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A Mathematical Study of Tripolar Intuitionistic Fuzzy Graphs

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Abstract
Graph-based models provide a fundamental mathematical framework for representing relational systems in terms of vertices and their connections. Fuzzy sets extend classical set theory by assigning each element a membership degree in the interval [0,1], thereby allowing partial membership rather than restricting membership to binary values. An intuitionistic fuzzy set assigns both membership and non-membership degrees to each element, subject to the condition that their sum does not exceed one, thus explicitly representing the degree of hesitation. Similarly, an intuitionistic fuzzy graph assigns membership and non-membership degrees to vertices and edges, enabling uncertain relationships to be modeled while maintaining compatibility between vertex and edge information. Motivated by these frameworks, this paper investigates Tripolar Intuitionistic Fuzzy Sets and Tripolar Intuitionistic Fuzzy Graphs, which extend intuitionistic fuzzy models by incorporating a tripolar representation of uncertain information.

Keywords: Tripolar Fuzzy Graph, Intuitionistic Fuzzy Set, Tripolar Intuitionistic Fuzzy Set, Tripolar Intuitionistic Fuzzy Graph

1. Introduction

1.1. Intuitionistic Fuzzy Sets

A fuzzy set assigns to each element a membership degree in the interval [0,1], thereby allowing partial membership rather than restricting membership to the binary values used in classical set theory [1]. Since their introduction, fuzzy sets have provided a fundamental framework for representing vagueness and uncertainty, and numerous extensions have subsequently been developed. Representative examples include Neutrosophic Sets [2–4], HyperFuzzy Sets [57,58], Plithogenic Sets [5,6], and other generalized uncertainty models.

Among these extensions, a Bipolar Fuzzy Set represents information from two opposite viewpoints, typically through positive and negative membership components [7–9]. Related frameworks include Bipolar Neutrosophic Sets [9,10],

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m-Polar Fuzzy Sets [11,12], and various other multipolar uncertainty models. A Tripolar Fuzzy Set may be regarded as a particular m-polar structure with three poles [13–16]. By incorporating three distinct membership perspectives, it extends the bipolar representation and provides a richer framework for situations in which information cannot be adequately described using only two opposing poles. It should nevertheless be noted that several formulations of Tripolar Fuzzy Sets have been proposed in the literature, depending on the interpretation and mathematical treatment of the three components.

Fuzzy Sets have also been extended to graph-based and higher-order relational structures, leading to concepts such as Fuzzy Graphs [18,19], Fuzzy HyperGraphs [20-22], and Fuzzy SuperHyperGraphs [23,24]. Several related graph frameworks have also been studied, including Bipolar Fuzzy Graphs [25,26], Bipolar Neutrosophic Graphs [27-29], Neutrosophic Graphs [30-32], Interval-Valued Neutrosophic Graphs [33,34], Plithogenic Graphs [35,36], Neutrosophic HyperGraphs [37,38], and Neutrosophic SuperHyperGraphs [39,40]. These extensions provide mathematical tools for representing uncertain, vague, indeterminate, or multipolar relationships in increasingly complex network structures.

Another important extension of the Fuzzy Set is the Intuitionistic Fuzzy Set [41,42], in which membership and non-membership degrees are considered simultaneously. Related concepts include Bipolar Intuitionistic Fuzzy Sets [43,44] and Intuitionistic Neutrosophic Sets [45,46]. The intuitionistic fuzzy framework has also been incorporated into graph-theoretic structures, giving rise to Intuitionistic Fuzzy Graphs [47,48], Intuitionistic Fuzzy HyperGraphs [49,50], and Intuitionistic Fuzzy SuperHyperGraphs [51,52]. These developments demonstrate that intuitionistic fuzzy information can be naturally combined with graph, hypergraph, and higher-order network structures to represent relational systems involving both positive membership and explicit non-membership information.

1.2. Our Contributions

Accordingly, the study of Intuitionistic Fuzzy Sets and their related concepts constitutes an important and necessary area of research. Although various extensions of these concepts have attracted increasing attention, some aspects remain insufficiently explored. Motivated by this observation, this paper investigates Tripolar Intuitionistic Fuzzy Sets and Tripolar Intuitionistic Fuzzy Graphs. A Tripolar Intuitionistic Fuzzy Set assigns membership and non-membership degrees across three poles, thereby simultaneously representing hesitation and multi-perspective uncertainty. A Tripolar Intuitionistic Fuzzy Graph assigns tripolar membership and non-membership degrees to vertices and edges, providing a framework for modeling multi-perspective uncertain relational structures.

2. Preliminaries

This section introduces the fundamental concepts and notation required for the subsequent discussion. Unless otherwise specified, every graph considered throughout this paper is assumed to be finite.

2.1. Tripolar Fuzzy Set and Graph

Tripolar fuzzy modeling characterizes each object through three coordinated degrees corresponding to positive, neutral, and counter-property viewpoints [13-16]. This three-component description allows uncertainty to be expressed from perspectives that are not captured by a single fuzzy membership value. In the graph setting, the same tripolar information is attached to vertices and edges, together with incidence constraints that relate every edge to its endpoints.

Definition 1 (Tripolar Fuzzy Set) (cf. [13-16]). Let X be a nonempty set. Under the Rao-type convention, a Tripolar Fuzzy Set (TFS) A over X is specified by the structure

$$A = \{(x, \rho_A(x), \nu_A(x), \delta_A(x)) : x \in X\},$$

where the three component mappings are given by

$$\rho_A, \nu_A : X \to [0,1], \qquad \delta_A : X \to [-1,0],$$

and the following admissibility requirement is imposed for each $x \in X$.

$$0 \le \rho_A(x) + \nu_A(x) \le 1.$$

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In this representation, $\rho_A(x)$ measures the positive membership component, $\nu_A(x)$ records the neutral, indeterminate, or irrelevant component, whereas $\delta_A(x)$ quantifies membership in the corresponding counter-property. Consequently, every element $x \in X$ is encoded by the tripolar triple

$$A(x) = \big(\rho_A(x), \nu_A(x), \delta_A(x)\big).$$

Given two admissible tripolar triples

$$a = \big(a_p, a_0, a_c\big), \qquad b = \big(b_p, b_0, b_c\big),$$

we employ the following natural partial order on tripolar information:

$$a \preccurlyeq_T b \quad \Leftrightarrow \quad a_p \leq b_p, \qquad a_0 \geq b_0, \qquad a_c \geq b_c.$$

The meet induced by this order is expressed as

$$a \wedge_T b = \big(\min\{a_p, b_p\}, \max\{a_0, b_0\}, \max\{a_c, b_c\}\big).$$

Definition 2 (Fuzzy Graph) . Let $G^* = (V,E)$ be a finite simple graph. A Fuzzy Graph supported on G^* is described by a pair $G_F = (\sigma, \mu)$, where

$\sigma : V \to [0,1]$,

$\mu : E \to [0,1]$,

and, for every edge $\{u,v\} \in E$,

$\mu(\{u,v\}) \leq \min\{\sigma(u), \sigma(v)\}$.

For a vertex v, $\sigma(v)$ specifies its membership grade, while $\mu(\{u,v\})$ gives the membership grade associated with the edge $\{u,v\}$. The incidence restriction therefore ensures that an edge can never receive a membership value larger than that of either endpoint.

Definition 3 (Tripolar Fuzzy Graph). Consider

$$G = (V, E), \qquad E \subseteq \binom{V}{2},$$

as a finite simple graph. Suppose that

$$A_V = (\rho_V, \nu_V, \delta_V)$$

is a Tripolar Fuzzy Set defined over V, and suppose that

$$A_E = (\rho_E, \nu_E, \delta_E)$$

is a Tripolar Fuzzy Set defined over E. Accordingly,

$$\rho_V, \nu_V : V \to [0,1], \qquad \delta_V : V \to [-1,0],$$

$$\rho_E, \nu_E : E \to [0,1], \qquad \delta_E : E \to [-1,0],$$

where

$$0 \leq \rho_V(v) + \nu_V(v) \leq 1 \quad (v \in V),$$

and

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$$0 \leq \rho_E(e) + \nu_E(e) \leq 1 \quad (e \in E).$$

We call the resulting structure

$$\mathcal{G} = (G, A_V, A_E)$$

a Tripolar Fuzzy Graph (TFG) whenever, for each edge $e = \{u, v\} \in E$, the following compatibility condition holds:

$$A_E(e) \preccurlyeq_T A_V(u) \wedge_T A_V(v).$$

In componentwise form, for every $\{u, v\} \in E$, this requirement is equivalent to

$$\rho_E(\{u, v\}) \leq \min\{\rho_V(u), \rho_V(v)\},$$

$$\nu_E(\{u, v\}) \geq \max\{\nu_V(u), \nu_V(v)\},$$

together with

$$\delta_E(\{u, v\}) \geq \max\{\delta_V(u), \delta_V(v)\}.$$

The positive coordinate satisfies the standard fuzzy-graph incidence bound. By contrast, the neutral and counter-property coordinates are ordered in the reverse direction under the tripolar order introduced above; hence their corresponding inequalities are reversed. These three componentwise restrictions ensure that the tripolar value assigned to every edge remains consistent with the values attached to both of its incident vertices.

2.2. Intuitionistic Fuzzy Set

An intuitionistic fuzzy set extends a fuzzy set by assigning to each element both a membership degree and a non-membership degree. The two degrees are constrained so that their sum does not exceed one, and the remaining part represents hesitation or indeterminacy.

Definition 4 (Intuitionistic Fuzzy Set). Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A on X is a structure

$$A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle : x \in X\},$$

where

$$\mu_A, \lambda_A : X \to [0,1]$$

satisfy, for every $x \in X$,

$$0 \leq \mu_A(x) + \lambda_A(x) \leq 1.$$

Here, $\mu_A(x)$ and $\lambda_A(x)$ denote the membership and non-membership degrees, respectively. The associated hesitation degree is

$$\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x) \in [0,1].$$

Several tripolar intuitionistic formulations have appeared in the literature [17]. To remain consistent with the tripolar value and the tripolar order used in Definition 1, this paper adopts the following coordinatewise intuitionistic refinement.

Definition 5 (Tripolar Intuitionistic Fuzzy Set). Let X be a nonempty set. A *Tripolar Intuitionistic Fuzzy Set* (TIFS) A on X is a structure

$$A = \{\langle x, \mathbf{m}_A(x), \mathbf{n}_A(x)\rangle : x \in X\},$$

where the tripolar membership and non-membership profiles are

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$$\mathbf{m}_A(x) = \big(\rho_A^m(x), \nu_A^m(x), \delta_A^m(x)\big), \qquad \mathbf{n}_A(x) = \big(\rho_A^n(x), \nu_A^n(x), \delta_A^n(x)\big).$$

For each $s \in \{m, n\}$,

$$\rho_A^s, \nu_A^s : X \to [0,1], \qquad \delta_A^s : X \to [-1,0],$$

and, for every $x \in X$,

$$0 \le \rho_A^s(x) + \nu_A^s(x) \le 1.$$

In addition, the membership and non-membership profiles satisfy the intuitionistic compatibility conditions

$$0 \le \rho_A^m(x) + \rho_A^n(x) \le 1, \qquad 0 \le \nu_A^m(x) + \nu_A^n(x) \le 1,$$

and

$$-1 \le \delta_A^m(x) + \delta_A^n(x) \le 0.$$

Consequently, the three hesitation magnitudes

$$\pi_{A,\rho}(x) = 1 - \rho_A^m(x) - \rho_A^n(x), \qquad \pi_{A,\nu}(x) = 1 - \nu_A^m(x) - \nu_A^n(x),$$

and

$$\pi_{A,\delta}(x) = 1 + \delta_A^m(x) + \delta_A^n(x)$$

all belong to [0,1]. If the non-membership profile is identically zero, the construction reduces to the Tripolar Fuzzy Set of Definition 1. If the neutral and counter-property coordinates are suppressed, the positive coordinate reduces to an ordinary Intuitionistic Fuzzy Set.

3. Results: Tripolar Intuitionistic Fuzzy Graph

A tripolar intuitionistic fuzzy graph assigns membership and non-membership degrees across three poles to vertices and edges, modeling multi-perspective uncertainty.

For tripolar values $a = \big(a_p, a_0, a_c\big)$ and $b = \big(b_p, b_0, b_c\big)$, define the tripolar join corresponding to the order $\preccurlyeq_T$ by

$$a \vee_T b = \big(\max\{a_p, b_p\}, \min\{a_0, b_0\}, \min\{a_c, b_c\}\big).$$

This is the least upper bound of a and b under $\preccurlyeq_T$.

Definition 6 (Tripolar Intuitionistic Fuzzy Graph). Let $G^* = (V, E)$ be a finite simple graph. Let

$$A_V = (\mathbf{m}_V, \mathbf{n}_V)$$

be a Tripolar Intuitionistic Fuzzy Set on V, and let

$$A_E = (\mathbf{m}_E, \mathbf{n}_E)$$

be a Tripolar Intuitionistic Fuzzy Set on E. The structure

$$\mathcal{G}_{\mathrm{TIF}} = (G^*, A_V, A_E)$$

is called a *Tripolar Intuitionistic Fuzzy Graph* (TIFG) if, for every edge $e = \{u, v\} \in E$,

$$\mathbf{m}_E(e) \preccurlyeq_T \mathbf{m}_V(u) \wedge_T \mathbf{m}_V(v), \qquad \mathbf{n}_E(e) \succcurlyeq_T \mathbf{n}_V(u) \vee_T \mathbf{n}_V(v).$$

Equivalently, the tripolar membership profile satisfies

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$$\rho_E^m(e) \le \min\{\rho_V^m(u), \rho_V^m(v)\}, \qquad \nu_E^m(e) \ge \max\{\nu_V^m(u), \nu_V^m(v)\},$$
$$\delta_E^m(e) \ge \max\{\delta_V^m(u), \delta_V^m(v)\},$$

whereas the tripolar non-membership profile satisfies

$$\rho_E^n(e) \ge \max\{\rho_V^n(u), \rho_V^n(v)\}, \qquad \nu_E^n(e) \le \min\{\nu_V^n(u), \nu_V^n(v)\},$$

and

$$\delta_E^n(e) \le \min\{\delta_V^n(u), \delta_V^n(v)\}.$$

Thus, the first condition generalizes the edge-compatibility requirement of a Tripolar Fuzzy Graph to the membership profile, while the second is its order-dual condition for non-membership. Restricting the model to the positive coordinate yields the usual intuitionistic fuzzy graph compatibility inequalities.

Theorem 1 (Well-definedness of the Tripolar Intuitionistic Fuzzy Graph). Let $G^* = (V, E)$ be a finite simple graph with $E \neq \emptyset$, and let $A_V = (\mathbf{m}_V, \mathbf{n}_V)$ be a Tripolar Intuitionistic Fuzzy Set on V in the sense of Definition 5. For every edge $e = \{u, v\} \in E$, define

$$\mathbf{m}_E(e) = \mathbf{m}_V(u) \wedge_T \mathbf{m}_V(v), \qquad \mathbf{n}_E(e) = \mathbf{n}_V(u) \vee_T \mathbf{n}_V(v).$$

Equivalently,

$$\mathbf{m}_E(e) = (\min\{\rho_V^m(u), \rho_V^m(v)\}, \max\{\nu_V^m(u), \nu_V^m(v)\}, \max\{\delta_V^m(u), \delta_V^m(v)\}),$$

and

$$\mathbf{n}_E(e) = (\max\{\rho_V^n(u), \rho_V^n(v)\}, \min\{\nu_V^n(u), \nu_V^n(v)\}, \min\{\delta_V^n(u), \delta_V^n(v)\}).$$

Then $A_E = (\mathbf{m}_E, \mathbf{n}_E)$ is a Tripolar Intuitionistic Fuzzy Set on E, and

$$G_{TIF} = (G^*, A_V, A_E)$$

is a Tripolar Intuitionistic Fuzzy Graph in the sense of Definition 6. Hence, the canonical construction of a Tripolar Intuitionistic Fuzzy Graph from vertex data is well-defined.

Proof. Fix an arbitrary edge $e = \{u, v\} \in E$. First, the construction is independent of the order in which the endpoints are written. Indeed, min and max are symmetric in their arguments, so replacing (u, v) by (v, u) leaves both $\mathbf{m}_E(e)$ and $\mathbf{n}_E(e)$ unchanged. Thus the two profiles are well-defined functions on the set of unordered edges E.

Since all vertex components ρ_V^s and ν_V^s, $s \in \{m, n\}$, take values in $[0,1]$, their minima and maxima also belong to $[0,1]$. Similarly, because δ_V^s takes values in $[-1,0]$, both the minimum and maximum of two such values remain in $[-1,0]$. Hence all six edge components have the required ranges.

We next verify the admissibility condition inside the membership profile. Choose $w \in \{u, v\}$ such that

$$\nu_V^m(w) = \max\{\nu_V^m(u), \nu_V^m(v)\}.$$

Then

$$\rho_E^m(e) + \nu_E^m(e) = \min\{\rho_V^m(u), \rho_V^m(v)\} + \nu_V^m(w) \le \rho_V^m(w) + \nu_V^m(w) \le 1.$$

Nonnegativity is immediate. Therefore

$$0 \le \rho_E^m(e) + \nu_E^m(e) \le 1.$$

For the non-membership profile, choose $w \in \{u, v\}$ such that

$$\rho_V^n(w) = \max\{\rho_V^n(u), \rho_V^n(v)\}.$$

Since $\nu_E^n(e) = \min\{\nu_V^n(u), \nu_V^n(v)\} \leq \nu_V^n(w)$, we obtain

$$\rho_E^n(e) + \nu_E^n(e) \leq \rho_V^n(w) + \nu_V^n(w) \leq 1,$$

and hence

$$0 \leq \rho_E^n(e) + \nu_E^n(e) \leq 1.$$

It remains to verify the intuitionistic compatibility between the membership and non-membership profiles. For the positive coordinate, let $w \in \{u, v\}$ attain $\max\{\rho_V^n(u), \rho_V^n(v)\}$. Then

$$\rho_E^m(e) + \rho_E^n(e) = \min\{\rho_V^m(u), \rho_V^m(v)\} + \rho_V^n(w) \leq \rho_V^m(w) + \rho_V^n(w) \leq 1.$$

For the neutral coordinate, let $w \in \{u, v\}$ attain $\max\{\nu_V^m(u), \nu_V^m(v)\}$. Since $\nu_E^n(e) \leq \nu_V^n(w)$,

$$\nu_E^m(e) + \nu_E^n(e) \leq \nu_V^m(w) + \nu_V^n(w) \leq 1.$$

Finally, choose $w \in \{u, v\}$ such that

$$\delta_V^n(w) = \min\{\delta_V^n(u), \delta_V^n(v)\}.$$

Because $\delta_E^m(e) = \max\{\delta_V^m(u), \delta_V^m(v)\} \geq \delta_V^m(w)$, Definition 5 gives

$$\delta_E^m(e) + \delta_E^n(e) \geq \delta_V^m(w) + \delta_V^n(w) \geq -1.$$

Both terms are nonpositive, so also

$$\delta_E^m(e) + \delta_E^n(e) \leq 0.$$

Thus

$$-1 \leq \delta_E^m(e) + \delta_E^n(e) \leq 0.$$

All conditions of Definition 5 are therefore satisfied by A_E. Moreover, by construction,

$$\mathbf{m}_E(e) = \mathbf{m}_V(u) \wedge_T \mathbf{m}_V(v) \quad \text{and} \quad \mathbf{n}_E(e) = \mathbf{n}_V(u) \vee_T \mathbf{n}_V(v),$$

so the two compatibility inequalities in Definition 6 hold with equality. Consequently, G_{TIF} is a Tripolar Intuitionistic Fuzzy Graph. Since e was arbitrary, the conclusion holds for every edge in E. ▫

4. Conclusion

This paper investigates Tripolar Intuitionistic Fuzzy Sets and Tripolar Intuitionistic Fuzzy Graphs and establishes a mathematical framework for representing uncertainty from three distinct perspectives while incorporating both membership and non-membership information. These concepts provide a natural extension of intuitionistic fuzzy structures to tripolar settings and offer a basis for modeling more complex forms of uncertain and multi-perspective information.

Future research may further develop these concepts by incorporating higher-order relational structures, particularly HyperGraphs [55.56] and SuperHyperGraphs [23], thereby enabling the representation of more complex and hierarchical interactions under tripolar intuitionistic fuzzy information. It would also be worthwhile to investigate analogous extensions in other areas of mathematics, including topology, algebra, and group theory. Such developments may contribute to a broader theoretical framework for Tripolar Intuitionistic Fuzzy Structures and facilitate their application to increasingly complex mathematical and real-world systems.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer (Conversion from LaTeX to Word)

Since this manuscript was generated by converting LaTeX source into a Word document, minor formatting discrepancies may have occurred. We have made every effort to eliminate errors as much as possible, but we kindly ask for your understanding in advance.

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