



Review

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The tables give $0.698 = \log 4.998,884,875$, so that 0.698 are the first three figures of $\log 5$ with a remainder in the antilogarithm of $0.011,155,125$. If we use the $\log a$ method *either* with Table A *or* with Table B there is not much difficulty in obtaining the next three figures 970 , but the table does not contain sufficient data to find the final remainder required to give the last three figures. The only way is to use the tables *backwards* in order to obtain the antilogarithm of $0.698,970$ correct to 9 decimals. This involves a second use of the tables with the assistance of the $\log a$ column, and leads to the result

$$\log 4.999,999,954 = 0.698,970.$$

The final remainder is $0.000,000,046$, from which a repetition of the first operation gives the last three figures of the logarithm, namely 004 .

It will thus be seen that the calculation of logarithms to 9 figures is three times as difficult as their calculation to 6 figures. It involves three distinct operations instead of one, and the author is therefore justified in calling the tables A and B respectively single and triple extension tables. It is not often that logarithms are actually required to 9 places, and the emergency may be provided for by keeping a copy of the tables and learning to work Table B when necessity arises, rather than by keeping a big cumbersome book of tables which is never to be found when it is wanted, and still involves cumbersome work in the matter of interpolation.

It should be pointed out that the class teacher who sends the copies of the explanatory notice to the bookbinder to have the pages cut in order to prevent his pupils from wasting their time, will find the pockets for holding the tables cut off. French people are as a rule mortally afraid of the guillotine, but unless this is used the pages should be made larger than the pockets, which they are not.

G. H. BRYAN.

Einführung in die Theorie der Differentialgleichungen mit einer unabhängigen Variablen. Von DR. LUDWIG SCHLESINGER. (*Sammlung Schubert*, XIII.) (Leipzig, 1904.)

Gewöhnliche Differentialgleichungen beliebiger Ordnung. Von DR. J. HORN. (*Sammlung Schubert*, L.) (Leipzig, 1905.)

The subject of differential equations has grown to such an extent that it is necessary for the writer of a text-book to confine himself to a particular aspect of the subject. The works that we have before us deal with what might be called the formal side of the subject, that is the considerations connected with the existence of solutions, the nature of the integrals in the vicinity of the singularities, and the form of the equations when the singularities are of a specified type.

Dr. Schlesinger's treatise is a good introduction to the higher work on the subject. The writer, aiming at simplicity, considers only equations of the first two orders, a feature which will be welcome to many readers since it enables them to obtain a grip of the essential ideas without having to master any difficult analysis.

It is a significant fact that the differential equations which arise out of problems in geometry and physics are no other than the equations

with the simplest types of singularities; this point is brought out clearly in Schlesinger's treatise, Riccati's equation for instance being obtained when the differential equation

$$\frac{dy}{dx} = f(x, y)$$

is supposed not to possess an integral with a variable branch point.

Another good feature which might be noticed is that in obtaining the definite integral solutions of the various equations the writer calls attention to the fact that in each case the method depends upon the construction of a relation of the form

$$D_x\{u(x, z)\} = \Delta_z\{v(x, z)\},$$

and we believe that it will be through the development of the theory of relations of this type that further progress in this branch of the subject may be expected.

Dr. Horn's aim has been to write a more elaborate treatise on the subject avoiding as much as possible a repetition of the portions treated by Schlesinger. The characteristics of the work are the frequent occurrence of systems of linear differential equations and the use that has been made of the canonical forms of a linear substitution.

In dealing with a system of ordinary linear differential equations with constant coefficients

$$\frac{dy_r}{dx} + \sum_{s=1}^n a_{rs} y_s = f_r(x) \quad (r=1, \dots, n)$$

the writer employs Weierstrass' method which depends upon the reduction of the system to the canonical form

$$\begin{aligned} \frac{dY_1}{dx} &= rY_1 + F_1(x), \\ \frac{dY_2}{dx} &= rY_2 + Y_1 + F_2(x), \\ \frac{dY_n}{dx} &= rY_n + Y_{n-1} + F_{n-1}(x). \end{aligned}$$

This process is more scientific than the usual one since it avoids the use of operators and is in keeping with the general plan of the work; we do not think, however, that it is quite so convenient in actual practice as the symbolic method.

A good feature of the book is that it gives an account of much recent work that has been done on the subject; thus, besides the valuable chapters on the asymptotic representation of an integral and infinite determinants, we find that mention is made of Painlevé's canonical forms of differential equations of the second order and of a certain type whose integrals are uniform functions.

The writer also does well to call attention to theorems that are related to some of the recent mathematical work, as for instance the theorem that if the coefficients $P_i(x, \mu)$ ($i=1, \dots, n$) of the linear differential equation

$$\frac{d^n y}{dx^n} + P_1(x, \mu) \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_n(x, \mu) y = 0$$

are continuous functions of the real variable x in the interval $|x-a| \leq r$ and whole rational functions of the complex parameter μ . A solution y which is such that the initial values of $y, y' \dots y^{(n-1)}$ for $x=a$ are independent of μ is a whole transcendental function of μ for values of x within the given range (see p. 297), a theorem which corresponds to an important result in the theory of integral equations.

The printing is good in both cases and I have not noticed any misprints.

H. BATEMAN.

MATHEMATICAL NOTES.

205. [I. 2. b.]. [A solution, not by elliptic functions is wanted of the following :

Given five lines $abcde$ in a plane, it is known that the pairs of points ab, ce ; bc, da ; cd, eb ; de, ac ; ea, bd are in a collineation. Prove that the fixed triangle of this collineation is self polar as to both the conic on (ab, bc, cd, de, ea) and the conic on (ac, ce, eb, bd, da) . (F. MORLEY).]

Solution by H. BATEMAN.

Lemma. Let a collineation referred to its fixed triangle be given by the equations

$$x' = ax, \quad y' = by, \quad z' = cz,$$

then the conic

$$S' \equiv Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy = 0$$

will correspond to

$$S \equiv Aa^2x^2 + Bb^2y^2 + Cc^2z^2 + 2Fbcyz + 2Gcaxx + 2Habyx = 0,$$

and a point $P(\xi, \eta, \zeta)$ will correspond to the point $P'(\frac{\xi}{a}, \frac{\eta}{b}, \frac{\zeta}{c})$ on the one hand and to $P'(a\xi, b\eta, c\zeta)$ on the other.

The polar of P' with regard to S is

$$Aax\xi + Bby\eta + Ccz\zeta + F(c\eta z + b\xi y) + G(c\xi z + a\xi x) + H(a\eta x + b\xi y) = 0,$$

and the polar of P with regard to S' is

$$Aax\xi + Bby\eta + Ccz\zeta + F(b\eta z + c\xi y) + G(a\xi z + c\xi x) + H(b\eta x + a\xi y) = 0.$$

These lines will be the same if

$$\frac{Aa\xi + Ha\eta + Ga\xi}{Aa\xi + Hb\eta + Gc\xi} = \frac{Hb\xi + Bb\eta + Fb\xi}{Ha\xi + Bb\eta + Fc\xi} = \frac{Gc\xi + Fc\eta + Cc\xi}{Ga\xi + Fb\eta + Cc\xi} = \lambda, \text{ (say).}$$

Eliminating ξ, η, ζ we get a cubic for λ , hence there will in general be three positions of P for which the lines are the same; if there are more than three positions of P , we must have $F=G=H=0$, and then the conics S and S' will have the fixed triangle as a self-conjugate triangle, and the two lines will be the same for all positions of P .

To apply this lemma to the theorem in question, let $ABCDE$ be the pentagon formed by the five lines $abcde$, $A'B'C'D'E'$ the pentagon formed by the alternate sides. We can show at once that $A, A'; B, B'; C, C'; D, D'; E, E'$ are in a collineation, by comparing the cross-ratios of the pencils $A\{BCDE\}$, $A'\{B'C'D'E'\}$, and the corresponding pencils for the other corners.

Let S be the conic $ABCDE$, S' the conic $A'B'C'D'E'$. Now the two points which correspond to A in the collineation are A'' on the one hand (for EC corresponds to $E'C'$ and BD to $B'D'$) and A' on the other; also the polar of A'' with regard to S is easily seen to be the line $A'G$, and by considering