

On the Graduation of the Sonometer

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XXV. *On the Graduation of the Sonometer.* By J. H. POYN-
TING, M.A., *Fellow of Trinity College, Cambridge**.

It seems likely that such valuable results will be obtained by means of Professor Hughes's sonometer, that it is desirable that some method should be employed to turn its at present arbitrary readings into absolute measure, so that, for instance, the induced currents caused by different metals in the induction-balance may be measured and compared with each other.

In Maxwell's 'Electricity,' vol. ii. chap. xiv., the general formula is given for the coefficient of induction of one circular circuit on another. Adapting this to the case where two equal circular circuits are on the same axis at a distance apart greater than the radius of the coils, the following formula is obtained.

Let a = distance between centres,

b = radius of either circle,

c = distance of either circumference from centre of other,

M = coefficient of induction.

Then

$$M = -\frac{4\pi^2 b^4}{c^3} \left\{ \frac{1}{2} - \frac{3b^2}{4c^2} + \frac{15b^4}{8c^4} - \frac{35b^6}{8c^6} + \frac{2835b^8}{256c^8} - \&c. \right\} \quad (1)$$

or

$$= -\frac{4\pi^2 b^4}{a^3} \left\{ \frac{1}{2} - \frac{3b^2}{2a^2} + \frac{75b^4}{16a^4} - \frac{490b^6}{32a^6} + \frac{24570b^8}{256a^8} - \&c. \right\} \quad (2)$$

Of these the latter uses directly the distance between the centres, the observed quantity—but is not nearly so convergent as the former, in which c may be at once deduced from $c = \sqrt{a^2 + b^2}$.

To obtain formulæ which might be strictly applied to the sonometer, we should have to consider the more general case of two coils of unequal radii b and β , for which I have found the formula corresponding to (2), viz.

$$M = \frac{4\pi b^2 \beta^2}{a^3} \left(\frac{1}{2} - \frac{3}{4} \frac{b^2 + \beta^2}{a^2} + \frac{15}{16} \frac{b^4 + 3b^2 \beta^2 + \beta^4}{a^4} - \frac{35}{32} \frac{b^6 + 6b^4 \beta^2 + 6b^2 \beta^4 + \beta^6}{a^6} + \&c. \right) \quad (3)$$

* Read December 13th, 1879.

We should then have to take the finite integrals of each term between the limiting values of b and β . But this would be exceedingly complicated and would require a knowledge of all the details of construction; and we may at least get a first approximation to the true result by replacing the coils by a single one of a radius intermediate between the greatest and least radii.

In Prof. Hughes's paper (Phil. Mag. July 1879) he gives the internal and external radii of his coils as 15 millims. and 27.5 millims. respectively. I have considered, then, that 25 millims. will give results not very far from the truth; and as it makes the calculations considerably easier, I have taken that as the value of b and applied the formulæ to the numbers given in the paper. The resultant current in the middle coil was zero when it was distant 47 millims. from one end and 200 from the other. This enables us to find the ratio between the number of turns in the two ends at least sufficiently nearly to apply to some of the results.

Let M_1 be the coefficient of induction of the larger coil on the movable one, M_2 that of the smaller, the former having m turns, the latter n . When the movable coil was 200 millims. from the large and 47 millims. from the small coil, since there was no induced current,

$$mM_1 = nM_2.$$

Applying formula (1), we have c for the larger coil

$$= \sqrt{200^2 + 25^2} = 201.5,$$

and for the smaller coil

$$c = \sqrt{47^2 + 25^2} = 53.2,$$

b being the same for both. Then

$$\begin{aligned} & \frac{m}{(201.5)^3} \left\{ \frac{1}{2} - \frac{3}{4} \left(\frac{25}{201.5} \right)^2 + \frac{15}{8} \left(\frac{25}{201.5} \right)^4 - \&c. \right\} \\ &= \frac{n}{(53.2)^3} \left\{ \frac{1}{2} - \frac{3}{4} \left(\frac{25}{53.2} \right)^2 + \frac{15}{8} \left(\frac{25}{53.2} \right)^4 - \frac{35}{8} \left(\frac{25}{53.2} \right)^6 \right. \\ & \quad \left. + \frac{2835}{256} \left(\frac{25}{53.2} \right)^8 - \&c. \right\} \end{aligned}$$

Multiplying each side by 2 and finding the successive terms,

$$m \times \frac{122}{10^9} \{1 - \cdot 02308 + \cdot 00088 - \&c.\}$$

$$= n \times \frac{6645}{10^9} \{1 - \cdot 33123 + \cdot 18286 - \cdot 09422 + \cdot 02633,$$

or

$$\frac{m}{n} = 43\cdot6.$$

I have applied the formula to the results for various metals given by Prof. Hughes in a table in his paper. In the table below, in the first column are Prof. Hughes's numbers, *i. e.* distances from the point of no induction. In the second are numbers proportional to $mM_1 - nM_2$; where M_1 , M_2 are the coefficients of induction of two simple coils calculated on the above hypothesis, m and n the number of turns in the two respectively. In the third column are the resistances for bars of the metal 100 millims. long and 1 millim. in diameter (Jenkin, p. 249). In the last column are the products of the numbers in the two preceding columns.

Metal.	Distance from point of no induction.	$mM_1 - nM_2$, proportional to	R.	$(mM_1 - nM_2)R.$
Silver	125	178	·21	37·4
Gold	117	135	·27	36·5
Aluminium...	112	116	·375	43·5
Copper	100	84	·21	17·6
Zinc	80	50·1	·72	36·1
Tin	74	44·6	1·70	75·8
Iron	45	22·46	1·25	28·1
Lead	38	18·87	2·5	47·2
Antimony	35	17·35	4·5	78·1
Bismuth	10	5·75	16·8	96·6

Mercury has been omitted, as it gives a very much higher value than any of the others. Were the induced currents in the induction-balance proportional to the resistances given in the table, the numbers in the last column would of course be all the same. The deviations from equality are far greater than could be accounted for by errors in the approximations I have adopted, especially for the metals not at the beginning or end of the list. Hence we are driven to conclude, either that the resistances of the metals given in the tables are not

the same as the resistances of the metals used by Prof. Hughes, or that the induced current is not proportional to the conductivity of the metal.

It should be noticed that the method of measuring currents by the sonometer assumes that the telephone integrates, as it were, the current; *i. e.* the loudness of the sound depends only on the total current, not on the time during which the current is passing, provided that the time be very short. I do not know whether this point has been investigated; but if not, it would probably be easy to examine it by means of the sonometer. It would be advisable to modify the instrument in such a way that the formulæ might be more easily employed, and that the approximations might be nearer to the truth.

The formulæ used in this paper may be obtained as follows, the method being adapted from that given in Maxwell.

The potential of a circular unit current at any point is the same as that of a magnetic shell of unit strength bounded by the circuit. This, again, is the same as the attraction of a thin plate of matter of unit surface-density in a direction perpendicular to the plane of the plate. If ω be the attraction of a plate of radius b , at a point distant b from the plate along its axis,

$$\begin{aligned}\omega &= 2\pi \left(1 - \frac{1}{\sqrt{1 + \frac{b^2}{c^2}}} \right) \\ &= 2\pi \left\{ \frac{1}{2} \frac{b^2}{c^2} - \frac{1 \cdot 3}{2 \cdot 4} \frac{b^4}{c^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{b^6}{c^6} - \&c. \right\}.\end{aligned}$$

If we introduce zonal harmonics as coefficients, this becomes

$$\omega = 2\pi \left\{ \frac{1}{2} \frac{b^2}{c^2} P_1 - \frac{1 \cdot 3}{2 \cdot 4} \frac{b^4}{c^4} P_3 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{b^6}{c^6} P_5 - \&c. \right\}.$$

This is now the potential at any point in space where $b < c$.

If there be a second circular circuit of radius β on the same axis, we may suppose it replaced by a magnetic shell bounded by the current and lying on the sphere, with centre at the centre of the first current, the radius of the sphere being c .

This shell may be considered to consist of two layers of

matter of equal and opposite densities, μ and $-\mu$, at distances c and $c+dc$ from the centre. The potential on the second layer is

$$\iint \mu \omega dS,$$

where the integration is taken over the shell. The potential on the second layer is

$$-\iint \mu \left(\omega + \frac{d\omega}{dc} dc \right) dS,$$

the sum being

$$-\iint \mu \frac{d\omega}{dc} dc dS ;$$

but since the strength $=1$, $\mu dc=1$, and we have the mutual potential

$$M = - \iint \frac{d\omega}{dc} dS.$$

Replacing the element dS by $c^2 d\mu d\phi$, the limits will be for ϕ from 0 to 2π , and for μ from 1 to μ .

Integrating with respect to ϕ , and remembering that c is constant in integrating for μ , we have

$$\begin{aligned} M &= 2\pi c^2 \int_{\mu}^1 \frac{d\omega}{dc} d\mu \\ &= -4\pi^2 c^2 \left\{ \frac{b^2}{c^3} \int_{\mu}^1 P_1 d\mu - \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{b^4}{c^5} \int_{\mu}^1 P_3 d\mu + \&c. \right\}. \end{aligned}$$

But we have the relation for zonal harmonics,

$$\int_{\mu}^1 P_n d\mu = \frac{1-\mu^2}{n(n+1)} \frac{dP_n}{d\mu}.$$

Substituting, we obtain

$$\begin{aligned} M &= -4\pi^2 (1-\mu^2) \left\{ \frac{b^2}{2c} \frac{dP_1}{d\mu} - \frac{1 \cdot 3}{2 \cdot 4} \frac{b^4}{3c^3} \frac{dP_3}{d\mu} \right. \\ &\quad \left. + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{b^6}{5c^5} \frac{dP_5}{d\mu} - \&c. \right\}. \end{aligned}$$

The following are the values for the coefficients (Ferrer's 'Spherical Harmonics,' p. 23), both in terms of μ and when

we substitute $\mu^2 = 1 - \frac{\beta^2}{c^2}$:—

$$\frac{dP_1}{d\mu} = 1,$$

$$\frac{dP_3}{d\mu} = \frac{3}{2}(5\mu^2 - 1) = \frac{3}{2}\left(4 - 5\frac{\beta^2}{c^2}\right),$$

$$\frac{dP_5}{d\mu} = \frac{15}{8}(21\mu^4 - 14\mu^2 + 1) = \frac{15}{8}\left(8 - 28\frac{\beta^2}{c^2} + 21\frac{\beta^4}{c^4}\right),$$

$$\frac{dP_7}{d\mu} = \frac{3003\mu^6 - 3465\mu^4 + 945\mu^2 - 35}{16} = \frac{448 - 3024\frac{\beta^2}{c^2} + \&c.}{16},$$

$$\begin{aligned} \frac{dP_9}{d\mu} &= \frac{109395\mu^8 - 180180\mu^6 + 90090\mu^4 - 13860\mu^2 + 315}{128} \\ &= \frac{5760 + \&c.}{128}. \end{aligned}$$

Substituting these values and putting $c^2 = a^2 + \beta^2$,

$$\begin{aligned} \therefore M = -4\pi \frac{\beta^2 b^2}{a^3} \left\{ \frac{1}{2} - \frac{3}{4} \frac{b^2 + \beta^2}{a^2} + \frac{15}{16} \frac{b^4 + 3b^2\beta^2 + \beta^4}{a^4} \right. \\ \left. - \frac{35}{32} \frac{b^6 + 6b^4\beta^2 + 6b^2\beta^4 + \beta^6}{a^6} + \&c. \right\}, \end{aligned}$$

The more useful form is obtained by retaining c . If we take the two circles of equal radius (i. e. $b = \beta$), we obtain

$$M = -4\pi^2 \frac{b^4}{c^3} \left\{ \frac{1}{2} - \frac{3}{4} \frac{b^2}{c^2} + \frac{15}{8} \frac{b^4}{c^4} - \frac{35}{8} \frac{b^6}{c^6} + \frac{2835}{256} \frac{b^8}{c^8} - \&c. \right\}.$$

XXVI. *On a new Form of Resistance-Balance adapted for comparing Standard Coils.* By J. A. FLEMING, D.Sc. (Univ. Lond.), Scholar of St. John's College, Cambridge*.

[Plate XIX.]

1. THE British Association Committee on electrical standards concluded their valuable labours on the unit of resistance by constructing copies of the selected standard. Certain of these coils, some fourteen in number, are at present preserved in the Cavendish Laboratory, Cambridge. It is

* Read December 13th, 1879.