	27.3	17.1
.....	55	76	47
72	0.02	14	45	66.5	94.4	60	3.92	12.2	1.31	0.88	0.055	0.16
72	0.02	91	284	43.5	95.5	60	4.00	12.5	1.33	0.89	0.055	0.16
72	0.02	142	435	42	101	63	4.25	13.0	1.40	0.87	0.059	0.17
72	37.6	81	51	1.12	0.71
.....	15.3	37.3	23.5
.....	12.0	30.2	19.0
72	0.02	95.5	295	31.0	74.0	46.6	3.1	8.9	1.03	0.66
.....	112	215	136
88	0.084	134	293	61	118	74	9.81	21.5	1.34	0.84	0.011	0.24
200	0.052	222	602	43.5	108	67	5.61	15.3	0.54	0.34	0.023	0.078
236	0.01	542	3400	90	324	202	3.24	20.2	1.37	0.86	0.014	0.086
.....	53.1	190	119
0.29	1.5	38	19.6	3.82	5.6	3.5	8.4	4.3	19.3	12.1	29.0	14.8
0.29	0.7	19.4	14.5	4.2	6.1	3.8	4.27	3.2	21	13.1	14.7	11.0
2.0	21	1.91	2.61	4.0	4.7	2.94	98	1.35	2.35	1.47	0.49	0.67
51	50	3200	283	0.37	1.04	0.6	52	4.6	0.02	0.012	0.10	0.09
36	17	1070	165	1.0	2.26	1.4	39	6.0	0.06	0.04	1.1	0.17
72	0.3	0.78	0.49	0.0108	0.0068
2.8	3.0	7800	895	9.5	18.4	11.5	55.1	6.3	6.6	4.1	19.8	2.2
.....	26	44.3	27.9
0.6	5.5	230	62	21.5	25	15.5	134	36.3	42	26	225	61
0.96	13	2360	410	39.5	54	33.6	700	122	56	35	730	126
2	1.5	254	129	10.2	17.7	11.2	27	13.7	88.5	56.0	135	68.5
.....	14.6	25.5	16.1
3.2	25	1820	228	5.4	9.0	5.6	225	28	2.8	1.74	70	8.7
.....	6.7	131	82
.....	60.6	85	15.1	27.6	17.3	5.5	7.75	1.26	0.79	0.25	0.35
22	2	29.2	41	7.8	14.0	8.8	2.82	3.94	0.64	0.40	0.13	0.18
.....	25	35	15.6	23.8	14.9	4.75	6.65	1.08	0.68	0.21	0.30
.....	2.96	4.1	7.1	8.25	5.18	1.65	2.28	0.37	0.23	0.07	0.10
.....	7.04	9.9	17.0	19.6	12.3	3.92	5.5	0.85	0.56	0.18	0.25
.....	1.4	1.9	7.3	7.2	4.53	1.49	2.02	0.32	0.21	0.07	0.09
.....	3.3	4.7	18	17.9	11.1	3.5	5.0	0.82	0.50	0.15	0.23
.....	1.1	23.4	14.7
.....	0.9	2.0	12.6

in question became a tolerably easy matter." On examining, however, the results obtained in the actual measurements with the sulphur (p. 471), we find that the sensibility of the galvanometer was not as above stated, 500,000 divisions per micro-ampere, but 34,200. And, as his scale was 3 metres away from the galvanometer, and his scale-divisions 1 millim., this number 34,200 reduced to our standard becomes 22,800*.

EXPLANATION OF TABLE.

r =resistance in ohms.

M =millimetres deflexion per micro-ampere when the period is 10 seconds, and the scale is placed as in the actual use of the instrument.

D =deflexion in scale-divisions per micro-ampere when the period is 10 seconds, and the scale-distance is equal to 2000 scale-divisions.

S =swing produced per micro-coulomb under the same conditions as in last.

V =volume occupied by the convolutions of wire, in cubic centimetres approximately.

I =moment of inertia of the suspended system, in C.G.S. units approximately.

DI =deflexion in scale-divisions (scale-distance equal to 2000 scale divisions) per micro-ampere for constant controlling moments, and for a periodic time equal to $10\sqrt{I}$ seconds.

$SI\frac{1}{2}$ =swing per micro-coulomb under the same conditions as in last. Columns 9, 10, and 11 give the deflexion per micro-ampere, and the swing per micro-coulomb, when the period is 10 seconds and the resistance of each instrument one ohm.

* Prof. Threlfall says, in a letter published in the *Phil. Mag.* for June 1890, in reply to these criticisms, that his galvanometer was not in a state of maximum sensibility when used for these experiments on sulphur. Granting this, and looking at the observations given on page 472 of the *Phil. Mag.*, vol. xxviii., from which the constant of the galvanometer was determined, we find that there was considerable vagueness of the zero, amounting in some cases to more than 4 per cent. of the double deflexion, as well as a gradual change in the position of the zero from 105 to 78. If, then, the control were diminished to about $\frac{1}{17}$ of the value it then had, and which would be necessary to give the galvanometer the sensibility claimed for it, the vagueness of the zero would probably become too serious for the instrument to be used for any but the very roughest measurements.

Columns 12 and 13 give the deflexion per micro-ampere and the swing per micro-coulomb, for the same controlling moment, and for the resistance of each instrument equal to one ohm.

Columns 14 and 15 give the deflexion per micro-ampere and the swing per micro-coulomb per cubic centimetre of coil, when the period is 10 seconds and the resistance of each galvanometer one ohm.

Columns 16 and 17 give the deflexion per micro-ampere and the swing per micro-coulomb per cubic centimetre of coil, for the same controlling moment, and the resistance of each galvanometer equal to one ohm. Periodic time of any instrument is $10\sqrt{I}$ seconds.

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F.T.C., Finsbury Technical College.

† The constants of these instruments have been kindly furnished by the makers.

‡ These instruments were tested after being in use for some time and without their needles being remagnetized.

* These numbers are calculated from those given in column 3.

VIII. *Self-Induction of Galvanometer-Coils.*

We have already seen that the sensibilities of two precisely similar galvanometers, which differ only in resistance, cannot easily be predicted from their resistances, since, in consequence of the varying thickness of the insulating covering of the wire, there is no simple connexion between resistance and the number of convolutions of wire occupying a given space. It therefore occurred to us to consider whether, since the self-induction of a coil of a given size and shape depends mainly on the square of the number of convolutions, and but little on the thickness of the wire or of the insulating covering, the square root of the self-induction would be proportional to the magnetic constant of the coil. The following table gives the results of the tests on the three astatic reflecting-galvanometers already referred to :—

Galvanometer.	Resistance, in ohms. r .	Self-Induction, in secohms. L .	Time Constant, in seconds.	D.
A	26	0.052	0.002	349
B	2457	1.75	0.00071	2160
C	6410	4.72	0.00074	3360

D, as before, standing for the number of scale-divisions of deflexion produced by the same current with the same distance of the scale, the same system of mirror and magnets being used in each case, and the controlling magnet adjusted to give the same periodic time of oscillation. The following gives the ratio of the values of D, of the square roots of the self-inductions, of the square roots of the resistances, and of the resistances to the power $\frac{2}{3}$:—

$$\frac{D_B}{D_A} = 6.190 ; \sqrt{\frac{L_B}{L_A}} = 5.801 ; \sqrt{\frac{r_B}{r_A}} = 9.721 ; \left(\frac{r_B}{r_A}\right)^{\frac{2}{3}} = 6.167.$$

$$\frac{D_C}{D_B} = 1.555 ; \sqrt{\frac{L_C}{L_B}} = 1.643 ; \sqrt{\frac{r_C}{r_B}} = 1.615 ; \left(\frac{r_C}{r_B}\right)^{\frac{2}{3}} = 1.468.$$

$$\frac{D_C}{D_A} = 9.627 ; \sqrt{\frac{L_C}{L_A}} = 9.526 ; \sqrt{\frac{r_C}{r_A}} = 15.70 ; \left(\frac{r_C}{r_A}\right)^{\frac{2}{3}} = 9.051,$$

From the preceding list it appears that the ratio of the square roots of the self-inductions gives the ratio of the galvanometer-constants of coils, not only much more accurately than the ratio of the square roots of the resistances, but, on the whole, even more accurately than the ratio of the resistances to the power $\frac{2}{3}$.

In view, then, of the importance of knowing the self-induction of galvanometer-coils, we give the results of some measurements made by means of the secohmmeter :—

Reflecting-Galvanometer with One Pair of Coils.			
	Resistance, in ohms.	Self-Induction, in secohms.	Time-Constant, in seconds.
Front coil	3007	3.75	0.00125
Back coil	2830	3.16	0.00112
Front and back coils, joined up as in actual use	5837	9.6	0.00164
Mutual induction of the two coils in position	1.35	
Reflecting-Galvanometer with Variable Damping.			
Four coils in series, but separated so as have little Mutual Induction	9680	12.48	0.00129
Reflecting High-Resistance Galvanometer (fig. 7).			
Top front coil	105,600	40.72	0.000386
Top back coil	101,200	39.50	0.000390
Bottom front coil	90,000	27.51	0.000306
Front top and back coils joined up in position	207,000	109.8	0.000531
Reflecting Ballistic Galvanometer (fig. 11).			
One Coil only	1003	0.166	0.000166
Flat Single-Coil Galvanometer. (Nobili type.)			
The one Coil	64	0.0324	0.000506

From the above it appears that a coil of the ordinary size and shape used in Thomson galvanometers has a coefficient roughly equal, numerically, to one thousandth of its resistance in ohms. If the coils of the instrument are in series and placed near each other, as in actual use, the coefficient is increased by about half its former amount, owing to mutual induction; so that, for Thomson galvanometers whose resistances vary between 1000 and 10,000 ohms, the time-constant is about 0.0015 second. With very high-resistance instruments the time-constant falls to 0.0005 second, in consequence of the increasing importance of the insulating covering of the wires.

IX. *Effect of Time and Use on Galvanometer-Needles.*

The following give the values of *D*, the deflexion in divisions of a scale, placed at a distance from the galvanometer equal to a length occupied by 2000 scale-divisions, produced by one micro-ampere, the controlling magnet being adjusted to give a periodic time of oscillation equal to 10 seconds.

Double-Coil Astatic Reflecting-Galvanometer.

Resistance, 686 ohms.

	<i>D</i> .
Needles magnetized and tested the same day.....	1600
After about two years' use	366
Needles remagnetized and then used for about one year .	826

Single-Coil Astatic Reflecting-Galvanometer.

Resistance, 26 ohms.

As obtained from the makers	198
After about three years' use	170
Needles remagnetized and tested the same day	243

Double-Coil Astatic Reflecting-Galvanometer ; Variable Damping. Resistance, 9744 ohms.

As obtained from the makers	4200
After about three years' use	3000
Needles remagnetized and tested the same day	4280

Rosenthal Non-astatic Reflecting Micro-Galvanometer.

Resistance, 44 ohms. (Figs. 8 and 9.)

As obtained from the makers	25·4
After four years' use	5·16

The needles of the first and last of these four galvanometers have lost much more magnetism than have the needles of the second and third galvanometers. As regards the Rosenthal instrument it might have been expected, in view of the powerful nature of its magnetic needle (see fig. 9) and of the fact that the current only acts on small projections of this magnet, that the magnetism would not be much diminished by use. But it has to be remembered that, whereas with the ordinary reflecting-galvanometers the zero-position of the needles is that in which the lines of force due to the coil are at right

angles to the axes of the magnetic needles, and therefore the needles are not in a position to have their magnetism much altered by the passage of a current, unless that current be very strong, the suspended magnet of the Rosenthal galvanometer will tend to have its magnetism diminished by every current sent through the instrument in such a direction as to push the poles of the suspended magnet out of the coil, since the lines of force of the two coils will pass along the axis of the suspended magnet. Care not having been taken to send the current round the Rosenthal galvanometer, so as to always suck the ends of the magnet into the coils, is probably the explanation of the weakening of its suspended magnets during four years in the ratio of 5 to 1; whereas the falling-off in the sensibility of the first of the four galvanometers in the preceding list in the ratio of 4·3 to 1 during a shorter time is probably due to inferior steel having been used in the construction of its magnetic needles.

In view of the great difference which exists between various specimens of magnet-steel, it is most important that only the very best magnet-steel should be employed in the construction of galvanometers which are intended to have a high sensibility, since the factor of merit of the instrument depends largely on the goodness of the magnetic needles.

Mr. C. V. Boys, F.R.S., thought that the factor of merit of galvanometers should not be given in scale-divisions per micro-ampere under the condition of constant controlling moment. This gave too great an advantage to instruments of the Gray or Rosenthal type. Great sensibility could be obtained by diminishing the moment of inertia of the suspended parts, the practical limit being determined by the trouble due to the silk fibre. Spider lines, when used in place of silk fibres, gave better results. It was possible, by using a good suspending arrangement, to use needles $\frac{1}{8}$ " long, and a period of twenty seconds, and to gain a sensibility far greater than those indicated in the paper. Ballistic galvanometers should be made with needles as light as possible. The method proposed of winding the central part of the coil in the opposite sense to the rest would probably not be good, owing to the unevenness of the field produced. The conclusion come

to by the authors that D'Arsonval galvanometers of great sensibility should be made with small coils placed in a very strong field, was one he had himself come to, but had finally abandoned, owing to difficulties caused by diamagnetism in the copper and to excessive damping due to Foucault currents.

Mr. Swinburne thought that the factor of merit of a galvanometer should be determined differently, according as it was to be used for the measurement of current, or quantity, or for null methods merely. He saw no great advantage in making practical instruments proportional. The name D'Arsonval should be dropped, as the instrument denoted by it was invented by Varley years ago. He would like to know the relative sensibility of the telephone and the Lippmann galvanometer.

Prof. Fitzgerald stated that Lord Rayleigh had shown that the microscope-method of observing angular deflexions was as sensitive as the ordinary method of mirror and scale, even when only the mirror was used as a pointer, so that if a pointer were attached it would be far more sensitive. The drawback, however, was that it was impossible to distinguish, with the microscope, between lateral displacements of the needles and the angular motion whose measurement was required. To get over this error it was necessary to read both ends of the pointer ; but this was hard to do.

LVI. *On Huygens's Gearing in illustration of the Induction of Electric Currents.* By LORD RAYLEIGH, Sec. R.S., Professor of Natural Philosophy in the Royal Institution*.

As a mechanical model of the electric machinery at work in the induction of currents Maxwell employed differential gearing ; and an apparatus on this principle, designed by him, is in use at the Cavendish Laboratory. Wishing to show something similar in a recent course of lectures, and not having differential gearing at my disposal, I designed more than one combination of pulleys, the action of which should be analogous to that of electric currents. These eventually resolved themselves into Huygens's gearing, invented, I

* Read May 16, 1890.