



XXXIV. Further notes on the formulæ of the electromagnet and the equations of the dynamo

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it the general. But this does not stand examination; the work has to be done, whether we derive the special results from the general, or conversely.

In the solid wire case

$$\begin{aligned} \text{or} \quad C_r &= \frac{r}{a_1} \frac{J_1(s_1 r)}{J_1(s_1 a_1)} C, \\ C_r &= \frac{r^2}{a_1^2} \left\{ 1 + \frac{1}{2}(\pi \mu_1 k_1 p + \frac{1}{4} m^2)(r^2 - a_1^2) \right. \\ &\quad + \frac{1}{\frac{3}{8} 1^2 2^2} (\pi \mu_1 k_1 p + \frac{1}{4} m^2)^2 (r^2 - a_1^2)(r^2 - 2a_1^2) \\ &\quad \left. + \frac{1}{\frac{1}{4} 1^2 2^2 3^2} (\pi \mu_1 k_1 p + \frac{1}{4} m^2)^3 (r^2 - a_1^2)(r^4 - 5r^2 a_1^2 + 7a_1^4) \right\} + \dots \} C. \end{aligned}$$

Or, use the M and N functions of Part I., equations (42). For we have

$$J_0(s_1 r) = (M + iN)(s_1 r^{\frac{1}{2}}),$$

where $s_1 r^{\frac{1}{2}}$ takes the place of the y in those equations. M contains the even and N the odd powers of $(p + m^2/4\pi\mu_1 k_1)$.

We have also

$$\Gamma_r = J_0(s_1 r) \Gamma_0 = \frac{s_1}{2\pi a_1} \frac{J_0(s_1 r)}{J_1(s_1 a_1)} C,$$

Γ_0 being Γ at $r=0$; and, since by the first of these

$$\Gamma_{a_1} = J_0(s_1 a_1) \Gamma_0$$

connects the boundary and axial current-densities, we see that the ratio of their amplitudes in the S.H. case is

$$(M^2 + N^2)^{\frac{1}{2}},$$

using the $r=a_1$ expressions, with $m=0$.

I hope to be able to conclude this paper in a third part.

XXXIV. *Further Notes on the Formulæ of the Electromagnet and the Equations of the Dynamo.* By PROFESSOR SILVANUS P. THOMPSON, D.Sc., B.A.*

1. The Lamont-Frölich Formula.

DR. O. FRÖLICH has done me the honour of replying† to a certain point in my former communication to the Physical Society, "On the Law of the Electromagnet and the Law of the Dynamo"‡. In that communication I pointed

* Communicated by the Physical Society: read June 26, 1886.

† *Elektrotechnische Zeitschrift*, vii. p. 163, May 1886.

‡ *Phil. Mag.* vol. xxi. p. 1, January 1886.

out that Lamont had in 1867 published a rational theory of the electromagnet, based upon the assumption that the permeability of the iron was at every stage of the magnetization proportional to the deficit of saturation, leading him to an exponential expression,

$$m = M(1 - e^{-kx}),$$

where m is the magnetism present at any stage, M its maximum value, k the ratio of the permeability to the deficit of saturation, and x the magnetizing force proportional (approximately) to the number of ampere-turns of the magnetizing current. This formula more correctly expressed the facts than either of the commoner formulæ of Lenz and Jacobi and of Müller.

I further pointed out that Lamont had himself* given, as a sufficient approximation to the formula, the simpler expression,

$$m = \frac{aMx}{M + ax},$$

which formula is mathematically identical with that now commonly attributed to Dr. Frölich. For, writing $a = kM$, we get at once

$$m = M \frac{kx}{1 + kx},$$

which is the formula claimed by Frölich.

Lamont having developed his exponential expression in a series of ascending powers of kx , I did the same for the simpler formula for the purpose of comparison, and showed that, *neglecting the fourth and higher terms* of each series, the expansions are very nearly equal for all values of kx except for very large ones, and are identical for the value $kx = \frac{2}{5}$. Dr. Frölich, overlooking the words I have above italicized, commits the mistake of supposing that I had said that Lamont's exponential expression is identical in value with the simpler formula when $kx = \frac{2}{5}$. I have said nothing of the kind.

Further, when Dr. Frölich says, "Hiernach ist die Aussicht vorhanden dass nicht die Lamontsche sondern die von mir benutzte Formel die wahre Gesetz der Elektromagnete enthält," he is forgetting that the formula used by him is also Lamont's. He has proved, in his most recent communication, that the differences between the calculated and the observed values are about half as great when calculated by the simpler formula. The second and simpler formula suggested by Lamont appears therefore to be better than the first and more

* Lamont, *Magnetismus*, p. 41.

complex formula which he suggested. It has been recently shown by Mr. Bosanquet* that if we take as expressive of the permeability, not the instantaneous value dm/dx , but the integral value m/x , and treat this on Lamont's plan as proportional to the deficit of saturation,

$$\frac{m}{x} = k(M - m),$$

we deduce at once the formula in question. Dr. Frölich's researches upon the dynamo have given us the most complete and perfect proofs of the adequacy of the formula to represent the facts of the electromagnet as it is used in practice. My former communication was indeed mainly written to point out the extreme value and interest of Dr. Frölich's work from this point of view.

2. Frölich's simplified Formula of the Electromagnet.

Since my former communication to the Physical Society was made, a further work on the theory of the dynamo by Dr. Frölich has appeared†. In this work he carries the simplification of the formulæ of magnet and of dynamo one stage further by introducing considerations somewhat closely connected with those which entered into my own work of 1883-4.

The form adopted by me in 1883 for the Lamont-Frölich formula was

$$H = \frac{G\kappa Si}{1 + \sigma Si};$$

where H is the resulting average intensity of the magnetic field (in which the armature rotates), κ the initial value of the magnetic permeability, G a coefficient depending upon such purely geometrical quantities as the form and size of core, pole-pieces, and coils, S the number of windings of the magnetizing coil, and σ the "saturation-coefficient." This can be transformed at once to Frölich's form by writing $M = G\kappa/\sigma$; $k = \sigma$; $x = Si$. In 1884 I pointed out‡ the nature of this saturation-constant and its importance in the resulting equations of the dynamo. It is the reciprocal of that number of ampere-turns which, to mark its importance, I ventured to term "diacritical," namely *that number of ampere-turns which will reduce the instantaneous value of the magnetic permeability to half its initial value*, or which, in the formula used, will give

* 'Electrician,' vol. xvi. p. 247, February 1886.

† *Die dynamoelektrische Maschine. Eine physikalische Beschreibung für den technischen Gebrauch*, von Dr. O. Frölich (Berlin, 1886).

‡ 'Dynamoelectric Machinery,' first edition, p. 221; also Report Brit. Assoc., Montreal Meeting, 1884.

to the magnet exactly *half its maximum magnetism*. I further pointed out in my lectures on the dynamo that year, that, if the number of windings of the coil S is given, there will be a "diacritical" current, namely a particular value of current which will exactly half-saturate the magnet. Dr. Frölich has independently made use of this conception, and has applied it to the formula of the electromagnet. The argument is his, but I retain the notation I have used.

Writing $(Si)'$ for the diacritical number of ampere-turns, we have (as I showed in 1884) $(Si)' = 1/\sigma$.

Taking the expression

$$H = \frac{G\kappa Si}{1 + \sigma Si} = \frac{G\kappa}{\sigma} \cdot \frac{Si}{\frac{1}{\sigma} + Si},$$

and writing

$$Y = \frac{G\kappa}{\sigma},$$

we have

$$H = Y \frac{Si}{Si + (Si)'}$$

where Y is obviously the limiting maximum value of H when the excitement is infinitely great. If S is given, then i' is the diacritical current, and the expression becomes

$$H = Y \frac{i}{i + i'},$$

which is true for every electromagnet excited by a single current. Two observations made on any electromagnet will determine the two constants Y and i' . Further, if r be the resistance of the magnetizing coil, since $ir = e$ (the potential requisite to send the current through the coil), we may obviously write the equation

$$H = Y \frac{e}{e + e'};$$

where e' is the diacritical difference of potential, namely that difference of potential which, applied to the coil of resistance r and of S convolutions, will half-saturate the core.

The extreme convenience of this form of the law of the electromagnet must be at once apparent, since it enables the equation of a given magnet to be instantly adapted to the case of any given current or potential, and is equally applicable to express either the intensity of the field or the magnetic moment of the magnet. To put the matter in a more general way, let ψ represent current, or potential, or ampere-turns, and let ψ'

be the diacritical value of the same for the given magnet; let ϕ be the intensity of the field, or the strength of the pole, or the magnetic moment, or the integral of the magnetic induction, and Φ its maximum value; then

$$\phi = \Phi \frac{\psi}{\psi + \psi'}$$

This being the general equation of the electromagnet, it remains to be shown how excessively simple become the equations of the various kinds of dynamo.

3. *Equations of the Series-wound Dynamo.*

If A is the "equivalent area" of the coils of the armature, and H the average strength of the field in which it turns, the number of lines of force cut in each quadrant is AH ; hence the average electromotive force at the speed n is

$$E = 4nAH. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But

$$H = Y \frac{i}{i + i'}$$

and, writing $B = 4AY$, and remembering that, if ΣR be the sum of the resistances in the circuit, $E/\Sigma R = i$ (by Ohm's law), we get

$$i = \frac{nB}{\Sigma R} - i'.$$

But Y being the maximum value of H , it is obvious that nB is the maximum value that E could possibly have (at that speed) even if the magnets were separately excited to saturation. Hence $nB/\Sigma R$ is the maximum value that i could have if the magnets were thus separately saturated and the armature, driven at speed n , were in a circuit the total resistance of which was equal to ΣR . Adopting Frölich's notation here, we will write as \bar{i} this current; and as it is important to distinguish the current generated under such conditions, I propose to call it the "maximal" current*. The equation of the series dynamo now becomes

$$i = \bar{i} - i'; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or, multiplying each term by ΣR ,

$$E = \bar{E} - E', \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* The *maximal* current must not be confused with the *maximum* current. The latter would be obtained by rotating the armature in the saturated field at a very high speed in a circuit of resistance so small that the current did just not fuse the conductors. The *maximal* current is that obtained at speed n in circuit of resistance ΣR when the magnets are separately excited to saturation.

where \bar{E} is the "maximal" value of E at that speed with saturated magnets. And, again, writing e for the difference of potentials at the terminals of the machine, since e , multiplied by the external resistance $R=i$ under all circumstances, we have

$$e = \bar{e} - e'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It appears, then, that in the case of the series-wound dynamo, each of the single electrical quantities is equal to the difference between the "maximal" value which that quantity could have at that speed if the field-magnet were separately saturated and the "diacritical" value of the same quantity. This important result was announced by Frölich* in 1885.

It may be remarked that, since we may write

$$H = Y \frac{E}{E + E'},$$

we may deduce from (1) the result

$$E = nB - E',$$

and from this derive equation (3).

It may be noted, in passing, that B is the electromotive forces that would be generated in the armature at speed = 1 if the field-magnets were separately excited to absolute saturation. It is the maximal value of E at unit speed.

4. *Expressions for the "Dead Turns."*

It is known that in every dynamo the current (with a given resistance) is not proportional to the speed, but is proportional to the speed less a certain number of revolutions per second. This latter number is known familiarly as the "dead turns." It is also known that (with given resistance) there is a certain speed below which the dynamo does not excite itself. This least speed of excitement (with given resistance) is the same as the "dead turns." It is called by some the "critical" speed; though that name is preferably reserved for the speed that is critical for self-regulation, and which is (unlike the least speed of excitation) independent of the resistance.

We can find an expression for the dead turns as follows:—

Taking the expression for the current,

$$i = \frac{nB}{\Sigma R} - i',$$

equate it to zero; the current of the machine (series-wound) being *nil* when n is reduced sufficiently to n' (= the dead

* *Elektrotechnische Zeitschrift*, vi. p. 133, March 1885.

$$i' = \frac{n'B}{\Sigma R},$$

and

$$n' = \frac{i' \Sigma R}{B} \dots \dots \dots (5)$$

It appears, then, that the dead turns are proportional to the diacritical current and to the resistance; and inversely proportional to A and Y , the factors of B . It will be noted that $i' \Sigma R = E'$, the diacritical electromotive force.

Again, we have found

$$E = nB - E';$$

and as $E' = n'B$, it follows that

$$E = (n - n')B \dots \dots \dots (6)$$

This result is interesting in itself, and might have been used as a starting-point for the equations of the dynamo, inasmuch as it is readily found by experiment as a fundamental relation between speed and electromotive force. Following the analogy and the nomenclature adopted, we may regard n' (the dead turns) as the diacritical speed. In other words, it is that speed at which, with magnets separately excited to saturation, the induced electromotive force will be diacritical, and will, with the given resistances ΣR , give the diacritical current. This equation (6) gives by far the best method of determining the important constant B . Two experiments to observe a pair of values of E and n will suffice to determine both n' and B .

5. *Equations of the Shunt Dynamo.*

Here we use r_s and i_s for the resistance and current of the shunt-coil, and r_a and i_a for those of the armature. We may then calculate the potential at terminals as follows. Writing \mathfrak{R} for the resistance of the whole system of machine and its circuits, as measured from brush to brush*,

$$E = e + r_a i_a = e \frac{r_a}{\mathfrak{R}} = 4nAH,$$

$$H = Y \frac{e}{e + e'},$$

$$e \frac{r_a}{\mathfrak{R}} = nB \frac{e}{e + e'};$$

whence

$$e = \frac{nB\mathfrak{R}}{r_a} - e' \dots \dots \dots (7)$$

* See my 'Dynamoelectric Machinery,' second edition, p. 298.

armature part and a field-magnet part. This does not quite correspond to the case. The "diacritical" term appertains to field-magnet primarily; but since the number of ampere-turns of current that will produce a half-saturated magnetic field depends also on the quantity and quality of the iron in the machine, it is impossible to regard it as independent of the iron masses of the armature. The "maximal" term is proportional both to A , the total effective area of the armature-coils, and to Y , the maximum value of the magnetic field. The "diacritical" term for currents is lowered by increase in the number of magnetizing coils, and for potentials is raised by increase in the resistance of the magnetizing coils. The "maximal" term is not altered by altering the magnetizing coils, but increases with an increase in the number of coils of the armature, and, for currents, decreases with increasing resistance. If the iron parts of a dynamo be given of a certain form, size, and quality, then it may in general be said that ψ' depends only on the windings (and, for currents, on the resistance) of the field-magnet, and $\bar{\psi}$ depends only on the windings of the armature (and, for currents, on its resistance) and not upon the windings of the field-magnet, or on their resistance except (and this only for currents) so far as their resistance contributes to the resistance of the whole circuit through which the current generated in the armature flows.

7. Conditions of Self-regulation for Compound-wound Dynamos.

In this case we write the formula of the electromagnet in terms of the ampere-turns; $S i_a$ for the excitation due to the coils in the armature part of the circuit, $Z i_s$ for that due to the shunt, and ϕ' as the diacritical number of ampere-turns. Then, writing

$$E = 4nAH;$$

$$e = E - (r_a + r_m)i_a;$$

$$H = Y \frac{Z i_s + S i_a}{Z i_s + S i_a + \phi'} = Y \frac{Z i_s + S i_a}{\phi}, \text{ by equation (11) ante;}$$

$$B = 4AY;$$

we have

$$e\phi' = i_s Z(nB - e) + i_a \{S(nB - e) - (r_a + r_m)\bar{\phi}\}. \quad (12)$$

The condition sought is to be such as to make e constant. Now ϕ' is a constant, so is i_s if e is, and n may be made constant at any value we please. Hence, as i_a is a variable, the condition of constancy can only be attained by giving the

dynamo such a speed n_1 that the coefficient of i_a shall be zero, or that

$$S(n_1 B - e) = (r_a + r_m) \bar{\phi}. \quad . \quad . \quad . \quad . \quad (13)$$

But $\bar{\phi}$, the maximal number of ampere-turns, is not itself a constant, since it contains as one of the three terms in its sum the quantity $S i_a$. Hence we gather that absolute self-regulation is physically impossible; and it approaches to perfection as $Z i_s + \phi'$ are great as compared with $S i_a$. In other words, there must be so much iron in the machine that the diacritical excitement is very great, and it must have a small armature-resistance; otherwise S (and $S i_a$) cannot be small as compared with Z (and $Z i_s$). This is known already to electric engineers. Assuming that the dynamo is well designed in these respects, $\bar{\phi}$ will be very nearly constant, and the equation of condition may be accepted as adequately true. This leaves equation (12) in the form

$$e \phi' = i_s Z (n_1 B - e),$$

$$\frac{e \phi'}{i_s Z} = n_1 B - e,$$

$$\frac{r}{Z} \phi' = n_1 B - e.$$

Putting in this value of $n_1 B - e$ into equation (13), we have

$$\frac{r_s}{Z} \phi' = \frac{r_a + r_m}{S} \bar{\phi}. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Now ϕ' may be written either as $S i_a'$ or as $Z i_s'$, and $\bar{\phi}$ may be written either as $S \bar{i}_a$ or as $Z \bar{i}_s$. Choosing the second form in the first case and the first form in the second case, we may obtain

$$r_s i_s' = (r_a + r_m) \bar{i}_a;$$

or, finally,

$$e_s' = \bar{e}_{a+m}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or, the diacritical value of the potential at terminals for the shunt-wound part of the circuit must be equal to the maximal value of the potential at terminals for the armature and series-wound part. The equation (14) also gives

$$\frac{Z}{S} = \frac{r_s}{r_a + r_m} \cdot \frac{\phi'}{\bar{\phi}}, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which is more correct than the formula usually given hitherto for the ratio of the shunt- and series-windings, and which assumes absence of saturation terms.

The simplicity of these results, no less than that of the processes by which they are derived, lends additional value to the new formula of Dr. Frölich, whose work deserves to be more widely known and recognized than it now is.

City and Guilds of London Technical College,
Finsbury, June 1886.

XXXV. *Electromagnets*.—V. *The Law of similar Electromagnets. Saturation, &c.* By R. H. M. BOSANQUET, *St. John's College, Oxford.*

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

IN previous papers on Electromagnets I have (1)* indicated the general nature of a formula for the moment and induction in electromagnets, and in subsequent papers † given the details of the complete examination of the permeabilities of many specimens of iron and steel, together with an attempt at a molecular formula for these permeabilities worked out in some detail. In the present communication I propose to give as shortly as possible an abstract of the results of a great number of experiments having special reference to the magnetic resistance of cylindrical bars of length equal to twenty times their diameter, with and without pole-pieces.

The datum in question (magnetic resistance) is that needed to define the magnetism under given electromagnetic excitation.

The experiments cover the whole region from small magnetic inductions up to saturation, or what would be commonly called so.

The experiments have been made on bars of different sizes of the proportions in question, so as to furnish for the first time an experimental examination of the law of the magnetism of similar solids. It appears to be of great interest to ascertain how far this law can be depended upon in practice.

The result is that, while in the main the law is conformed to, the irregularity shown by different specimens, especially in the extreme regions of small inductions and saturation, is very great. General deductions therefore, such as have been recently published, depending on the behaviour of single

* Phil. Mag. xvii. p. 531.

† Ibid. xix. pp. 73, 333; xx. p. 318.