

XXVI.—*The 364 Unifilar Knots of Ten Crossings, Enumerated and Described.*

By Rev. THOMAS P. KIRKMAN, M.A., F.R.S.

(Read July 20, 1885.)

1. The 119 subsolids (marked *ss*) and the 244 unsolids (marked *us*), of these unifilars are here arranged in lists according to their flaps.  $F_e$  is the number of flaps of  $e$  loops upon a knot; and the headings of the lists, as, *e.g.*  $_{10}\mathbf{I}$ ,  $F_2=1$ ,  $F_1=3$ , describe so far all the knots in the lists. Thus in  $_{10}\mathbf{I}$  each has one 2-ple flap and three single ones. After the number in the list comes always the base on which the knot is constructed by the rules of my paper, XVII. in vol. xxxii. part ii. of the *Trans. R. S. E.*; and the reader who has that paper before him will find it easy to draw any knot on its base, nearly always there figured, by the first given flap, which is the leading one of the knot described; thus in  $_{10}\mathbf{I}$  the first written flap is the double one, and in  $_{10}\mathbf{P}$ ,  $F_2=1$ ,  $F_1=2$ , it is the triple one. The leading flap of a subsolid is always followed by a colon.

It will be seen that no two subsolids nor unsolids have the same description. A flap AB, CD is generally given by its collaterals AB only; but the coverticals CD are added when required for distinguishing the knots from each other.

To me it appears that this tabulation of these knots will be more useful than the engraving of the figures; for the student who draws a 10-fold unifilar will hereby more readily satisfy himself that it is found or not found in my census, than if he had 364 knots projected before him, in the manner of the plates of my former paper. One solid knot makes up 364 unifilars.

2. I am indebted to Professor TAIT for the detection of several bifilars which I had passed as unifilars, and for the addition of four to my list of unifilars,—namely,  $_{10}\mathbf{B}, 21$ ;  $_{10}\mathbf{D}, 17$ ;  $_{10}\mathbf{I}, 30$ ; and  $_{10}\mathbf{L}, 8$ ; and I may obtain from him farther contributions before he has performed on the figures all his surprising feats of twisting, which add a charm of conjuring to this curious and difficult inquiry.

The abbreviations which mark the symmetry are those used and explained in my paper above mentioned. My linear drawings of these unifilars of ten, as well as the more numerous figures of the unifilars of eleven crossings, will be found in the archives of the Royal Society of Edinburgh.

I have to acknowledge two omissions in my census of the knots of nine crossings. One is that of the bifilar  ${}_9A\zeta^2$  referred to as a base under  $_{10}\mathbf{I}$ . This ought to have been formed in art. 55 by drawing from the point  $c$  the flap 63,44.

The other knot omitted is a unifilar ( ${}_9A\gamma^2$ ) which should have been formed

by drawing in art. 53, in the base under 7, the flap 35,44. But no unifilar of ten crossings can be made on this  ${}_9A^2$ .

3. As Professor TAIT excludes all compound knots, *i.e.*, all that can be cut by a closed curve in two mid-edges only, the name of a fixed flap ought to be given to every flap whose deletion lays bare such a compound, *i.e.*, such a section through two edges only. As the deletion of such a flap is forbidden, so must be the drawing of it; and it cannot compete for the leadership with a flap drawn or about to be drawn.

A correction is to be made also in (2) of art. 27, which ought to stand thus:—

(2) If neither  $\epsilon$  nor  $\epsilon'$  be zoned polar, but be (*a*) one zoneless polar and the other epizonal, or (*b*) one zoneless polar and the other zonal, or (*c*) one epizonal and the other zonal, only one resulting configuration is possible: in all other cases, when neither  $\epsilon$  nor  $\epsilon'$  is zoned polar, and not both are asymmetric, two and only two configurations can and must be made by the above variation of posture of the charge.

In my plates in volume xxxii. part ii. a few errors require correction. In Pl. XLI., for  ${}_9H$ ; 8, 10; read  ${}_9H$ ; 4, 4, 10: in Pl. XLII., for  ${}_9By$ , 18; read  ${}_9By$ ; 8, 10: for  ${}_9Db$ ; 4, 14; read  ${}_9Db$ , 4, 4, 10: for  ${}_9Ch$ , 18; read  ${}_9Ch$ ; 4, 14: for  ${}_9Dk$ , asym.; read  ${}_9Dk$ , Moz: after  ${}_9Dl$  and after  ${}_9Dm$  write 18. In Pl. XLIII. under  ${}_9Gl$ , for asym. write 2zo. Mox. Het.

*Postscript, July 13, 1885.*—This day I see for the first time that when the problem is to construct, not all the knots of  $n$  crossings, but only the non-compound unifilars, in which the tape passes over and under itself alternately at successive crossings, there is no need to discuss at all marginal dissections, nor marginal charges, nor any use of bifilar bases. This is shown as follows:—

Let  $K_n$  be any non-compound unifilar of  $n$  crossings alternately under and over all through the circuit. Going round the circle, plant at every mid-edge between two crossings a dot on the right of the thread.

Every flap will have two dots, both inside, or both outside, or one inside and the other outside of it. In the last case call the flap odd; in the others, even.

The following theorems are easily proved:—

Theorem A.—If  $K_n$  above defined has an even single flap (of one loop only), it can be reduced to an unifilar, solid or unsolid, of  $n-1$  crossings by shrinking up that flap to a point.

Theorem B.—If  $K_n$  has an odd single flap, it can be reduced to an unifilar, solid or unsolid, of  $n-2$  crossings by effacing the two edges and the two summits of that flap.

Theorem C.—If  $K$  has a double flap, of two loops, the two terminal con-

tiguous loops of a  $(2+i)$ -ple flap ( $i \geq 0$ ), the knot can be reduced to an unifilar, solid or unsolid, of  $n-2$  crossings by shrinking up those two loops to a point.

It is evident that, if any clear definition of a leading flap and of a fixed flap be made and stuck to, the constructing converses (easily defined) of these three theorems must completely solve the following problem :—

The non-compound unifilars of  $n-1$  and of  $n-2$  crossings, alternately over and under, being given, to construct all the unifilars  $K_n$  above defined of  $n$  crossings, without risk of repeating a result in any posture, or of making a plurifil knot.

All that we have to do in reducing  $K_n$  is to do that at a leading or co-leading flap. All that we have to do in constructing  $K_n$  on a base, is to see that we do it by drawing or completing a flap which shall be the leader, or a co-leader on  $K_n$ . And we shall of course define that a plural flap leads any single one.

Thus, by theorem A,  ${}_4A$  (*vide* Pl. XL. vol. xxxii. *Trans. R. S. E.*) reduces to  ${}_8A$ , on which it is regularly built by its even flap.

By theorem B,  ${}_6A$  reduces to  ${}_4A$ , on which  ${}_6A$  is properly constructed by its odd flap.

By theorem C,  ${}_6F$  and  ${}_6G$  reduce to  ${}_4A$ , on which by a double flap either is correctly formed.

By these little examples the constructing converses are plainly suggested.

This appears to make an end of the puzzle of unifilar knots whose crossings are all through alternately under and over, so far as their construction upon lower non-compound unifilars is desired, as a preparation for the curious transformations and reductions by twisting of LISTING and TAIT.

I fear that my distinction of subsolids and unsolids is of little value, as a subsolid can often be twisted into an unsolid, and *vice versâ*.

I have had theorems B and C for nearly a year. Had I obtained theorem A earlier, my tasks on the unifilars of 8, 9, 10, and 11 crossings would have been much easier, and under less risk of error.

The simplicity of the three theorems is provoking enough, as usual, after the labour spent with clumsier tools, which looked so much more learned.

<b><math>{}_{10}A</math>.</b>			3. ss. ${}_8C$ ; 63,43: 43; asym.
$F_1=1$ .			4. " " 63,53: 43; "
1. ss. ${}_8W$ ; 44,43:	Moz.		5. " " 54,43: 44; "
2. ss. ${}_8Ag$ ; 55,33:	2zo. Mox. Het.		6. " " 53,43: 2p. Mox. Het.
3. ss. ${}_8Ap$ ; 43,54:	asym.		7. " " 43,53: "
<b><math>{}_{10}B</math>.</b>			8. " ${}_8D$ ; 53,53: 43; asym.
$F_1=2$ .			9. " " 53,43: 43; "
1. ss. ${}_8A$ ; 63,63:	2p. Mox. Het.		10. " " 43,53: 43: "
2. ss. " 63,64: 53;	asym.		11. " " 43,43: 2p. Mox. Het.
			12. " ${}_8E$ ; 54,33: "
			13. " ${}_8M$ ; 44,43: 2zo. Mox. Het.

14. ss.  ${}_8U$ ; 54,33: 44; asym.
15. "  ${}_8W$ ; 43,54: 43; "
16. " " 53,44: 43; "
17. " " 44,44: 2p. Mox. Het.
18. " " 43,54: Moz.
19. " " 43,44: 2p. Mox. Het.
20. " " 53,54: "
21. " " 53,54: Moz.
22. "  ${}_8At$ ; 64: Moz.
23. us.  ${}_8A$ ; 53,63: 33; asym.
24. "  ${}_8B$ ; 44: 33; "
25. "  ${}_8C$ ; 43: 33; "
26. " " 53,43: 33; "
27. "  ${}_8D$ ; 43: 33; "
28. "  ${}_8T$ ; 64: 33; "
29. "  ${}_8X$ ; 34: 33; "
30. "  ${}_8Ag$ ; 73: 33; Moz.
31. "  ${}_8Ah$ ; 53: 33; "
32. "  ${}_7C$ ; 63: 53; asym.
33. " " 73: 73; "
34. "  ${}_7D$ ; 63: 43; "
35. " " 53: 43; "
36. "  ${}_4A$ ; 53: 53; 2p. Mox. Het.

<sup>10</sup>**C.**

$F_1 = 3.$

1. ss.  ${}_8A$ ; 63,64: 64: 63; asym.
2. "  ${}_8C$ ; 63,54: 63: 53; "
3. " " 63,54: 63: 54; "
4. " " 54,55: 53: 53; "
5. " " 53,64: 34,35: 54; "
6. " " 43,64: 64: 43; "
7. "  ${}_8D$ ; 53,55: 53: 53; "
8. " " 53,54: 53: 53; "
9. " " 53,54: 54: 43; "
10. " " 53,54: 53: 43,44; "
11. " " 43,54: "
12. "  ${}_8E$ ; 63,43: 53: 53; "
13. " " 63: 54: 53; "
14. " " 54,53: 55: 53; "
15. "  ${}_8G$ ; 63,53: 53: 43; "
16. " " 63: 44: 43; "
17. " " 53: 64: 43; "
18. " " 53: 44: 43; "
19. " " 44: 54: 43; "

20. ss.  ${}_8G$ ; 54,43: 44: 43; asym.
21. " " 53,34: 54: 43; "
22. "  ${}_8I$ ; 54: 54: 53; "
23. " " 53: 53: 43,55; "
24. "  ${}_8P$ ; 54,53: 44: 43; "
25. "  ${}_8Q$ ; 53: 53: 43,54; "
26. "  ${}_8R$ ; 55,43: 54,43: 53,44; "
27. " " 55,53: 53: 53; Moz.
28. "  ${}_8S$ ; 63: 63: 53; "
29. " " 54,53: 53: 53; asym.
30. " " 63,43: 63: 53; "
31. "  ${}_8V$ ; 63,53: 63: 44; Moz.
32. " " 63,43: 63: 44; "
- 2p. Mox. Het.
33. " " 54: 53: 44; asym.
34. "  ${}_8X$ ; 53,54: 53: 43,55; "
35. " " 63,44: 43: 43; "
36. " " 63,54: 53: 43; "
37. " " 53,44: 43: 43; "
38. " " 54,44: 44: 43; "
39. "  ${}_8Aa$ ; 54: 54: 43,55; "
40. "  ${}_8Ac$ ; 64,43: 65: 43; "
41. "  ${}_8Ad$ ; 64,43: 63: 43; "
42. " " 74,43: 73: 73; "
43. "  ${}_8Ae$ ; 74,33: 73: 73; "
44. "  ${}_8Ad$ ; 65,33: 63: 53; "
45. " " 55,43: 53: 43; "
46. "  ${}_8Af$ ; 55,33: 53: 43; "
47. "  ${}_8Af$ ; 64,33: 63: 43; "
48. "  ${}_8Ag$ ; 73: 73: 73; Moz.
49. " " 55,44: 53: 53; "
- 2p. Mox. Het.
50. "  ${}_8Ah$ ; 63,54: 63: 53; asym.
51. " " 54,44: 53: 53; "
52. us.  ${}_6F$ ; 75: 33: 33; asym.
53. ss.  ${}_8At$ ; 55,43: 54,33: 53,44; "
54. "  ${}_8Au$ ; 54: 54: 43,44; "
55. "  ${}_8Av$ ; 64,33: 65: 43; "
56. "  ${}_8Ba$ ; 64: 64: 43; "
57. us.  ${}_8E$ ; 63,53: 54: 33; asym
58. "  ${}_8F$ ; 63: 44: 33; "
59. "  ${}_8C$ ; 54: 43: 33; "
60. "  ${}_8I$ ; 54: 33: 33; "
61. " " 53: 53: 33; "
62. "  ${}_8P$ ; 53: 43: 33; "
63. "  ${}_8Q$ ; 44: 43: 33; "

64. us.  ${}_8R$ ; 73; 73; 33; Moz.  
 65. " " 74; 73; 33; asym.  
 66. "  ${}_8S$ ; 65; 53; 33; "  
 67. " " 63; 63; 33; 2p. Mox. Het.  
 68. "  ${}_8U$ ; 63, 43; 54; 33; asym.  
 69. " " 55; 53; 33; "  
 70. "  ${}_8Ac$ ; 65; 63; 33; "  
 71. "  ${}_7E$ ; 63; 63; 43; "  
 72. " " 54; 53; 53; "  
 73. " " 64; 53; 43; "  
 74. " " 73; 73; 43; "  
 75. " " 63; 54; 53; "  
 76. " " 74; 73; 43; "  
 77. "  ${}_6A$ ; 63; 44; 43; "  
 78. "  ${}_6C$ ; 53; 33; 33; 2zo. Mox. Het.  
 79. "  ${}_6A$ ; 53; 44; 43; "

 ${}_{10}D$ .

$$F_1=4.$$

1. ss.  ${}_8E$ ; 63; 64; 54; 53; asym.  
 2. " " 54; 54; 54; 54; 2p. Mox. Het.  
 3. "  ${}_8G$ ; 63; 64; 53; 44; asym.  
 4. " " 53; 55; 53; 53; "  
 5. " " 54, 55; 54; 53; 53; "  
 6. "  ${}_8I$ ; 63; 63; 54; 54; "  
 7. " " 63; 63; 55; 53; "  
 8. " " 54, 54; 54; 53; 53; 2p. Mox. Het.  
 9. "  ${}_8L$ ; 63; 64; 55; 53; asym.  
 10. "  ${}_8P$ ; 54; 53; 53; 44; "  
 11. " " 63; 63; 54; 43; "  
 12. "  ${}_8Q$ ; 53; 53; 44; 44; 2p. Mox. Het.  
 13. " " 53; 54; 44; 43; asym.  
 14. "  ${}_8V$ ; 63; 65; 54; 53; "  
 15. " " 63; 65; 54; 43; "  
 16. us. " 65; 55; 43; 33; "  
 17. ss.  ${}_8P$ ; 53; 54; 53; 43; "  
 18. "  ${}_8Y$ ; 54; 54; 53; 43; "  
 19. "  ${}_8Ab$ ; 65; 64; 53; 43; "  
 20. " 74; 74; 73; 43; "  
 21. "  ${}_8Ac$ ; 73; 75; 73; 43; "  
 22. " " 55; 55; 53; 43; "  
 23. "  ${}_8Ac$ ; 64; 64; 63; 63; 2p. Mox. Het.

24. ss.  ${}_8Af$ ; 64; 63; 55; 53; asym.  
 25. "  ${}_8Al$ ; 64; 64; 63; 63; "  
 26. "  ${}_8Ar$ ; 63; 66; 63; 44; 2p. Mox. Het.  
 27. " " 73; 73; 75; 44; Moz.  
 28. " " 55; 55; 53; 53; asym.  
 29. " " 63; 66; 63; 43; "  
 30. "  ${}_8Aw$ ; 55; 54; 54; 43; "  
 31. " " 73; 74; 74; 53; "  
 32. "  ${}_8Ax$ ; 53; 55; 55; 53; 2p. Mox. Het.  
 33. " " 54; 54; 43; 43; Moz.  
 34. "  ${}_8Bf$ ; 65; 64; 53; 53; asym.  
 35. "  ${}_8Bj$ ; 55; 55; 55; 44; Moz.  
 36. "  ${}_8Bm$ ; 55; 54; 54; 44; 2p. Mox. Het.  
 37. us.  ${}_8Aw$ ; 75, 43; 74; 43; 33; asym.  
 38. " " 75, 33; 74; 43; 33; "  
 39. "  ${}_8Az$ ; 65, 33; 64; 43; 33; "  
 40. " " 65, 34; 64; 43; 33; "  
 41. "  ${}_8Bj$ ; 76; 76; 43; 33; "  
 42. "  ${}_7L$ ; 66; 63; 63; 44; "  
 43. " " 75; 73; 73; 44; "  
 44. "  ${}_6G$ ; 66; 66; 33; 33; 2zo. Mox. Het.  
 45. "  ${}_6H$ ; 55; 55; 33; 33; "  
 46. ss.  ${}_8Aj$ ; 64; 63; 54; 43; asym.  
 47. " " 55, 44; 54; 53; 43; "

 ${}_{10}E$ .

$$F_1=5.$$

1. ss.  ${}_8An$ ; 55; 65; 65; 43; 43; 2p. Mox. Het.  
 2. "  ${}_8Bk$ ; 55; 65; 65; 55; 53; Moz.  
 3. us.  ${}_8Bl$ ; 76; 76; 73; 44; 33; "

 ${}_{10}F$ .

$$F_2=1; F_1=0.$$

1. us.  ${}_9B$ ; 55; Moz.  
 2. "  ${}_9F$ ; 65; "

 ${}_{10}G$ .

$$F_2=1; F_1=1.$$

1. us.  ${}_9L$ ; 64; 64; asym.  
 2. "  ${}_9V$ ; 54; 44; "

3.	us.	${}_9Ae; 54; 54, 33;$	asym.
4.	"	${}_9Ap; 54; 54, 35;$	"
5.	"	${}_9Br; 74; 33;$	"
6.	"	${}_9Bs; 44; 63;$	"
7.	"	${}_9Bs; 54; 33;$	"
8.	"	${}_9Bs; 44; 43;$	"
9.	"	${}_9By; 44; 53;$	Moz.
10.	"	${}_9By; 64; 33;$	"
11.	"	${}_9Ck; 64; 43;$	asym.
12.	"	${}_9Cl; 74; 43;$	"

 ${}_{10}H.$ 

$$F_2 = 1; F_1 = 2.$$

1.	us.	${}_9M; 54; 55; 53;$	asym.
2.	"	${}_9M; 65; 63; 53;$	"
3.	"	${}_9N; 74; 73; 73;$	Moz.
4.	"	${}_9R; 54; 53; 53;$	asym.
5.	"	${}_9S; 64, 54; 63; 43;$	"
6.	"	${}_9S; 64; 53; 44;$	"
7.	"	${}_9T; 64, 44; 63; 43;$	"
8.	"	${}_9T; 54; 53; 53;$	Moz.
9.	"	${}_9V; 64; 54; 43;$	asym.
10.	"	${}_9V; 54; 54; 53;$	"
11.	"	${}_9W; 55; 53; 43;$	"
12.	"	${}_9W; 54, 54; 54, 54; 43;$	"
13.	"	${}_9X; 55; 44; 44;$	"
14.	"	${}_9Aa; 54, 54; 54, 44; 43;$	"
15.	"	${}_9Aa; 54, 43; 54; 43;$	"
16.	"	${}_9Ac; 54; 64; 63;$	"
17.	"	${}_9Ac; 74; 74; 43;$	"
18.	"	${}_9Ad; 74; 74; 53, 74;$	"
19.	"	${}_9Ad; 64; 64; 63;$	"
20.	"	${}_9Af; 64; 64; 53;$	"
21.	"	${}_9Ag; 65; 64; 44, 54;$	"
22.	"	${}_9Ag; 55; 54; 54;$	Moz.
23.	"	${}_9An; 64; 44; 43;$	asym.
24.	"	${}_9Ar; 64; 64; 43;$	"
25.	"	${}_9As; 74; 74; 43;$	"
26.	"	${}_9At; 64; 65; 43;$	"
27.	"	${}_9Au; 74; 74; 53, 73;$	"
28.	"	${}_9Av; 64; 65; 53;$	"
29.	"	${}_9Ba; 55; 54; 54;$	"
30.	"	${}_9Ba; 65; 64; 44, 34;$	"
31.	"	${}_9Be; 65; 64; 44, 63;$	"
32.	"	${}_9Be; 55; 54; 54;$	Moz.

33.	us.	${}_9Bk; 66; 65; 65;$	Moz.
34.	"	${}_9Bl; 65; 65; 54;$	asym.
35.	"	${}_9Bm; 75; 74; 74;$	Moz.
36.	"	${}_9Bp; 65; 53; 33;$	"
37.	"	${}_9Bp; 54; 55; 33;$	"
38.	"	${}_9Bq; 66; 53; 33;$	"
39.	"	${}_9Bt; 54; 63; 33;$	asym.
40.	"	${}_9Bt; 44; 63; 43;$	"
41.	"	${}_9Bu; 75; 43; 33;$	"
42.	"	${}_9Bv; 44; 54; 53;$	"
43.	"	${}_9Bv; 65; 63; 33;$	"
44.	"	${}_9Bv; 64; 64; 33;$	"
45.	"	${}_9Bw; 44; 54; 43;$	"
46.	"	${}_9Bw; 65; 43; 33;$	"
47.	"	${}_9Bx; 55; 54; 33;$	"
48.	"	${}_9Cd; 44; 65; 43;$	"
49.	"	${}_9Cd; 76, 43; 43; 33;$	"
50.	"	${}_9Ce; 54; 65; 33;$	"
51.	"	${}_9Ce; 76, 33; 43; 33;$	"
52.	"	${}_9Ce; 44; 65; 43;$	"
53.	"	${}_9Ce; 54; 65; 33;$	"
54.	"	${}_9Cm; 74; 73, 33; 44;$	Moz.
55.	"	${}_9Cm; 74; 73, 44; 44;$	"
56.	"	${}_9Cn; 64, 44; 53; 44;$	"
57.	"	${}_9Cn; 64, 33; 53; 44;$	"
58.	"	${}_9Cp; 54; 63; 53;$	asym.
59.	"	${}_9Dk; 44; 55; 33;$	Moz.
60.	"	${}_9Dk; 66; 33; 33;$	"
61.	"	${}_9Dl; 84; 85; 83;$	asym.
62.	"	${}_9Dm; 74; 76; 63;$	"

 ${}_{10}I.$ 

$$F_2 = 1; F_1 = 3.$$

1.	us.	${}_9E; 64; 65; 53; 53;$	asym.
2.	"	${}_9Ah; 65; 63; 54; 44;$	"
3.	"	${}_9Ah; 55; 55; 53; 44;$	"
4.	"	${}_9Ai; 65; 64; 53; 53;$	"
5.	"	${}_9Ai; 54; 55; 54; 53;$	"
6.	"	${}_9Al; 75; 74; 43; 43;$	"
7.	"	${}_9Al; 75; 74; 53; 53;$	"
8.	"	${}_9Aw; 66; 63; 63; 54;$	"
9.	"	${}_9Aw; 64; 65; 54; 53, 54;$	"
10.	"	${}_9Aw; 64; 65; 54; 53, 53;$	"
11.	"	${}_9Ax; 74; 74; 73; 54, 44;$	"
12.	"	${}_9Ax; 74; 74; 73; 54, 43;$	"

13. us.  ${}_9A_x$ ; 75; 73; 73; 54; asym.
14. "  ${}_9A_z$ ; 65; 65; 43; 43; "
15. " " 54; 55; 54; 43; "
16. " " 65; 64; 43; 43; "
17. "  ${}_9Be$ ; 55; 55; 54; 53; "
18. " " 65; 63; 54; 54; "
19. "  ${}_9Bf$ ; 54; 54; 65; 63; "
20. " " 55; 53; 65; 63; "
21. "  ${}_9Bg$ ; 65; 65; 53; 53; "
22. " " 66; 64; 53; 53; "
23. " " 64; 63; 55; 54; "
24. "  ${}_9Cf$ ; 74; 75; 44; 33; "
25. " " 76; 73; 44; 33; "
26. " " 55; 65; 63; 33; "
27. "  ${}_9Ch$ ; 75; 76; 65; 33; "
28. "  ${}_9Cj$ ; 44; 76; 76; 73; "
29. " " 87; 87; 83; 33; "
30. "  ${}_9Az^2$ ; 54; 65; 63; 43; "

 ${}_{10}J.$ 

$$F_2=2; \quad F_1=0.$$

1. us.  ${}_8A$ ; 64; 64; 2p. Mox. Het.
2. "  ${}_8C$ ; 64; 54; asym.
3. "  ${}_8D$ ; 54; 54; 2p. Mox. Het.
4. "  ${}_8X$ ; 54; 44; asym.

 ${}_{10}K.$ 

$$F_2=2; \quad F_1=1.$$

1. us.  ${}_8E$ ; 74; 74; 44; asym.
2. " " 65; 64; 63,65; "
3. " " 65; 64; 63,63; "
4. "  ${}_8F$ ; 64; 55; 63; "
5. "  ${}_8G$ ; 65; 54; 43,63; "
6. " " 65; 54; 43,65; "
7. " " 54; 54; 54; "
8. "  ${}_8I$ ; 65; 64; 53; "
9. "  ${}_8P$ ; 54; 54; 53; Moz.
10. "  ${}_8Q$ ; 55; 54; 43; asym.
11. "  ${}_8R$ ; 85; 84; 83; "
12. "  ${}_8S$ ; 76; 74; 63; "
13. "  ${}_8U$ ; 64; 55; 65; "
14. " " 66; 65; 63; "
15. " " 75; 74; 54; "

16. us.  ${}_8Y$ ; 55; 44; 53; Moz.
17. "  ${}_8Z$ ; 54; 44; 53; "
18. " " 64; 64; 33; 2zo. Mox. Het.
19. "  ${}_8Aa$ ; 65; 44; 55; Moz.
20. "  ${}_8Ac$ ; 76; 44; 73; asym.
21. " " 74; 44; 75; "

 ${}_{10}L.$ 

$$F_2=2; \quad F_1=2.$$

1. us.  ${}_8J$ ; 55; 55; 54; 54;  
2p. Mox. Het.
2. "  ${}_8K$ ; 65; 64; 53; 55; Moz.
3. " " 54; 54; 55; 55;  
2zo. Mox. Het.
4. "  ${}_8L$ ; 65; 65; 63; 63;  
2p. Mox. Het.
5. "  ${}_8V$ ; 65; 54; 55; 65; asym.
6. " " 76; 75; 65; 43; "
7. "  ${}_8An$ ; 77; 44; 77; 33; Moz.
8. "  ${}_8L$ ; 75; 74; 54; 53; asym.

 ${}_{10}M.$ 

$$F_2=3; \quad F_1=0.$$

1. us.  ${}_7E$ ; 75; 74; 54; asym.
2. "  ${}_7F$ ; 65; 65; 54; Moz.

 ${}_{10}N.$ 

$$F_3=1; \quad F_1=1.$$

1. us.  ${}_8B$ ; 66; 44; asym.
2. "  ${}_8T$ ; 76; 74; "
3. "  ${}_8X$ ; 65; 33; "
4. "  ${}_8X$ ; 55; 34; "
5. "  ${}_8Am$ ; 55; 33; 2zo. Mox. Het.

 ${}_{10}P.$ 

$$F_3=1; \quad F_1=2.$$

1. us.  ${}_8E$ ; 75; 73,45; 54; asym.
2. " " 75; 73,53; 54; "
3. " " 76; 53; 53; "
4. "  ${}_8F$ ; 66; 53; 53; Moz.

5. us.  ${}_8F$ ; 75; 73; 44; asym.
6. „  ${}_8G$ ; 76; 43; 43; „
7. „ „ 65, 54; 54; 43; „
8. „ „ 65, 53; 54; 43; „
9. „  ${}_8Q$ ; 66; 43; 43; 2p. Mox. Het.
10. „ „ 65; 44; 43; asym.
11. „  ${}_8Y$ ; 66; 53; 33; Moz
12. „ „ 55; 53; 44; „
13. „ „ 75; 44; 33; „
14. „  ${}_8Ac$ ; 85; 85; 33; asym.
15. „ „ 87; 83; 33; „
16. „ „ 55; 65; 63; „

 ${}_{10}Q.$ 

$$F_3=1; F_1=3.$$

1. us.  ${}_8L$ ; 76; 73; 54; 53; asym.
2. „ „ 75; 74; 54; 53;

 ${}_{10}R.$ 

$$F_3=1; F_2=1; F_1=0.$$

1. us.  ${}_7C$ ; 85; 84; asym.
2. „  ${}_7D$ ; 65; 54; „
3. „  ${}_7G$ ; 75; 44; „

 ${}_{10}S.$ 

$$F_3=1; F_2=1; F_1=1.$$

1. us.  ${}_7E$ ; 76; 54; 73; asym.
2. „ „ 85; 85; 43; „
3. „ „ 75; 54; 74; „
4. „ „ 65; 65; 63; „
5. „  ${}_7F$ ; 76; 75; 43; „
6. „ „ 65; 55; 54; „
7. „  ${}_7J$ ; 77; 44; 33; Moz.
8. „ „ 55; 44; 55; „

 ${}_{10}T.$ 

$$F_3=1; F_2=2; F_1=0.$$

1. us.  ${}_6B$ ; 75; 74; 55; Moz.
2. „  ${}_6C$ ; 77; 76; 76; 2zo. Mox. Het.

 ${}_{10}U.$ 

$$F_3=2.$$

1. us.  ${}_6A$ ; 65; 65; 2p. Mox. Het.

 ${}_{10}V.$ 

$$F_4=1; F_1=1.$$

1. us.  ${}_7C$ ; 86; 83; asym.
2. „  ${}_7D$ ; 76; 43; „
3. „  ${}_7G$ ; 66; 53; Moz.

 ${}_{10}W.$ 

$$F_4=1; F_1=2.$$

1. us.  ${}_7E$ ; 87; 83; 43; asym.
2. „ „ 76; 54; 53; „
3. „  ${}_7F$ ; 77; 74; 43; „
4. „  ${}_7J$ ; 66; 55; 33; Moz.

 ${}_{10}X.$ 

$$F_4=1; F_2=1$$

1. us.  ${}_6A$ ; 76; 54; asym.

 ${}_{10}Y.$ 

$$F_4=1; F_2=1; F_1=1$$

1. us.  ${}_6B$ ; 77; 54; 53; Moz.
2. „  ${}_6C$ ; 87; 85; 75; „
3. „  ${}_6B$ ; 76; 73; 55; „

 ${}_{10}Z.$ 

$$F_4=1; F_3=1.$$

1. us.  ${}_5A$ ; 96; 95; Moz.

 ${}_{10}\Delta.$ 

$$F_5=1; F_1=1.$$

1. us.  ${}_6A$ ; 87; 43; asym.



$_{10}\Theta.$	$_{10}\Sigma.$
$F_5=1; \quad F_2=1.$	$F_6=1; \quad F_1=1.$
1. us. $_5A; 97; 94; \text{Moz.}$	1. us. $_4A; 98; 93; \text{Moz.}$
$_{10}\Xi.$	$_{10}\Phi.$
$F_5=1; \quad F_3=1.$	$F_7=1; \quad F_1=1.$
1. us. $_4A; 77; 55; 2\text{zo. Mox. Het.}$	1. us. $_4A; 99; 33; 2\text{zo. Mox. Het.}$

Finally, there is one solid unifilar knot, the  $_{10}B$  described in art. 68 of my memoir above referred to. In that article the quadrifilar (4466)  $_{10}A$  is wrongly designated a zoned triaxine; that  $_{10}A$  is a 4-zoned monarchaxine heterozone.

NOTE.—Two Appendices to this paper will be found in the *Proceedings of the Royal Society of Edinburgh*, vol. xiii. p. 359.