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Rev. Banen Powell M.A. F.R.S.

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XXXI. *An Abstract of the essential Principles of M. Cauchy's View of the Undulatory Theory, leading to an Explanation of the Dispersion of Light ; with Remarks. By the Rev. BADEN POWELL, M.A., F.R.S., Savilian Professor of Geometry, Oxford.*

[Continued from p. 113.]

IN the motions expressed by equation (40.), we may observe that the displacements and velocities depend on the sole variables ϱ and t ; and at the end of the time t , therefore, they are the same for all molecules situated at the same distance ϱ from the plane (16.) to which it is perpendicular.

We have thus far obtained expressions for $\xi \eta \zeta$, the resolved parts of the actual displacement of a molecule m in the directions of three rectangular axes in terms of $s' s'' s'''$, which represent three distinct absolute motions or displacements in the directions of three lines at right angles in space, determined by the circumstance of their coinciding with the axes of a given ellipsoid, and having determinate inclinations to a given plane dependent on the values of the arbitrary quantities which enter into the expressions. We have also general expressions for the velocities in those directions ; and in general some of the molecules may take each one of the three motions thus defined.

Now, if at the commencement of the motion the displacement of all the molecules take place in directions parallel to one of the three axes of the ellipsoid just referred to, and the whole velocities are consequently to be estimated in those lines, then the initial values, or the functions $\varpi(\varrho) \Pi(\varrho)$ expressed by equations (36.) and (37.), will vanish for two of the values of s . And consequently for any time t , the displacement s determined by equation (40.) will also vanish for the same two values of s : or, in other words, two of the displacements of the molecules will likewise always vanish, or the whole motion will continue always parallel to the same axis of the ellipsoid. We will take s as that one value which does not vanish. Except so far as the remark above made extends, viz. that the motions are the same at the same time for all molecules situated at the same distance from the plane (16.), the expression above given for the value s (40.) is not of such a nature that we can *directly* infer from it the actual conditions of the sort of displacement which a molecule undergoes, or the consequences which result ; but we may arrive at some conclusions of this kind if we can suppose the

functions to be subject to a particular condition, viz. that we may have a function $\varpi'(\varrho)$, such as to give

$$\Pi(\varrho) = \Omega \varpi'(\varrho); \quad (42.)$$

in which case it will be found that the expression (40.) will be reducible to

$$s = \varpi(\varrho + \Omega t). \quad (43.)$$

Now, from this form it follows that if ϱ and t receive the respective increments $\Delta\varrho$ and Δt , the value of s will remain the same, if we have

$$\Delta\varrho = -\Omega \Delta t; \quad (44.)$$

that is, the displacement s will be the same for a molecule situated at the end of the time t , at the distance ϱ , from the plane (16.), and for a molecule situated at the end of the time $t + \Delta t$ at the distance $\varrho + \Delta\varrho$.

The motion, then, of a molecule m is immediately transmitted to other molecules situated on the side on which the values of ϱ are *negative*; and the velocity with which the motion is propagated in the direction perpendicular to the plane (16.), which is expressed by the value of $\frac{\Delta\varrho}{\Delta t}$ given by equation (19.), will be exactly equal to the *positive* constant Ω .

Again, it is evident, from the form of the functions (36.) (37.), that they have the same *recurring values* when we suppose ϱ to increase by $\frac{2\pi}{k}$, and consequently the function (43.)

will do the same when ϱ is thus increased, and t by $\frac{2\pi}{k\Omega}$.

Let us assume

$$l = \frac{2\pi}{k} \quad (45.), \quad \text{and} \quad T = \frac{2\pi}{k\Omega} \quad (46.)$$

If now, at the end of the time t , we divide the space into an indefinite number of laminæ by parallel planes corresponding to the values of ϱ which reproduce the periodical equal values

of s or of $\frac{ds}{dt}$, then it will evidently represent the thickness

of each lamina, while T represents the time of the isochronous oscillations performed successively by a molecule. We will call these laminæ "plane waves," and we will suppose their thicknesses divided into two equal parts by that one of the parallel planes whose equation is

$$ax + by + cz = \varrho = -\Omega t. \quad (47.)$$

Then for the points through which these planes pass, we shall have constantly

$$s = \varpi(0), \quad \text{and} \quad \frac{ds}{dt} = \Omega \varpi'(0); \quad (48.)$$

or, what is the same thing, from (36.) (37.),

$$s = d_0 A + e_0 B + f_0 C \quad \text{and} \quad \frac{ds}{dt} = k \Omega (g_0 A + h_0 B + i_0 C) \quad (49.)$$

And for the planes bounding the waves successively,

$$s = \varpi\left(\frac{l}{2}\right) \quad \frac{ds}{dt} = k \Omega \varpi'\left(\frac{l}{2}\right); \quad (50.)$$

or, what is the same thing,

$$s = -[d_0 A + e_0 B + f_0 C]$$

$$\text{and} \quad \frac{ds}{dt} = -k \Omega (g_0 A + h_0 B + i_0 C) \quad (51.)$$

Further, the velocity of the propagation of a plane wave, or, in other words, the velocity of the displacement of the plane (47.) measured perpendicular to it, will be *constant* by virtue of the formula (47.), and represented by Ω .

If we suppose the functions such as to fulfill the same conditions as those of (42.) with only the difference of the sign, or

$$\Pi(\varrho) = -\Omega \varpi'(\varrho), \quad (52.)$$

the same considerations readily show that we should have

$$s = \varpi(g - \Omega t), \quad (53.)$$

and by consequence, in the same way as before,

$$\Delta \rho = \Omega \Delta t. \quad (54.)$$

The inference, then, will here be that the motion of m is immediately transmitted to molecules on the *positive* side, the velocity being still the positive constant Ω .

It may also be observed in either case, that the formula which determines s in functions of k for a given direction of the plane (16.), will also determine T or Ω in functions of l .

If, however, the functions $\Pi(\varrho)$ and $\varpi(\varrho)$ be such that the condition (42.) is not fulfilled either with the positive or negative sign, then we cannot proceed to determine the value of s by the conditions involved in the former investigation; that is, it will follow that the formula (39.), or the three similar formulas involving the three values of s , will not enable us to determine the nature and conditions of the three displacements in directions parallel to the axes of the ellipsoid. But we may consider the value of s as representing a motion produced by the composition of six motions (three on each side of the given plane), each corresponding to that represented by the equations (43.) and (53.), according to their signs.

The plane waves corresponding to each of these six mo-

tions will propagate themselves in space with velocities equal, two and two, but proceeding in opposite directions, and represented by Ω' , Ω'' , Ω''' .

We have already observed that from the form of the functions (36. 37.), s_0 and s_1 have recurring values when ϱ is increased by $\frac{2\pi}{k}$; and the similar remark made with respect to the function (43.), it will also be seen, is not restricted to that particular case, but applies equally to the general formula (38.), when t is increased by $\frac{2\pi}{k\Omega}$. Thus, then, adopting the notation of (45. 46.) for the intervals of recurrence in space and in time, we have directly from those expressions

$$\Omega = \frac{l}{T} \quad (55.)$$

Or we find in general that there is always a constant relation between the *length of a wave* and the velocity of its propagation; or, in other words, that the velocity of the propagation is directly as the lengths of the waves, and inversely as the times of the oscillations of the individual molecules of the ætherial fluid; or, what is the same thing, the interval of the time of the recurrence or arrival of two successive waves at the same point.

It must be recollected that, in order to simplify the investigation, we have proceeded solely with reference to a single displacement in the direction of each of the axes; or, more precisely, it has been conducted on the assumption made at first, that we might consider each of the expressions (17.) as reduced to a single term: those expressions, however, really involve the sum of a number of similar terms. In the expressions (23.), therefore, which represent the initial values of ξ , η , ζ and of their differentials, as well as in the equations (33.), the same consideration must be attended to, that is, we must take

$$\xi_0 = \Sigma [d_0 \cos k\varrho + g_0 \sin k\varrho] \quad (56.)$$

&c.

$$\xi_1 = \Sigma [d_1 \cos k\varrho + g_1 \sin k\varrho] \quad (57.)$$

&c.

$$\xi = \Sigma [A' s' + A'' s'' + A''' s'''] \quad (58.)$$

&c.

We shall then have only to introduce the values of s' , s'' , s''' as above found, and the motion of the system may be considered as produced by the combination of many, or even an infinity of similar motions, each the same as those represented by the equations (43.) and (53.).

Finally, in order to complete the analytical view of the sub-

ject, M. Cauchy proceeds to show how the sum of terms indicated by Σ may be changed into definite integrals. In this investigation it will not be necessary to our purpose to follow him, but we shall proceed to some remarks on the expressions above deduced and their physical applications.

[To be continued.]

XXXII. *Æconomical Means of procuring pure the Salts of Manganese, and of analysing the Minerals which contain Manganese and Iron, &c.* By THOMAS EVERITT, Esq., Professor of Chemistry to the Medico-Botanical Society, &c.*

HAVING had occasion for some pounds of pure salts of manganese for experiments on dyeing, my attention was turned to consider the convenience and æconomy of those processes prescribed in our systematic works. The process of Faraday by hydrochlorate of ammonia, is easy of execution, and perfect as to the results, but expensive; that of Turner, "by mixing the oxide left after procuring oxygen gas by heat with one sixth of charcoal, and exposing to a white heat for half an hour in a covered crucible, dissolving in hydrochloric acid, evaporating to dryness, and keeping the mass in perfect fusion for a quarter of an hour, &c." yields also good results, but is tedious in the execution, and expensive, if time and trouble be considered; moreover, by the first ignition, although we subsequently save a little hydrochloric acid (none being lost as chlorine) by reducing the manganese to protoxide, we also at the same time render the iron in such a state that on dissolving in hydrochloric acid, we have a protoxide, which is more difficult to get quit of by the second ignition than it would have been as a peroxide.

As I possessed a large quantity of hydrochlorate of manganese and iron, the accumulated solutions from preparing chlorine by hydrochloric acid and ordinary oxide of manganese, I was induced to make a variety of trials on this liquid with the view of separating the iron from the manganese; the results of which trials being entirely satisfactory, I venture to request a place for a short account of them in the London and Edinburgh Philosophical Magazine.

Method, No. I.—*Depending on the circumstance that when a solution of hydrochlorate of iron, strictly peroxide (which is always the case in the above liquid), is evaporated to dryness, and the heat afterwards slightly elevated, a small portion sub-*

* Communicated by the Author.