

VIII.—*Experimental Inquiry into the Laws of the Conduction of Heat in Bars.*
 Part II. *On the Conductivity of Wrought Iron, deduced from the Experiments of 1851.* By JAMES D. FORBES, D.C.L., LL.D., F.R.S., V.P.R.S. Ed., Principal of St Salvator and St Leonard's College, St Andrews, and Corresp. Member of the Institute of France. (Plates I., II., III., IV., and V.)

(Read 20th February 1865.)

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INTRODUCTION.

The Articles are numbered in continuation of those in the First Part of this Paper.

39. In the first part of this paper, read to the Royal Society of Edinburgh in April 1862, and published in their Transactions,† I explained the principles of a method devised by me in 1850 for ascertaining the absolute conducting power of substances capable of being formed into long bars; and I also stated the general results of experiments made in 1851 on the Conductivity for heat of wrought Iron.

40. I explained in Art. 14 of that paper, that the publication of the results had been for ten years withheld, partly in consequence of the state of my health which completely interrupted the experiments, but still more from the defective graduation of some of the thermometers used, which made it necessary to submit the instruments to a careful scrutiny, and to repeat with the duly corrected numbers the whole of the elaborate projections of the curves and calculations from them, on which the accuracy of the final results of course depends.

41. I stated that the friendly aid and exemplary patience of the late Mr WELSH of the Kew Observatory had supplied me with data for correcting the readings of the most important, and at the same time the most inaccurately graduated of the series of French thermometers employed in these experiments.

* Sections III. and VII. have been added to this paper since it was read.

† Vol. XXIII. p. 133.

Without his help the present corrected reduction of the observations could never have been made; and even with the aid of the tables kindly prepared by him, it has been a work of no small labour and anxiety to bring to one strictly accordant scale the whole of the observations made with eight or ten thermometers, none of them deserving of being called standards, and in most of which the zero appears to have oscillated at different periods.

42. I have thought it unnecessary, as it would certainly have been most tedious, to print in this paper the crude observations and the numerous tables of reduction formed for the scales of the several thermometers. I have thought it sufficient to give the corrected results, which, in many cases, are the mean of independent readings of different thermometers.

43. Besides the correction of scale errors, an important correction required to be applied in order to reduce the readings to what they would have been had the column of the mercury in the thermometer partaken of the temperature of the bulb. Owing to the small transverse dimensions of the bars, whose temperatures were to be ascertained, the bulb of the thermometers was often little more than covered by the mercury with which the holes in the bars were filled (Art. 20). The stems were therefore necessarily exposed in their whole length to the temperature of the surrounding air. In the case of the higher temperatures to be measured, this correction was not only large (amounting sometimes to 3° Cent., always additive), but also in some degree uncertain, owing to the ascending currents of warm air in the neighbourhood of the heated bar, and enveloping the stem of the thermometer.* However, I believe that the formula in the note below leads to pretty accurate results, checked, as it has been, by occasional observations of a small auxiliary thermometer suspended in the air, touching the stem of the thermometer to be reduced, about its middle.

44. The hotter thermometers are probably slightly *over* corrected. I have stated that in extreme cases this correction amounts to about 3° Cent., a quantity which may possibly be erroneous in some cases to one-tenth of its amount, but

* The form of the correction is very simple, being

$$\frac{\text{Degrees exposed} \times \text{Excess of Temp. shewn over air.}}{\text{Dilatation of Merc. in Glass for 1° Cent.}}$$

always additive. If T be the temperature as read, t the temperature of the air, and a the scale reading of the commencement of the stem of the particular thermometer, the correction is very nearly

$$+ \frac{(T - a)(T - t)}{6400}.$$

Since t and a are usually small numbers, the correction increases nearly as the square of the temperature to be measured.

Fortunately, the precision of this correction is not very important to the result. It chiefly affects the actual temperatures; for it will be more fully seen hereafter, that if the same instrument be used in the dynamical and statical experiments, being exposed in precisely the same way, the measures will be relatively correct, and the deduction of the conductivity will not thereby be sensibly affected.

I hope rarely so. Much larger corrections would have been inevitable at the highest temperatures (about 200° Cent.), had I not invariably employed for these a thermometer in which about 110° of the mercury was expelled from the bulb into the cavity at the top of the stem. The corrections for the reading of this thermometer were determined by Mr WELSH with extraordinary care. As its indications only commenced about the boiling-point of water, the length of the column exposed to the air was comparatively short.

45. For the principles on which the experimental investigations are founded, I refer to Art. 5, &c., of the former part of this paper. It will be recollected that there are two distinct classes of experiments, in one of which (the statical) the permanent temperatures at different points of a bar are to be observed; in the other (the dynamical) the velocity of cooling of a short bar of similar section, uniformly heated at first, is to be ascertained. I shall now proceed to describe these experiments severally more in detail than I have yet done, and to classify and discuss the results.

§ I. *Statical Experiments.*

46. *The Apparatus.*—A general account of this has been given in Arts. 17–20. It will, however, be rendered more intelligible by a reference to Plate I., fig. 1. The long wrought-iron bar AB was supported on a wooden frame CD by means of one fixed support E, and two moveable props F, G, which were all of wood, and were brought to a blunt edge at top, on which the bar rested, at about 15 inches above the top of a massive table, which stood in a spacious apartment attached to the Natural Philosophy Class-room (Edinburgh University). No fire was allowed during the experiments, and the south shutters being closed, the room was lighted from the north. At the end of the bar, towards the left side of the figure, was attached the heating apparatus, a cast-iron crucible H, usually filled with just-melting lead. It was kept hot by means of the powerful gas-furnace I, with a double metal chimney and two concentric rows of burners. The gas was derived from the main pipe by a flexible tube L, and passed through one of Milne's patent gas regulators, K, with a view to obtaining a uniform flame, which, however, remained subject to occasional fluctuation. The connection of the crucible with the conduction bar will be best understood from the sectional diagram in Plate II. fig. 1. An internal flange *a a'* was cast on the crucible, leaving a square cavity 2·5 inches long, into which the extremity A of the conduction bar was thrust, and was retained there by friction only. The exterior face of the crucible *b c* is almost vertical, and determines the position from which the distances of the thermometers along the bar are reckoned. Supposing the crucible itself to be maintained at the constant temperature of melting lead, it seems reasonable to assume that the bar A, so far as encased within it—that is, up to the zero line *b c*—may be regarded as having nearly

Fig. 1

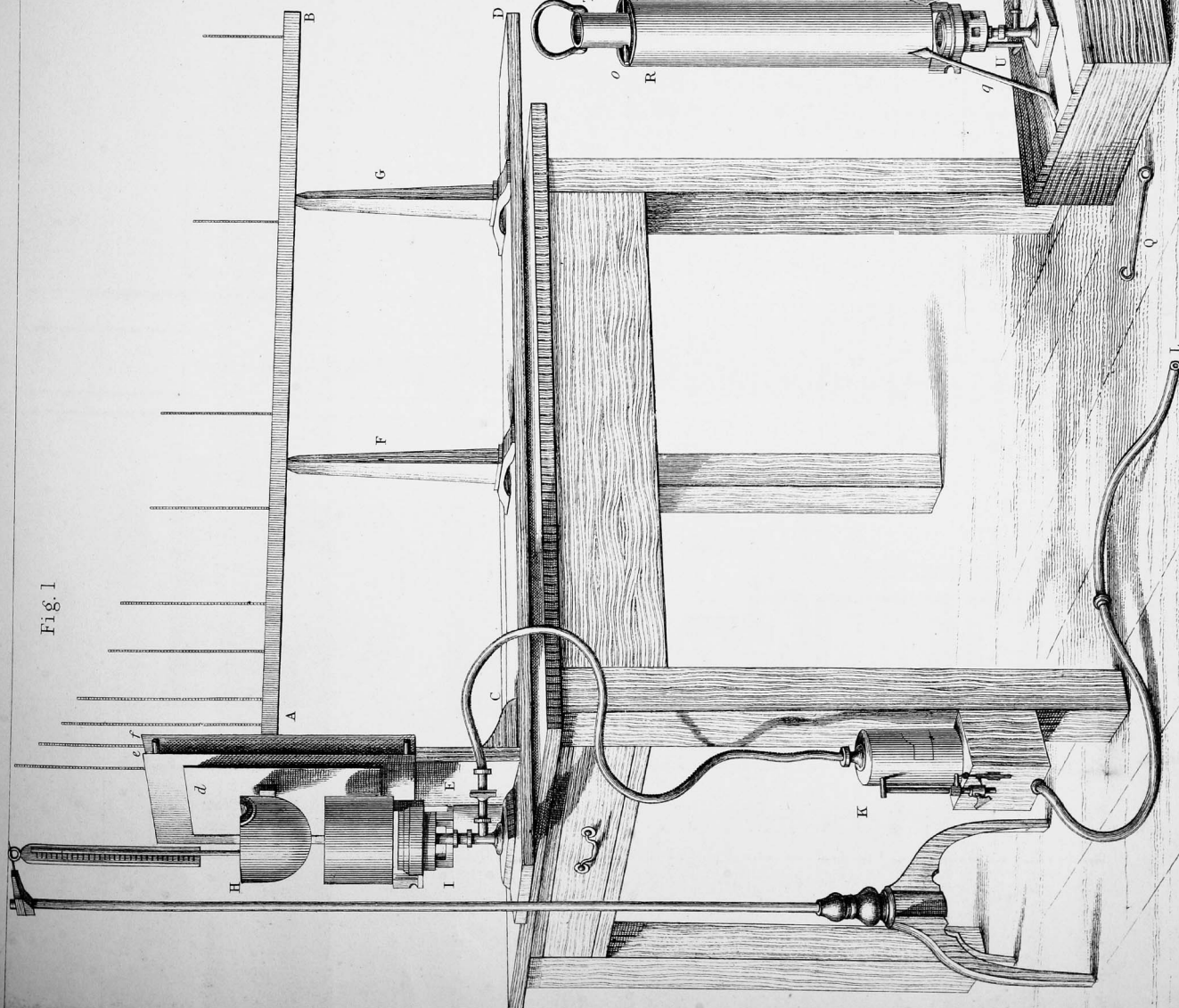


Fig. 2

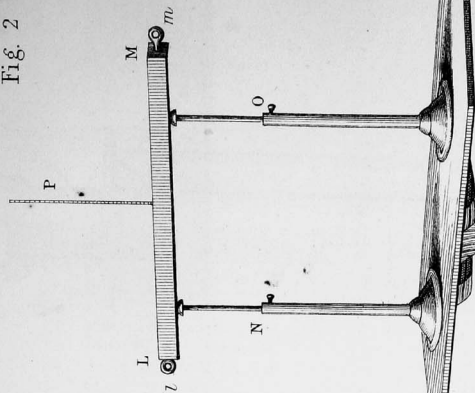


Fig. 3

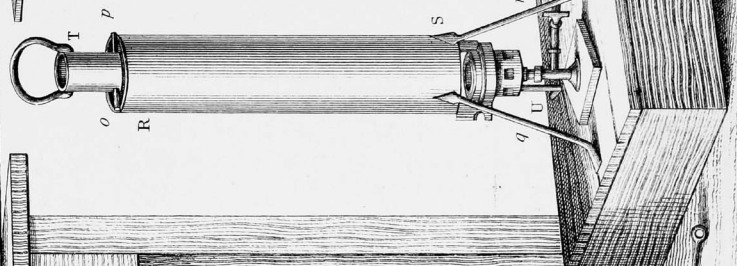


Fig 1

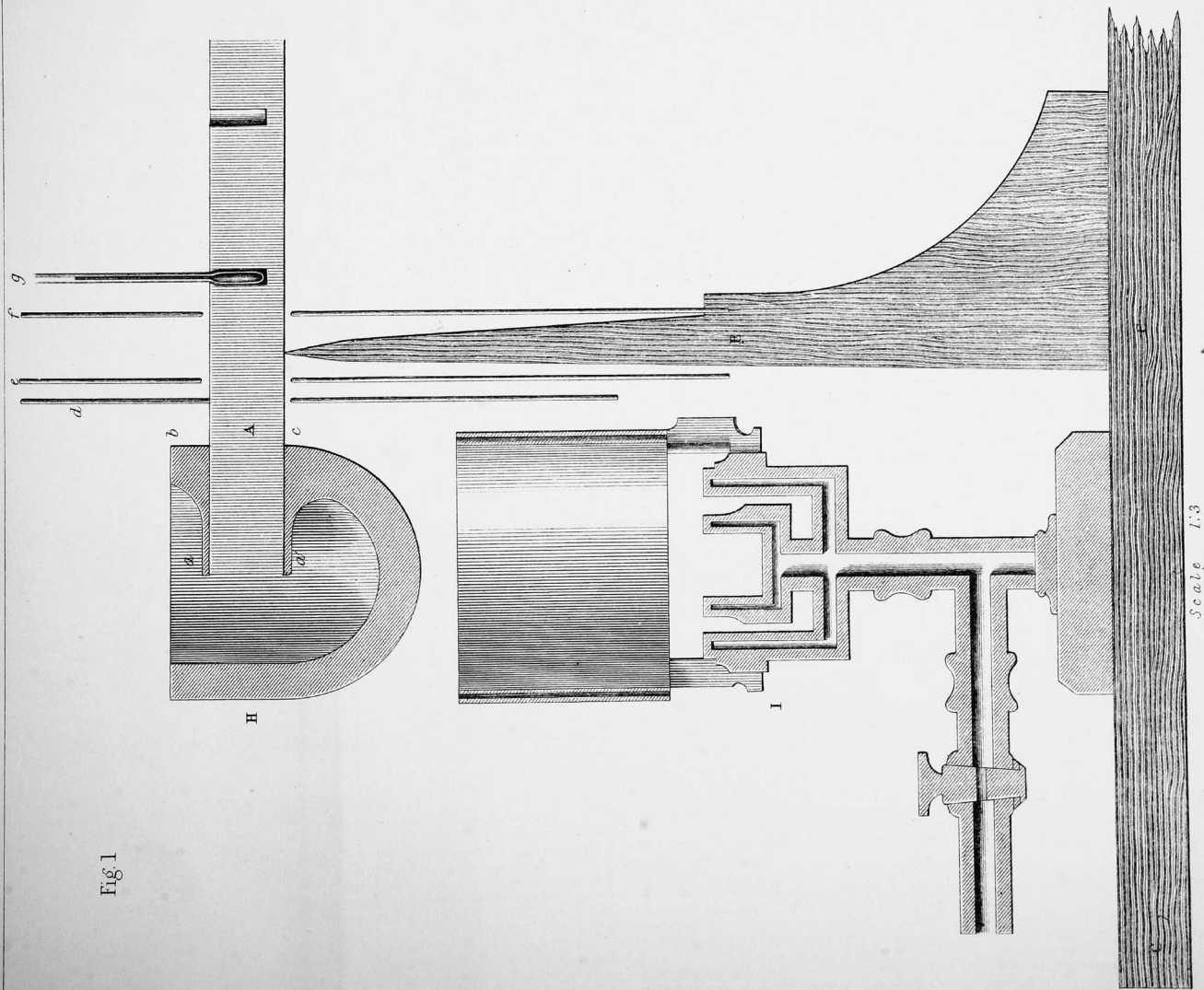
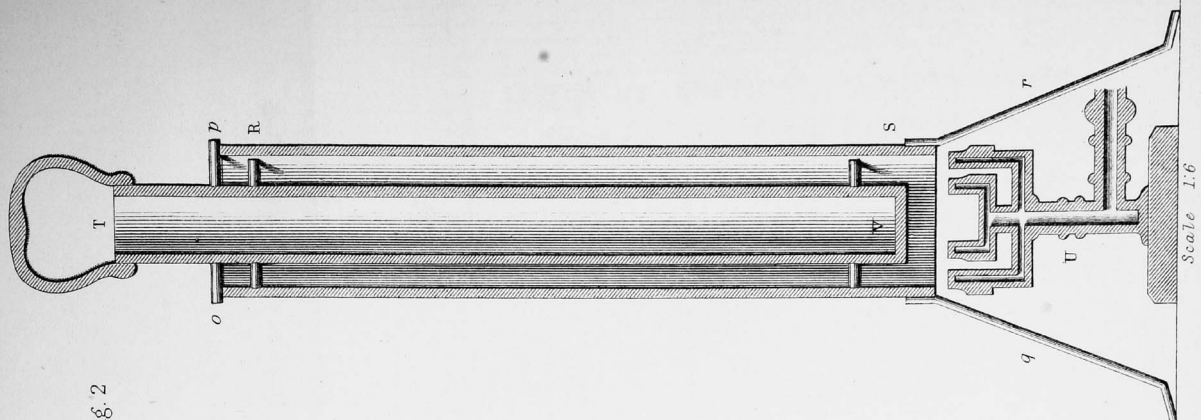


Fig. 2



the same temperature.* The gas flame and the violently heated currents of air thence arising were prevented from playing against any part of the bar by a piece of metal plate fastened by wire to the crucible against the face *ab* (but to prevent confusion not shown in the figure), while the whole conduction bar was farther protected from heated currents, and from radiation from the crucible and gas chimney, by three polished metal screens *d, e, f*, placed parallel to one another, two to the left and one to the right of the wooden support E. The square apertures in the screens were 0·25 inch wider than the dimensions of the bar, and they were supported in such a manner as not usually to touch it in any part. These screens very effectually defend the thermometers, as well as the bar, from extraneous heat. The first thermometer only—that at 3 inches distance from the zero line *bc*—is seen at *g* in the section, fig. 1.

47. The conduction-bars have already been described in Arts. 17, 18. The more perfect one was 1·25 inch square, and fully 8 feet long, reckoning from the zero line above mentioned. It was used in two states, *first*, with a naked polished surface, and, *secondly*, when coated with thin paper. The other bar, also of wrought-iron, was 1 inch in the side and 7 feet long. In the present paper I shall discuss separately the results obtained with these two bars, presenting three quite independent cases, but which, as they ought to lead to an identical value of the conductivity of iron (assuming the quality of the bars to be alike), put the method here proved to a severe trial.

48. Throughout the remainder of this paper, when I speak of Case I., I mean the 1½-inch bar with moderately *polished* surface; Case II. is the same bar with *paper* surface; Case III. is the 1-inch bar with naked, but less brightly polished surface.

49. The thermometers were inserted in holes in the bar 0·28 inch diameter and about $\frac{3}{4}$ inch deep. They were surrounded by mercury or (in the hotter holes) by fusible metal. (See Art. 20, and also Plate II. fig. 1.) Nine or ten thermometers were usually employed, and in the case of the principal bar (Cases I. and II.) they were usually (though not always) spaced as follows, reckoning from the zero line *ab* on the face of the crucible :—

0·25, 0·5, 0·75, 1, 1·5, 2 or 2·5, 4, 6, 8 feet.

50. The method of using a single standard thermometer for obtaining final results by the method of *stepping*, with its advantages, have been fully explained at Art. 22.

51. “The *free temperature*, or that to be deducted from the readings of the thermometers, in order to get the true excess of statical temperature along the bar, was obtained by inserting a well-compared thermometer into a hole containing mercury, drilled in a similar but short bar of iron, supported in the free air of the

* See, however, note to Art. 70, below.

room in the neighbourhood of the long bar, and similarly exposed, but without artificial heat." (Art. 23.) The arrangement is shown in Plate I. fig. 2.

52. The gas furnace was usually lighted about 8 A.M., when the lead in the crucible was gradually melted. The readings of the thermometers were not recorded until about four hours had elapsed, and the experiment seldom lasted altogether less than eight hours, generally ten or eleven hours. It was difficult to keep the flame of the gas furnace steady, the "regulator" used for the purpose being apparently of little use. The lead in the crucible, after being quite fluid, sometimes solidified a little over the interior flange which grasps the bar. I at length found the best way of regulating the temperature to be, to keep the eye very constantly on the first thermometer in order, and whenever the slightest rise commenced, to dip a little cold lead into the crucible, and either let it melt or chill the mass by its contact, or the gas might be cautiously lowered. If the temperature was seen to be falling, the gas had to be raised. With experience I learned to keep the temperature of the three-inch hole within a range of 2° Cent. under favourable circumstances, the temperature being nearly 200° Cent. In some experiments in which solder was employed instead of lead, I used a thermometer whose bulb dipped into the crucible, where it stood about 460° Fahr.* My able assistant, Mr JAMES LINDSAY, learned to regulate this with great nicety.

53. When the temperature had been for a long time quite steady at the three-inch (or hottest) hole, the thermometers, disposed, as has been explained, along the bar, were successively read, and the readings recorded. This was done from left to right along the bar with all deliberation, without regard to any possible change during the process in the temperature at the hot end of the bar, since any such change is comparatively slowly propagated along the bar. In like manner, in "stepping" with one thermometer from point to point of the bar (Art. 2), a slight change in the temperature of the source is immaterial, since the "stepping" is performed faster than the wave of heat can follow.

54. A careful examination of the whole record of simultaneous readings was made, and those corresponding to the most stationary conditions of temperature were selected; and these being corrected for scale errors and temperature of column (Arts. 42, 43) were entered, after the free temperature indicated by the little bar (Art. 51) had been deducted, in the following tables as the Corrected Excesses of Statical Temperatures in centigrade degrees.

55. Looking cursorily over Table I., we may observe, *First*, that each day's observations are comparable only amongst one another, as no *exact* coincidence of the temperature of the crucible, or source of heat on different days, was attempted. *Secondly*, the bracketed observations are made with independent thermometers. *Thirdly*, comparing the first series for the bar covered with paper

* By an oversight in the first part of this paper (Art. 19, *note*), it was stated that in this instance the thermometer was dipped in melted lead.

TABLE I.—CORRECTED EXCESSES OF STATICAL TEMPERATURE, ORIGIN AT OUTER SURFACE OF CRUCIBLE—DEGREES CENTIGRADE.

(The numbers bracketed together were obtained with different thermometers.)

	3 inch.	6 inch.	9 inch.	1 ft. 0 in.	1 ft. 6 in.	2 ft. 0 in.	2 ft. 6 in.	3 ft. 0 in.	4 ft. 0 in.	5 ft. 0 in.	6 ft. 0 in.	7 ft. 0 in.	8 ft. 0 in.
CASE I.—1½ inch Iron Bar, naked. 1851, March 7 (Series A.)	191.1	134.6	97.7	72.2	March 7. Series B. { 40.9 { 40.6*	24.6	° ...	9.7	° ...	1.88	° ...	0.53	0.34
March 8,	192.3	135.3	97.95	{ 72.4 { 72.4	{ 40.9 { 40.6*	{ 24.1* { 24.15	° ...	9.5	° ...	1.78	° ...	0.43	0.28
March 14,	194.6	137.65	{ 99.65 { 99.45	73.4	41.35	{ 24.25 { 24.15	° ...	{ 9.05 { 8.9*	° ...	{ 1.70 { 1.61*	°	{ 0.25* { 0.35 { 0.45
(Standard) April 11,	191.0	{ 134.7 { 134.3	{ 97.15 { 97.4 { 97.25	71.85	{ 40.9 { 40.65*	° ...	{ 14.8 { 14.8	° ...	{ 4.05 { 4.05	° ...	{ 0.95 { 0.95	° ...	{ 0.30 { 0.28
CASE II.—1¼ inch Iron Bar, covered with Paper.													
(Standard) 1851, April 15, . .	162.9	{ 105.7 { 105.65	{ 71.45 { 71.45	49.35	{ 25.0 { 24.65*	[13.0?]	{ 7.08 { 6.98	[3.85?]	{ 1.28 { 1.28	0.50	{ 0.18 { 0.18	° ...	0.0
April 16,†	{ 120.55 { 120.3 { 120.5	{ 80.0 { 80.15	{ 54.95 { 54.65 { 54.8*	{ 38.4 { 38.15*	19.45	10.3	{ 5.65 { 5.65	° ...	{ 1.10 { 1.10	° ...	{ 0.15 { 0.15	° ...	0.0
CASE III.—1 inch Iron Bar, naked. 1850, Dec. 21,	149.9	100.6	69.95	...	25.62*	...	10.2	7.65	1.25	{ 1.0 { 1.0	0.78	0.07	
1851, Jan. 11 (Series a.)	156.6	105.1	72.42	50.8	...	19.0	...	7.62	1.02	{ 0.77 { 0.86	0.52	0.0	
(Standard) ... (Series b.)	159.0	{ 105.75 { 105.8	72.8	51.0	...	19.4	...	7.83	1.2	{ 0.92 { 0.92	0.67	0.0	

* Defective thermometer, or otherwise less reliable observations.

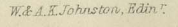
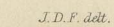
† The crucible was this day filled with a mixture of lead and tin (solder, which melts at a much lower temperature than pure lead), heated considerably above its melting-point.

(April 15) with the last of the same bar naked (April 11), which were made in almost similar circumstances, we notice the prodigious effect of the increased radiation due to the paper casing. Though the heat at the origin may be considered as the same, at a distance of only three inches the temperature in the second case was less by nearly 30° Cent.; at thirty inches distance, the proper heat of the bar was but *one-half* of what it was in Case I.; at four feet, *one-third*; and, at eight feet, it had vanished in the second case, while it was still sensible in the first. *Fourthly*, it may be remarked that in Case I. the bar scarcely fulfilled, as to length, the implied condition of the experiment, which assumes that the extreme end of the bar shall be sensibly of the temperature of the air. As $0^{\circ}.3$ of heat remained at eight feet, and as the bar extended only a few inches beyond that point, it would appear that the conducted heat was not absolutely dissipated by radiation. The effect, which is to render the decrement of heat in the extreme holes rather too slow, is, however, barely appreciable in the deductions.

56. *Graphical Interpolation of the Statical Experiments.*—As it was desirable to combine the results of the independent experiments in each of the three Cases, and to deduce the most probable temperature for any point of the bar, a graphical method was adopted as follows:—Large sheets of drawing-paper were provided covered with engraved squares one-tenth of an inch in the side. A horizontal line was taken to represent the distances reckoned along the bar on a scale of four inches to a foot, and at the proper intervals the observed temperatures (or rather excesses of temperature) were set off as vertical ordinates on a scale of 10° to 1 inch. The general arrangement of the observations in this way is shown in Plate III. on a reduced scale for Case I.

57. It is plain, however, that this primary projection could only apply to a single and comparable series of observations under each Case of Table I., since the temperature of the origin might vary from one experiment to another. One set under each Case was assumed as a standard series to which the others were to be referred. In Case I., April 11; in Case II., April 15; in Case III., January 11 (*b*). But as it is plain that for one and the same bar the curve of temperature has the same form, though it may deviate in position to the right or left along the bar, each of the other days' observations was separately projected on transparent cloth, and then laid on the engraved squares over the first projection. By moving the system of projected points to the right or left (taking care to keep the line of abscissæ in each case accurately coincident), a position was easily found where the points to be interpolated accommodated themselves best to the general curvature of the fundamental series.

58. This method of interpolating independent series belonging to different fundamental temperatures has very great advantages. Had circumstances allowed me to continue these observations, I should have applied it more extensively. A clear instance of its utility will be seen by comparing the first and second row



of figures in Case II. of Table I. The observations of April 16 were made with melted solder as a source of heat, which fuses at a much lower temperature than lead. The result is, that the temperatures in the 3, 6, 9 inch holes and those which follow, are *intermediate* between the temperatures shown in those holes in the other experiment. Thus, by varying the temperature of the source of heat, we may multiply indefinitely points in the curve without increasing the number of holes with which the bar is pierced, which is evidently undesirable. It would have been very serviceable for the interpolation of the numbers in Case I., had the temperature of the origin been expressly varied for this purpose.

59. The general agreement of the independent interpolated observations has been highly satisfactory, as may be seen from Plate III., where the several sets of temperatures belonging to Case I. are distinguished by marks.

60. A continuous curve was next to be drawn through the extremities of the ordinates, so as best to conciliate the whole of the observations. To draw this curve was a matter requiring great nicety and judgment, owing to the limited number of ordinates disposable. It is well known to every one who has used such projections, that to draw an interpolating curve advantageously requires that the rate of increment of the two variables shall not be *excessively* unequal. In curves like those of Plate III., which rise very rapidly at one end, and become almost or quite asymptotic at the other, it is indispensable to make subsidiary projections of different parts of the curve, in which the relative scale of the vertical and horizontal co-ordinates shall be altered. For the part of the curve between 0 and 2 feet, the temperatures had to be contracted in scale and the abscissæ expanded; while for the right-hand branch of the curve the contrary was done, even to the extent of magnifying the vertical scale of degrees tenfold, whilst the horizontal scale of feet was diminished fourfold, compared with the first projection. For each of the three Cases (Art. 48), the statical curve was thus subdivided and partially projected on four different scales, three of which are exhibited on the engraved Plate. The result of this close analysis and comparison has been highly favourable to the assurance of accuracy in the final results, since the interpolated temperatures for any abscissæ are the result of two, if not three, projections of the observations on different relative scales. No numerical or other casual error could thus possibly escape detection.

61. I believe that these curves, as now obtained with the ordinates immediately to be given, are favourable specimens of numerical accuracy and geometrical definition, considering the difficulties attending the experiments. Throughout a great part of the curves (and that by far the most important for the results), the temperature excesses of the bar, or vertical ordinates, may, I hope, be esteemed correct within a very small fraction of their amount. The following table (which may be regarded as summing up the whole Statical data) contains the ordinates of the curves corresponding to the three Cases of Art. 48.

TABLE II.—STATIONARY EXCESSES OF TEMPERATURE (FROM ALL THE PROJECTIONS)
ADOPTED.

Distance from Origin at Crucible.	EXCESS OF TEMPERATURE (CENTIGRADE) OF BAR ABOVE AIR.		
	CASE I. 1½ inch Bar, naked.	CASE II. 1½ inch Bar, covered.	CASE III. 1 inch Bar, naked.
0 inch	275·5 c.	260·5 c.	282·2 c.
1	242·9 c.	221·7 c.	243·2 c.
2	214·8 c.	189·5 c.	210·2 c.
3	190·5	162·9	182·2 c.
4	168·8*	140·0*	159
5	150·6*	121·5	137·5*
6	134·7	105·9	120·5*
7·5	114·1*	86·6	99·8
9	97·3	71·3	82·8*
10·5	84·0*	59·0	68·6
I. foot 0 inch	72·0	49·2	57·1
3	53·6	34·5	40·9*
6	40·8	24·6	29·5
9	31·0	17·7	21·6
II. feet 0	24·2	13·0	15·65*
3	18·9*	9·45	11·5*
6	14·8	7·0	8·55*
III. „ 0	9·33	3·8	4·95*
6	6·15*	2·1*	2·78*
IV. „ 0	4·0	1·28	1·56
V. „ 0	1·8	0·47	0·55
VI. „ 0	0·9	0·165†	0·13*
VII. „ 0	0·50
VIII. „ 0	0·28†

The numbers marked c. are derived from calculation. See Art 68 below.
The numbers marked thus * belong to points in the curve not closely adjacent to points of observation; and, therefore, are less certain than the others.
† 0·32 by mean of 7 observations. ‡ Mean of 4 observations.

Adjacent to the principal curve of Plate III. is a dotted curve, which exhibits the remarkable change of character in the curve when the bar is coated with a highly radiating surface of paper (Case II., Art. 48).

62. *Formulae of Interpolation for the Statical Curves.*—It was not originally my intention to have entered on the thorny enterprise of seeking equations to satisfy the statical curves of temperature. My original plan (Art. 6 of former paper) was to deal with Curves alone, or almost entirely. And when we do not wish to exceed the limits of direct observation, it is perhaps the safest, as well as by far the easiest plan. I wished, however, to throw all possible light on the problem, for the benefit of those who may hereafter extend these observations. I also wished to obtain the greatest amount of information from the data at my disposal; and by means of formulæ, to extend the results somewhat (though not far) beyond the limits of observation. It will

be seen farther on (Art. 71) that a formula, however empirical, enables us to execute promptly and with confidence the otherwise tentative and uncertain process of drawing tangents to the curve in its higher part, in other words, of obtaining values of $\frac{dv}{dx}$ upon which the final deductions mainly depend. (See Arts. 6 and 31.)

63. It had been acutely perceived by LAMBERT nearly a century ago,* that the temperatures of a bar heated in the manner of our experiment would diminish in a regular geometrical progression. A more rigorous analysis led BIOT† and FOURIER‡ to the same result, according to the physical assumptions with which they started. BIOT and DESPRETZ§ subjected various metallic bars to experiment, but they assumed the logarithmic law to be true, and endeavoured to accommodate their numerical results to it, as well as they might. BIOT, in particular, applied to his (apparently excellent) observations, the method of least squares to enable him to draw a logarithmic curve through his points of observation, giving no attention to the fact, that the temperatures found did not conform themselves by any means to the *a priori* geometrical law, and that the laws of Probability could not be applied to them without ascribing extravagant and improbable errors to a large part of the curve of observation.||

64. That the temperatures deviate systematically from the law of continued progression, will appear from the following table of the ratios of successive ordinates, taken three inches apart in the three Cases of Table II.

MEAN RATIO¶ BETWEEN TWO CONSECUTIVE ORDINATES 3 INCHES APART, FROM THE NUMBERS IN TABLE II.

INTERVALS.	CASE I.	CASE II.	CASE III.
3 to 6 inches, . . .	·707	·650	...
6 to 9 „	·722	·673	·687
9 inches to I. foot, . .	·733	·690	·690
I. foot to I. foot 6 inches,	·753	·707	·719
I. foot 6 inches to II. feet,	·770	·727	·728
II. to III. feet, . . .	·787	·735	·750
III. to IV. „	·809	·762	·755
IV. to VI. „	·830	·774	·731

* Pyrometrie. Berlin, 1779, p. 185.

† Traité de Physique, vol. iv. p. 669.

‡ Théorie Analytique de la Chaleur. 1822.

§ Traité Élémentaire de la Physique. 1836, p. 197.

|| Compare Note to Art. 3 of this paper.

¶ By “mean ratio,” I intend to express, that where more than one 3-inch space is included in the Interval specified in the first column, the number which follows is the average decrement throughout that space. Thus, in Case I. the whole interval from II. to III. feet, shows a decrement from 24°·2 to 9°·33, which would result from the mean ratio of 0·787, four times multiplied into itself.

65. It will be observed that in every instance, with the single exception of the final number of the Table under Case III., the decrement of temperature becomes materially slower as we recede from the heated end of the bar. The exception is of little weight, as it depends on the residual temperature of the one-inch bar, six feet from the crucible, at which point the warmth was barely perceptible. The statical temperatures of the bar therefore increase more rapidly than in a geometrical progression. I have found that there is a sufficient analogy between the curve of statical temperature which we are here investigating, and that of the tension of steam at different temperatures, to afford some assistance in the selection of empirical formulæ in the present instance; being in each case a modified geometrical progression, though here the progression of ordinates is more rapid than a simple continued proportion, while in the tension of steam it is less rapid.

66. The most complete discussion of this class of formulæ, and the methods calculating from them, is to be found in M. REGNAULT'S *Relation des Expériences sur la Vapeur*. The available formulæ are reducible to three; YOUNG'S,* ROCHE'S,† and BIOT'S.‡ The two former contain three constants, the latter five. The last has been found the most satisfactory for the empirical representation of the elasticity of steam; and it is the only one of the three which can be regarded as applicable throughout the entire limits of experiment. The same is, I believe, true for the Conduction-curves with which we are now occupied. With five constants, five points of the curve may be accurately represented, and the intermediate deviations are of course inconsiderable. As none of the formulæ (except the simple logarithmic which is found to be inapplicable) have any foundation in principle, the whole matter is purely one of convenience. For simplicity's sake, I used only the formulæ of YOUNG and ROCHE, but I now think that the greater labour involved in the application of BIOT'S formula would have been repaid by the directness and certainty of the results. M. REGNAULT has given rules to facilitate its numerical calculation.

67. I have found the formula,—

$$\log v = A - \frac{bx}{1 + cx} \quad . \quad . \quad . \quad . \quad \text{Eq. (1.)}$$

(where v is the excess of temperature above the air at a point of the bar, whose abscissa, *in feet*, is x , and A , b , and c are constants), to represent tolerably well the temperature curve of Case I., as represented by the numbers in Table II.,

* $p = A(1 + at)^n$, where p is the elasticity, and t the temperature.

† $\log p = \log a + \frac{bt}{1 + ct}$.

‡ $\log p = a + b\alpha' + c\beta'$.

throughout the whole extent of the experimental curve.* But in order to follow the observations more closely, it seemed desirable to divide the curve into two parts, one between 0 and 1·5 feet, and the other beyond 1·5 feet, and to employ distinct constants for each. A like process was applied to the numbers of Table II., for Cases II. and III. In these, the variations of temperature along the bar being more rapid, the approximation of the formulæ was less exact than in the first case, and a formula with three constants was insufficient to represent the curve throughout its whole extent. I found it advisable, in the case of the bar covered with paper (No. II.), to use modified formulæ for the upper, middle, and lower part of the curve. I may add that it was found convenient to adapt the formula (Eq. (1.) of this article) to the calculation of the lower temperatures, by changing the origin to an arbitrary point some feet to the right, and by reckoning the abscissæ in the opposite direction, thus rendering the second term of the equation positive instead of negative. To this end the equation was written,—

$$\log v = A + \frac{bz}{1 + cz},$$

when

$$z = n - x.$$

68. The coincidence of the various formulæ with the experimental numbers of Table II. is shown in the foregoing Table.

* The formula in this case would be,—

$$\log v = 272.7 - \frac{.63374x}{1 + .0956x}.$$

The following adaptation of YOUNG's formula also represents the observations in Case I. very approximately.

$$v = (.43027 + .09539x)^{-6.65}.$$

<i>z</i> in feet.	<i>v</i> by Experimental Curve.	<i>v</i> by Formula (<i>a</i> + <i>bx</i>) ^{<i>p</i>} .	Difference.	<i>v</i> by Formula $\log v = \log a - \frac{bx}{1+cx}$	Difference.
0	...	272.66	...	272.7	...
0.25	190.5	190.5	0.0	191.0	+ 0.5
0.5	134.7	135.5	+ 0.8	135.9	+ 1.2
0.75	97.3	98.04	+ 0.74	98.23	+ 0.93
1.0	72.0	72.0	0.0	72.0	0.0
1.25	53.6	53.6	0.0
1.5	40.8	40.41	− 0.39	40.21	− 0.59
2.0	24.2	23.75	− 0.45	23.53	− 0.67
2.5	14.8	14.52	− 0.28
3.0	9.33	9.18	− 0.15	9.08	− 0.25
4.0	4.0	4.0	0.0	4.0	0.0
5.0	1.8	1.91	+ 0.11	1.96	+ 0.16
6.0	0.9	1.00	+ 0.10
8.0	0.28	0.31	+ 0.03	0.36	+ 0.08

TABLE III.—SHOWING THE COMPARISON OF THE GRAPHICAL CURVES OF STATICAL TEMPERATURE WITH DIFFERENT FORMULÆ OF INTERPOLATION.

Distance from Origin in		CASE I.				CASE II.				CASE III.				
Inches and Feet.	Feet and Decimals = x.	ϕ by Exptm. Curve.	Form. A.	Form. B.	Differ-ence.*	ϕ by Curve.	Form. C.	Form. D.	Form. E.	Differ-ence.*	ϕ by Curve.	Form. F.	Form. G.	Differ-ence.*
0 inch	0	...	275.5	260.5	282.2
1 "	0.083	...	242.9	221.7	243.2
2 "	0.166	...	214.8	189.5	210.2
3 "	0.25	190.5	190.5	...	0.0	162.9	162.9	162.9	...	0.0	...	182.2
4 "	0.333	168.8	169.3	...	+0.5	140.0	140.6	140.6	...	+0.6	159.0	158.4	...	-0.6
5 "	0.416	150.6	150.9	...	+0.3	121.5	121.9	121.9	...	+0.4	137.5	138.2	...	+0.7
6 "	0.5	134.7	134.7	...	0.0	105.9	105.9	[106.0]	...	0.0	120.5	120.8	...	+0.3
7.5 "	0.625	114.1	114.2	...	+0.1	86.6	86.52	[86.53]	...	-0.08	99.8	99.36	...	-0.44
9 "	0.75	97.3	97.3	...	0.0	71.3	71.32	[71.14]	...	+0.02	82.8	82.18	...	-0.62
1 ft. 0 "	1.0	72.0	71.6	...	-0.4	49.2	[49.48]	49.08	...	-0.12	57.1	57.19	[56.8]	+0.09
3 "	1.25	53.6	53.6	53.6	0.0	34.5	...	[34.7]	34.4	-0.1	40.9	40.67	[40.9]	-0.23
6 "	1.5	40.8	40.77	40.75	-0.04	24.6	...	[25.1]	24.64	+0.04	29.5	29.52	[29.61]	+0.02
2 ft. 0 "	2.0	24.2	[24.61]	24.17	-0.03	13.0	12.94	-0.06	15.65	...	15.76	+0.11
6 "	2.5	14.8	...	14.80	0.0	7.0	7.00	0.0	8.55	...	8.55	0.0
3 ft. 0 "	3.0	9.33	...	9.33	0.0	3.8	3.89	+0.09	4.95	...	4.72	-0.23
4 ft. 0 "	4.0	4.0	...	4.00	0.0	1.28	1.29	+0.01	1.56	...	1.52	-0.04
5 ft. 0 "	5.0	1.8	...	1.87	+0.07	0.47	0.47	0.0	0.55	...	0.52	-0.03
6 ft. 0 "	6.0	0.9	...	0.95	+0.05	0.165	0.185	+0.02	0.13	...	0.189	+0.06
7 ft. 0 "	7.0	0.50	...	0.51	+0.01
8 ft. 0 "	8.0	0.28	...	0.279	0.00

* The bracketed numbers are not used in taking the differences.

* The bracketed numbers are not used in taking the differences.

69. The formulæ used in the preceding calculations are the following :—

$$\begin{aligned}
 \text{CASE I.} & \left\{ \begin{array}{l} \text{(A) } \log v = \log 275.5 - \frac{.66184x}{1 + .13093x} \\ \text{(B) } \log v = \log 4.0 + \frac{.3472z}{1 - .0556z}, \text{ where } z = 4 - x. \end{array} \right. \\
 \text{CASE II.} & \left\{ \begin{array}{l} \text{(C) } \log v = \log 260.5 - \frac{.85265x}{1 + .1819x} \\ \text{(D) } \log v = \log 259.08 - \frac{.83855x}{1 + .1606x} \\ \text{(E) } \log v = \log 0.47 + \frac{.4217z}{1 - .0405z}, \text{ where } z = 5 - x. \end{array} \right. \\
 \text{CASE III.} & \left\{ \begin{array}{l} \text{(F) } \log v = \log 282.2 - \frac{.7872x}{1 + .1275x} \\ \text{(G) } \log v = \log 0.52 + \frac{.4521z}{1 - .0282z}, \text{ where } z = 5 - x. \end{array} \right.
 \end{aligned}$$

70. With reference to the preceding numerical Table, I may remark, *First*, that the differences shown are not in all cases deviations from *direct* observations, but between the formulæ and the graphical interpolation of the data. There is a difficulty (which will be understood from Art. 57) in comparing compendiously the formulæ with the *single* data of Table I. When the points of the curve are somewhat distant from points of observation, the numbers in the preceding Table, obtained from the formulæ, may be, and probably are more reliable than those assigned from the curve. *Secondly*, the curve of Case I. appears to be the most reliable in all respects. And in particular I consider the portion of the curve which includes the highest temperatures, or those corresponding to points on the bar between 0 and 3 inches, to be very nearly accurate. From numerous independent calculations, I conclude that the value of v at the origin, or in contact with the crucible, is pretty exactly $275^{\circ}.5$ Cent., as there assigned. If we add to this $12^{\circ}.5$ for the approximate temperature of the apartment, we have 288° for that of the bar where it enters the crucible, and is supposed to have very nearly the temperature of melting lead. This is a considerably lower temperature than is usually attributed to melting lead.*

* Usually stated at from 320° to 330° Cent., 608° to 626° Fahr. Biot, indeed, gives it as only 260° , inferentially derived from his conduction experiments (*Traité de Physique*, iv. 677); but this is on the supposition of the logarithmic law prevailing. CRICHTON, junior, gives $606^{\circ}.5$ Fahr. (T. Thomson); DANIELL, 612° ; KUPFFER, 633° . Supposing any of these last numbers to be correct, the inference must be, that in the conduction experiments described in the present paper, the temperature of melting lead did not extend to the outside of the iron crucible when the origin of the co-ordinates has been taken, but must be sought somewhere in the interior. This conclusion is strengthened by some other, though indirect considerations.

71. I cannot too distinctly repeat that the formulæ adopted in the preceding Table are only to be regarded as a means of more conveniently grouping the observations. The most important use to be made of these formulæ, however, yet remains to be mentioned. It will be seen by reference to Arts. 6, 28, &c., of the former part of this paper, or to § IV. of the present paper, that it is not the ordinates themselves of the statical curve of cooling which are to be used in obtaining the conductivity of the bar, but the values of the differential coefficient $-\frac{dv}{dx}$ for each part of the bar. In other words, we must be able to draw a tangent to the curve of statical temperature at any point of the curve. This may be roughly done mechanically, or it may be done by dividing the curve into short elementary portions, and treating each portion as if it were part of a logarithmic curve (see below, Art. 82, on the Analogous Treatment of the Dynamical Curve); or, finally, it may be obtained from the equations above given. The two last methods have been used in the reductions, and especially the last of all, which is the only satisfactory one for the higher parts of the statical curve. The general form of the empirical equation being,

$$\text{Tab. log } v = A - \frac{bx}{1 + cx}$$

when reduced to Napierian logarithms, gives

$$0.4343 \text{ hyp. log } v = A - \frac{bx}{1 + cx}$$

and

$$\frac{dv}{dx} = -2.3036 \frac{bv}{(1 + cx)^2}$$

whence the numbers which will be given in § IV. of the present paper are computed, the values of b and c being taken from the formulæ of Art. 69.

§ II. *Experiments on Cooling.*

72. *The Apparatus.*—It will be seen, by reference to the former part of this paper (Arts. 5, 24), that, in order to interpret the indications of the permanent temperature of a bar, and to deduce its conductivity, we must have an independent set of observations on the cooling of a similar bar, or a portion of a similar bar. For this purpose, the apparatus shown in fig. 2 of Plate I. was employed. The same short bar, LM, which has been already referred to (Art. 51), as being used in the statical experiment for determining the temperature the bar would have had independently of the heat applied at one end, was supported on the props N, O, after being duly heated. It is now to be used to ascertain the rate of loss of heat from a bar having the Section and Surface proper to each of the three Cases of Art. 48, in terms of the scale of the thermometer P, inserted at or near its middle point.

73. I have so fully described, in Art. 24, the manner of performing the

Cooling experiment, that I need here do little more than refer to the figures by which it is now illustrated, and give the corrected results as to the "law of cooling."

74. Fig. 2 of Plate I. shows the small iron bar employed, which in Case I. and Case II. (Art 48.) was 20 inches long and $1\frac{1}{4}$ inch square, first naked and polished, and afterwards covered with paper; it was marked C. In Case III. it was a polished (or at least a bright) bar, 20 inches long, 1 inch square, and marked E. Each bar had a ring at each end, *l*, *m*, and could be handled by seizing either end by the hook Q, fig. 3. Having been covered with several folds of stout paper to prevent a sudden chill of the metal bath into which it was to be introduced, it was lowered vertically and lengthwise into the cylindrical iron vessel shown at fig. 3, and in section in Plate II. fig. 2. It consists of a stout iron tube TV, about two feet long, with a bottom at V, and a handle at T. It rests by means of two iron pins, *o*, *p*, on the upper edge of a cylindrical iron chimney RS, supported by three feet, of which two are seen at *q* and *r* over the gas furnace U, the powerful flame of which, playing between the two cylinders, keeps a quantity of solder or of "fusible metal" in the interior one, not only melted, but heated considerably above the melting point. The bar under experiment, after being coated with several folds of paper, having usually also been well warmed over a hot-air stove, was introduced by the hook Q into the metal bath, then turned end for end several times, until it was believed that the heat had well penetrated its entire thickness. It was then withdrawn, shaken, the paper covering rapidly cut off, the bar wiped with a cloth,* and placed horizontally on the two ivory-topped props N, O (Plate I. fig. 2), the thermometer P inserted in the central hole,† into which heated mercury had already been placed, and the reading of the thermometer from minute to minute immediately commenced, the times being given by an assistant. The free temperature was determined by a thermometer sunk in a cold bar in the neighbourhood, or by one suspended in the air, or by both.

75. *The Observations.*—As in the statical observations there are three cases.

Case I. Iron bar, $1\frac{1}{4}$ inch square, roughly polished.

Case II. Do. do. covered with paper.

Case III. Do. 1 inch square, roughly polished.

76. Two independent sets of observations of the law of cooling on different days have been obtained for each case. Moreover, as more than one thermometer was observed in the holes of each bar (as in the example which follows), except for the very highest temperatures, use has from time to time been made of these auxiliary series. The whole of these observations have been most carefully corrected for the

* The *wiping* of the bar I believe to have been unnecessary and injurious. It lowered the temperature, and interfered with the distribution of the heat in the bar.

† The $1\frac{1}{4}$ inch bar had a central hole, and others 1.5 inch distant, right and left. The 1 inch bar had only two holes equidistant from the centre of the bar.

scale errors and for the temperature of the stem. Where the temperature of the air of the room has not been quite steady, the variations have been interpolated and allowed for in deducing the excess of temperature of the bar above that of the room.

77. I shall give the details of one experiment as a specimen (all reductions being first made).

TABLE IV.—COOLING OF SHORT 14-INCH BAR, C.—26TH MARCH 1851.

Hour.	Hole (2), Centre. Corrected Excess.	Hole (1), to the Left. Corrected Excess.	Hole (3), to the Right. Corrected Excess.	Hour.	Hole (2), Centre. Corrected Excess.	Hole (1), to the Left. Corrected Excess.	Hole (3), to the Right. Corrected Excess.
h. m. s.	°	°	°	h. m. s.	°	°	°
12 59	170·7			2 1		55·20	
1 0	167·4			2 2	54·15		
1 1	164·0			3 3			53·2
2 2	160·85			4 4	52·4		
3 3	157·6			5 5		51·65	
4 4	154·5			6 6	50·6		
5 5	151·45			7 7			49·75
6 6	148·45			8 8	49·0		
				9 9		48·35	
				10 10	47·4		
12	131·7			21	40·1	40·15	
13	129·2			22	39·35		39·40
14	126·6			24	38·25	38·20	
15	124·2			26	37·10		37·10
16	121·7	119·85		28	36·0	36·05	
17							
18	116·95			48	26·9	26·95	
19			114·25	50	26·15		26·2
20	112·65			52	25·4	25·4	
21		110·85		54	24·6		24·6
22	108·35						
23			106·0	3 31	14·95	14·92	
24	104·5			34	14·35	14·32	
25		103·05		36	14·0		13·92
26	100·65			38 30	13·40		13·25
29			95·10				
30	93·75			4 10	8·98	9·00	8·92
31		92·05		15	8·48	8·50	8·41
32	90·25			20	7·98	7·9	7·8
33		88·9		25	7·53	7·55	7·44
34	87·0						
35			85·5	6 0*	2·79	2·6	
36	84·10			10*	2·54	2·4	
37			82·5	20*	2·15	2·1	
38	81·25			30*	2·0	1·9	
39		79·95					
40	78·50			8 10*	0·9	0·7	
				20*	0·9	0·7	
2 0	56·10			30*	0·85	0·7	

* Read by an assistant. The scale of the thermometer in hole (3) being liable to mistake in reading, its results are omitted.

78. *Graphical Interpolations.*—The observed excesses of temperature (as obtained, for example, in the preceding experiment for the central hole) were projected in a curve of which the times were taken as abscissæ, and the independent temperatures of the bar as ordinates. When more than one series of observations (on the same bar at different times) were to be combined, a procedure exactly similar to that described in Art. 57 for the stationary temperatures was employed; that is to say, one series being first projected on the engraved paper as fundamental, any other series was next similarly projected on tracing cloth, and the system of points thus obtained was moved to the right or left over the first, until the points in the two curves appeared to be superposed satisfactorily. The interpolated observations were then pricked through, and a curve drawn through the whole.

TABLE V.—CURVES OF COOLING (IN TERMS OF TIME).

Time from Arbitrary Origin.	CASE I. 1½ inch Bar.		CASE II. 1¼ inch Bar, covered.	CASE III. 1 inch Bar.
	March 26.	March 29.		
— 10 Min.	...	*242.3
— 5	...	*221.4	*263.9	...
— 2.5	...	*211.5	*243.6	...
0	...	*201.9	*225.0	*258.5
2.5	...	192.0	*207.7	*243.3
5	...	183.5	*191.8	*228.9
7.5	...	174.8	177.0	*215.3
10	167.3	166.5	163.6	202.4
12.5	159.1	158.2	150.6	190.05
15	151.4	150.6	139.1	178.7
20	137.0	136.5	119.25	157.55
25	124.2	124.2	102.95	138.9
30	112.6	112.8	88.8	122.45
35	102.6	102.5	77.1	108.6
40	93.7		67.0	96.45
50	78.6		50.95	77.25
60	65.9		39.4	62.2
70	55.8		30.65	50.5
80	47.3		24.25	41.3
90	40.5		19.2	33.95
100	34.9		15.27	28.2
125	24.2		8.9	18.0
150	17.1		5.45	11.95
175	12.3		3.42	8.1
200	8.95		2.15	5.55
300	2.95		0.4	1.4
400	1.12		...	0.4

The numbers marked thus * are deduced from the Equations of Art. 88.

79. A specimen of the Curves of Cooling is given in Plate IV. The subsidiary curves in the same plate show different sections of the main curve projected on different scales (as in the case of the Statical Curves, Art. 60), for convenience of interpolation. The main curve corresponds to Case I. The dotted line adjacent to the main curve in the Plate shows the modification of the law of cooling introduced by covering the bar with paper, as in Case II. The results of the whole are shown in the preceding Table. The origin of the abscissæ (the times) is of course wholly arbitrary in each case.

80. The continuity of the curves thus obtained was in general satisfactory, though in one or two instances it seemed desirable to project part of two curves as distinct, as in Case I.

81. It is of little use, however, to possess merely a knowledge of the free temperature of the bar in terms of the time. The valuable information which we require in the deduction of conductivity is the "rate of cooling," or the proportional momentary loss of heat corresponding to a given excess of temperature. This is expressed mathematically by $\frac{dv}{dt}$, and might be directly obtained by discovering the equation to the primary curve of cooling, and then differentiating it.

82. There is, however, not less difficulty in finding a formula of interpolation to represent the curve of Cooling throughout its extent, than we have already found in the case of the curve of Statical Temperature, and it would evidently require the introduction of as many constants. I therefore preferred, in the first instance, (seeing that from the multiplied observations of cooling, the ordinates of this curve are more perfectly known than in the other case), to subdivide it into elementary arcs, and treating each of these as a portion of a logarithmic curve (to which it approximates), to find the value of $\frac{dv}{dt}$, or the "rate of cooling," corresponding to successive values of v ,* and by projecting these in curves to study their inflections in detail in each of the three forms of experiment already often referred to.

83. The three upper figures of Plate V. represent the "rates of cooling" of each bar in terms of its temperature-excess. From the study of these the peculiarities of the law of cooling above adverted to will become evident, and the harmony of the three cases is exhibited to the eye.

84. *First*, For very small excesses of temperature, the rate of cooling is comparatively slow, but increases much more rapidly than the temperature. To illus-

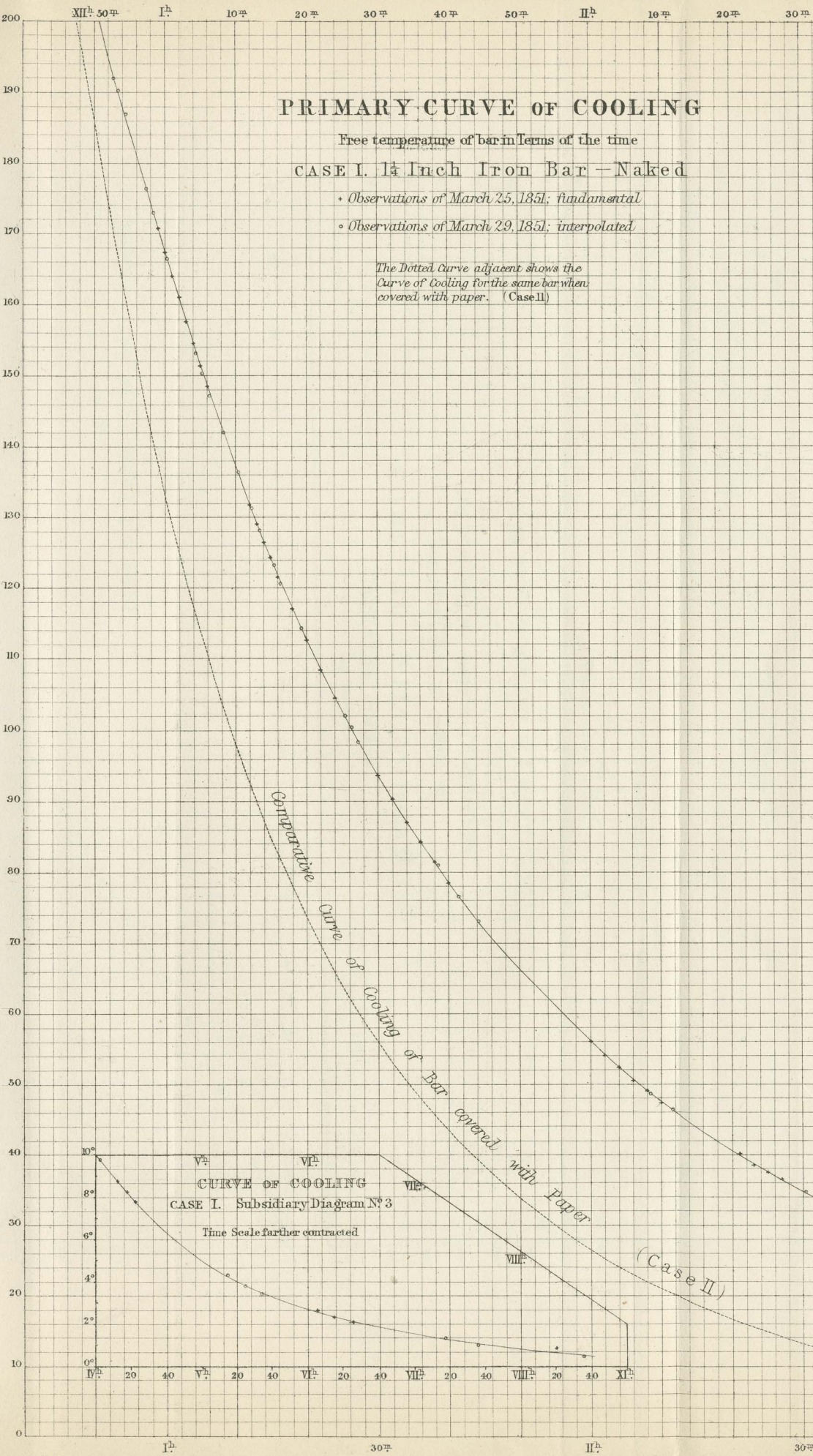
† By the formula $\frac{dv}{dt} = 2.3026 \frac{\log v' - \log v}{t - t'} \times \frac{v' + v}{2}$, where v and v' are the excesses of temperature corresponding to the times t and t' . $\frac{v' + v}{2}$ is the mean ordinate to which the result corresponds. The logarithms are tabular.

PRIMARY CURVE OF COOLING

Free temperature of bar in terms of the time
CASE I. 1½ Inch Iron Bar - Naked

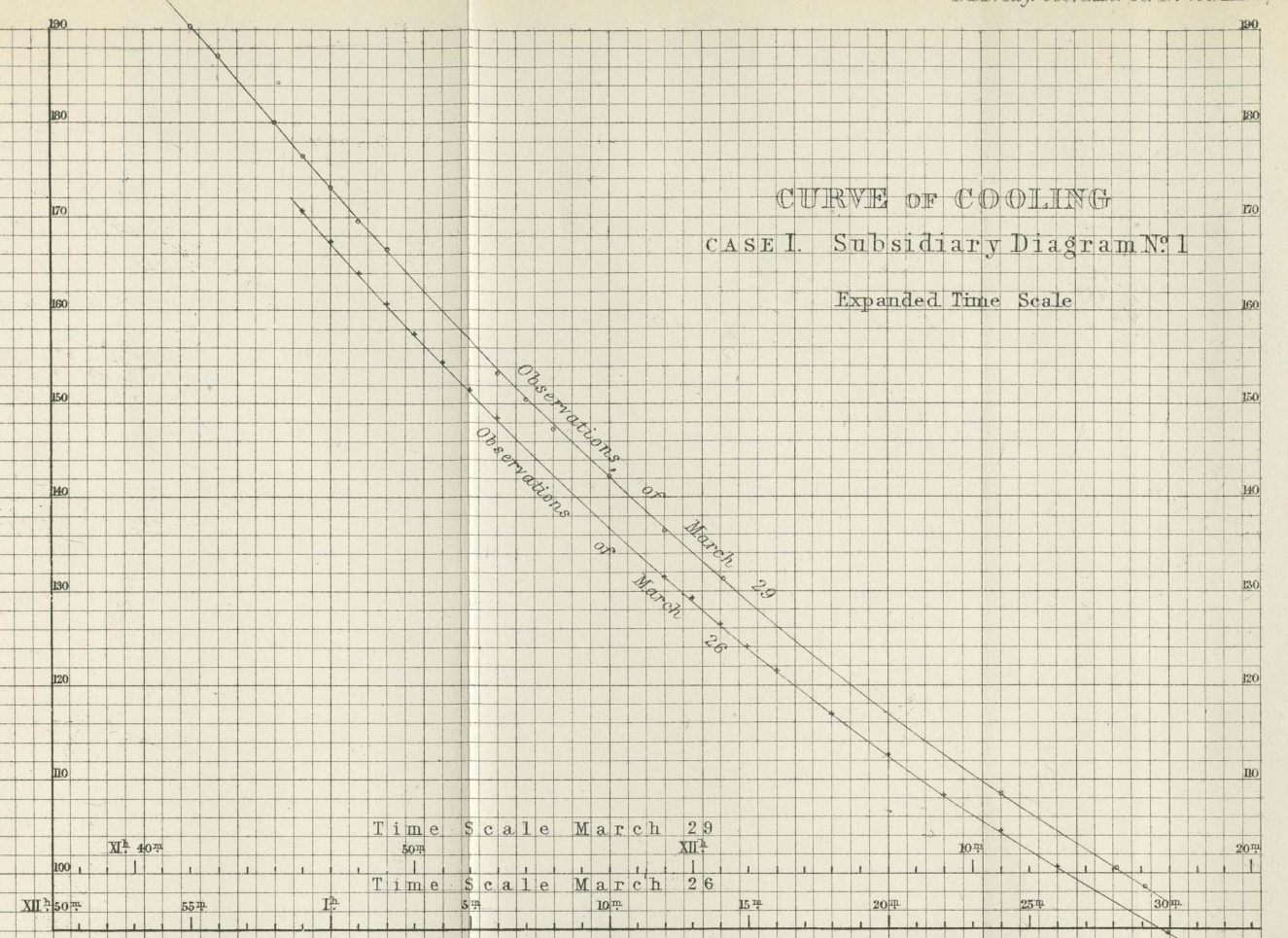
- Observations of March 25, 1851; fundamental
- Observations of March 29, 1851; interpolated

The Dotted Curve adjacent shows the Curve of Cooling for the same bar when covered with paper. (Case II)



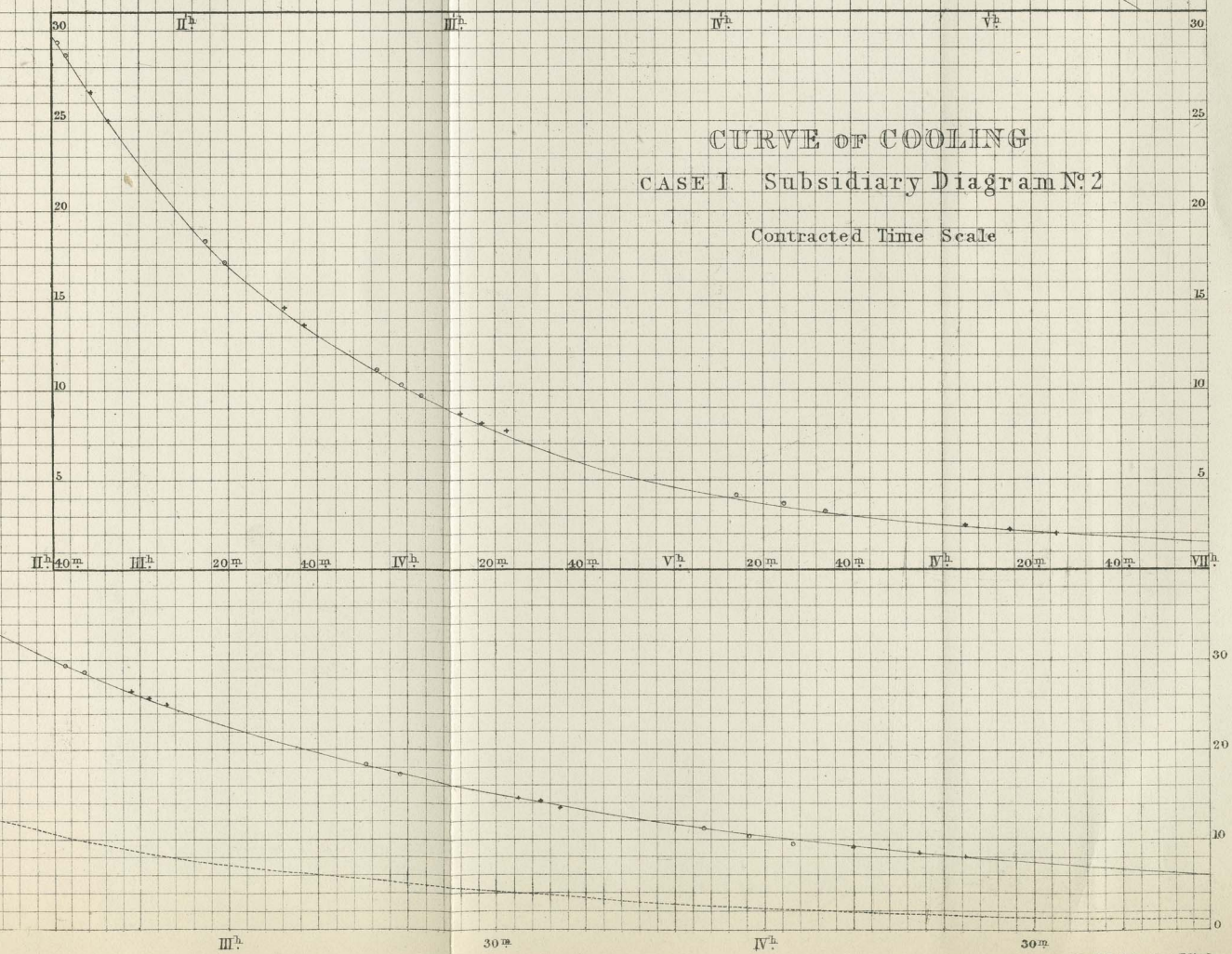
CURVE OF COOLING CASE I. Subsidiary Diagram N° 1

Expanded Time Scale



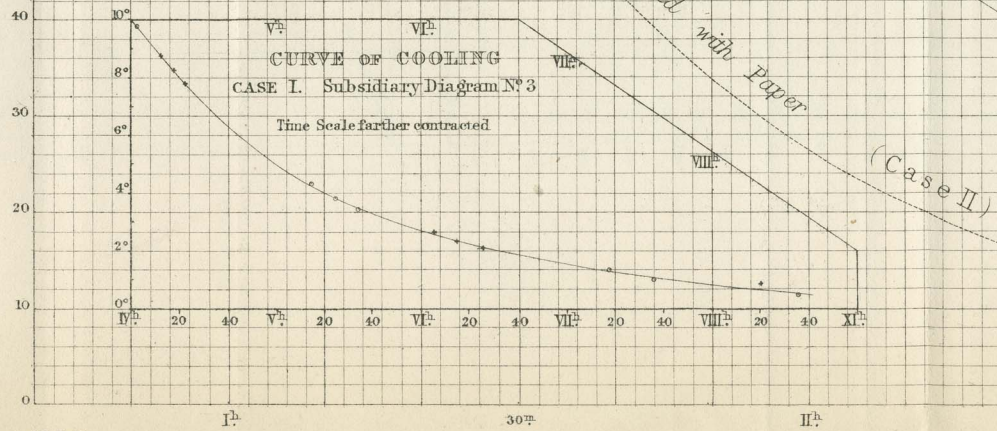
CURVE OF COOLING CASE I. Subsidiary Diagram N° 2

Contracted Time Scale



CURVE OF COOLING CASE I. Subsidiary Diagram N° 3

Time Scale farther contracted



SECONDARY CURVES OF COOLING

"Rate of Cooling" in terms of Free Temperature (art. 81)
 $-\frac{dy}{dt}$ in terms of v

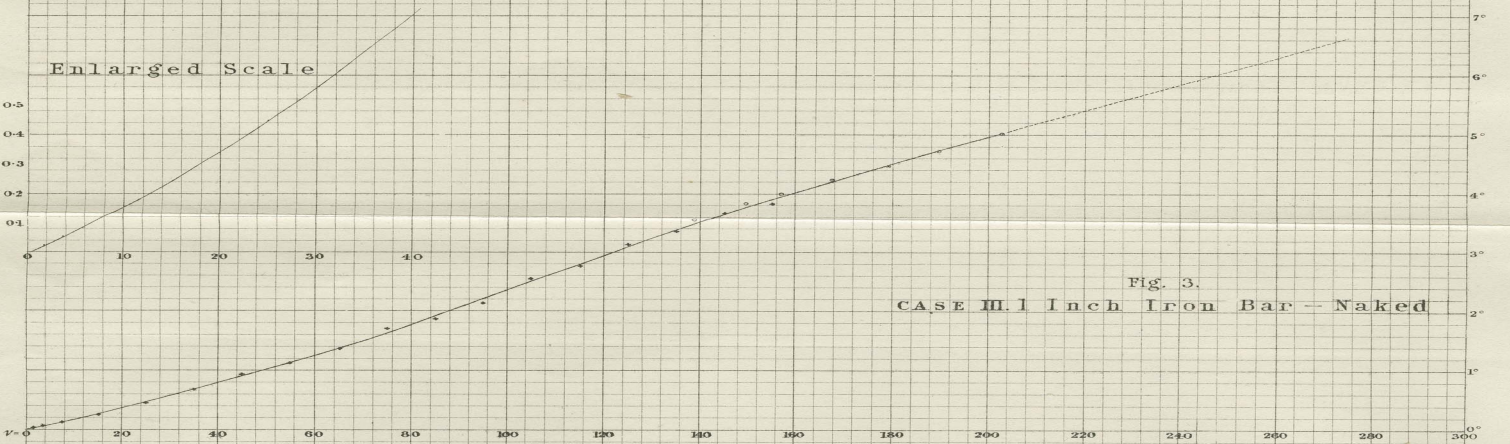
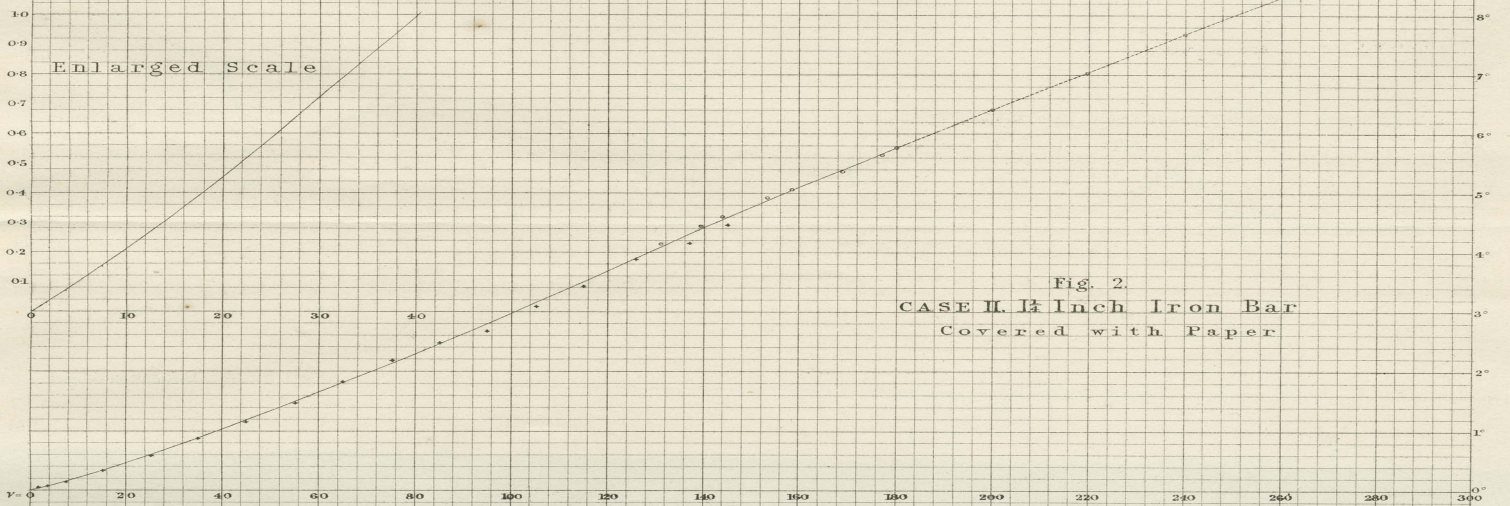
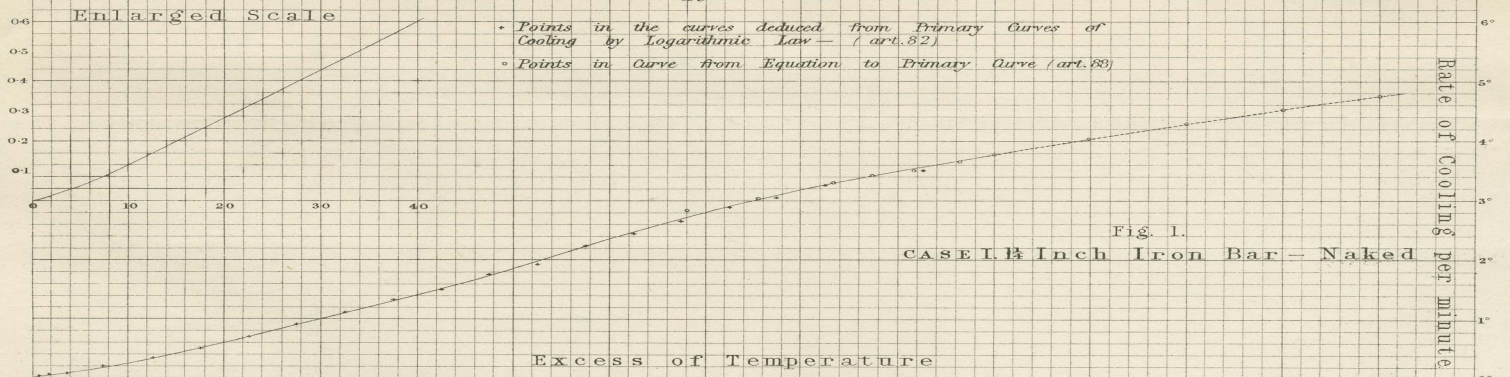
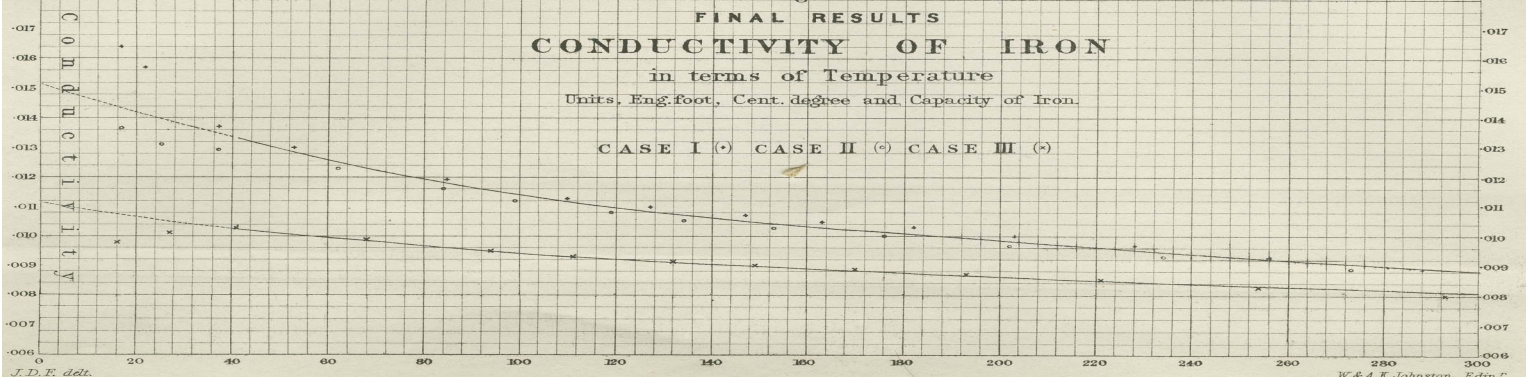


Fig. 4.
 FINAL RESULTS
 CONDUCTIVITY OF IRON
 in terms of Temperature
 Units, Eng. Foot, Cent. degree and Capacity of Iron.



trate this, a portion of each curve near its origin has been drawn separately on an exaggerated scale in a subsidiary figure, where its deviation from a straight line is abundantly manifest. In each case it may be adequately represented for the first 4° or 5° by an arc of a common parabola—more accurately perhaps by a semicubical parabola. However, taking the former as the simplest, I find the following equations to represent the part of the three curves nearest their origin:—

$$\text{Case I. } \frac{dv}{dt} = .00836 v + .000615 v^2$$

$$\text{Case II. } \frac{dv}{dt} = .01626 v + .00065 v^2$$

$$\text{Case III. } \frac{dv}{dt} = .01046 v + .00091 v^2$$

85. *Secondly*, The concavity upwards of these curves of the “rate of cooling”—showing that the cooling increases faster than the temperature rises—gradually diminishes; and in all the three curves we find between 110° and 120° (centigrade), a space nearly straight, indicating a point of contrary flexure. Above 150° the curve is in all the three cases slightly convex upwards, showing a rate of cooling slower in proportion than the rise of temperature.

86. *Thirdly*, This last circumstance appeared to me to be deserving of an elaborate verification. I therefore applied the same formula of interpolation which I had used with success to represent considerable arcs of the statical curve of temperature (see Art. 67, Eq. (1.)), being what has been called ROCHE’s formula, to represent the temperatures of the cooling bar in the higher parts of the primary curve of cooling.

87. In this I was successful, and I deem the matter of sufficient importance to show the coincidence between the original thermometric observations and the formulæ employed in each of the three cases. The times (t) are in each case reckoned from an arbitrary origin; and v is the excess of temperature above that of the air actually observed. [See Table VI.]

88. *Fourthly*, It will be seen that, within the limits of these tables, the observations are, upon the whole, well represented by the equations. Moreover, they confirm a result at which I had previously arrived from the projections, as to the law of cooling at higher temperatures, namely, that above 140° or 150° there is a gradual falling off in the rate of cooling, compared to the measure of temperature. For it is to be observed, that the equations employed to represent the primary curve of cooling coincide with the simple geometrical or logarithmic law, when the co-efficient of t in the *denominator* of the fraction (c of Art. 67) vanishes. When this co-efficient is positive, the progression is faster than geometrical; when negative, it is slower. Now, in each of the three cases c is negative,

TABLE VI.

CASE I.—1½-INCH BAR, NAKED.				CASE II.—1½-IN. BAR, COVERED.				CASE III.—1-INCH BAR, NAKED.			
Formula; $\log v = 2.30471 - \frac{.008133t}{1-.00262t}$				Formula; $\log v = 2.35215 - \frac{.01385t}{1-.00007t}$				Formula; $\log v = 2.41255 - \frac{.01051t}{1-.00113t}$			
<i>t</i>	<i>v</i> observed.	<i>v</i> calc.	Diff.	<i>t</i>	<i>v</i> observed.	<i>v</i> calc.	Diff.		observed.	<i>v</i> calc.	Diff.
minutes.	°	°	°	minutes.	°	°	°	minutes.	°	°	°
2.5	192.1	192.45	+0.35	7.5	177.0*	177.1	+0.1	9	207.4	207.5	+0.1
3	190.3	190.55	+0.25	8	174.3	174.3	0.0	10	202.4	202.4	0.0
4	187.15	187.0	-0.15	9	168.7	168.8	+0.1	11	197.45	197.4	-0.05
5	183.5*	183.45	-0.05	10	163.6	163.5	-0.1	12	192.65	192.6	-0.05
6	179.95	179.95	0.0	11	158.25	158.4	+0.15	13	187.9	187.9	0.0
7	176.55	176.55	0.0	12	153.4	153.4	0.0	14	183.25	183.3	+0.05
8	173.1	173.15	+0.05	13	148.4	148.5	+0.1	15	178.7	178.7	0.0
9	169.7	169.65	-0.05	14	143.75	144.0	+0.25	17.5	167.5*	167.8	+0.3
10	166.55	166.4	-0.15	15	139.3	139.3	0.0	20	157.55	157.55	0.0
14	153.45	153.7	+0.25	16	135.0	135.0	0.0	21	153.75	153.5	-0.25
15	150.45	150.55	+0.1	17	130.9	130.8	-0.1	22	149.9	149.7	-0.2
16	147.65	147.5	-0.15					23	146.1	146.0	-0.1
								25	138.9*	138.7	-0.2
* From interpolating curve.				* From curve.				* From curve.			

consequently the progression is slower than geometrical, and the curve of the “rate of cooling,” in terms of v , is convex upwards, as already stated.

89. *Fifthly*, By satisfying the observations by equations, we have farther these advantages—(1.) We can, with approximate accuracy, extend the law of cooling somewhat beyond the limits of observation, though with caution; (2.) We can also obtain the values of $\frac{dv}{dt}$ in a ready and continuous manner. The higher parts of the curves in Plate V. have been deduced in this way, and thus the “rate of cooling” has been tabulated for temperatures higher than those actually observed; but such numbers, being more or less hypothetical, are distinguished by asterisks in the following Table, which in other respects includes the results obtained from the observations treated as has been already described.

TABLE VII.—SHOWING THE “RATE OF COOLING,” $-\frac{dv}{dt}$ FOR DIFFERENT EXCESSES OF TEMPERATURE (v).

v .	CASE I. 1½ inch Bar, naked.	CASE II. 1½ inch Bar, papered.	Ratio to I.	CASE III. 1 inch Bar, naked.	Ratio to I.
1	0.009	.017	1.74	0.0115	1.28
2	.019	.035		.0245	
3	.031	.054		.0395	
4	.043	.075		.056	
5	.057	.096		.072	
10	0.124	.203	1.64	.158	1.27
20	.275	.44	1.60	.34	1.24
30	.43	.72	1.67	.55	1.28
40	.60	1.01	1.68	.78	1.30
50	.80	1.30	1.62	1.01	1.26
60	1.01	1.62	1.60	1.25	1.24
70	1.21	1.95	1.61	1.52	1.26
80	1.42	2.27	1.60	1.77	1.25
90	1.63	2.60	1.59	2.04	1.25
100	1.84	2.95	1.60	2.33	1.27
120	2.27	3.67	1.62	2.92	1.28
140	2.80	4.40	1.57	3.50	1.25
160	3.18	5.08	1.60	4.03	1.27
180	3.48	5.75	1.65	4.50	1.29
200	3.78	*6.38	1.69	4.95	1.31
220	*4.04	*7.00	1.73	5.40	1.34
240	*4.29	*7.65	1.78	*5.85	1.36
260	*4.52	*8.28	1.83	*6.30	1.39
280	*4.75	*8.90	1.88	*6.72	1.42

The numbers marked thus * being the results of calculation, are to be regarded as more or less hypothetical, and increasingly so at the higher temperatures.

90. *Sixthly*, I will not attempt to account for the inflections of the curves of Plate V. on physical principles, farther than to remark that the rapid increase in the velocity of cooling with temperature in the lowest part of the scale is perhaps owing to the separate effects of cooling by radiation, and cooling by convection. It seems probable that a certain excess of temperature of the bar above the air is necessary to determine efficient atmospheric currents, and thus to accelerate the rate of cooling; that, in fact, there is an amount of viscosity in air, which it requires a certain elevation of temperature properly to overcome. I would also observe, that the cooling in Case I. is (at higher temperatures) less regular than in the two other cases, while in Case III. the logarithmic law is almost accurately observed at those temperatures. This is no doubt to be ascribed to the greater mass of the Bar No. I., compared to its radiating power, occasioning probably

sensible irregularities, depending on the primitive distribution of heat in the bar, and on the want of uniformity in the temperature of its transverse section. The nearer that we approach to the ideal of an infinitely slender bar, the more shall we escape those periodical irregularities (see Art. 25 of the former part of this paper), arising from the primitive distribution of heat in its substance, which no doubt gives rise to some of the peculiarities of the inflections in the curves of "rates of cooling." In particular, we may naturally ascribe, in part at least, to the fact that the bar is heated first of all to a uniform temperature throughout in the fusible metal bath, the relatively diminished rate of cooling observed at the highest temperatures. At the same time I would repeat the caution, that the hypothetical or dotted portion of those curves cannot be relied on as expressing an actual fact, at least to more than a little way beyond the range of experiment.

§ III. *On the Proportion of Heat dissipated from the Bar by Radiation and Convection.*

91. Although not of direct importance to the determination of conducting power, I will indicate shortly how the numbers in Table VII., may be used to ascertain the relative amount of heat lost by radiation and convection at any or all points of the surface of the bar in Cases I. and II. The method was originally due to Sir JOHN LESLIE, but was stated more clearly by DALTON (System of Chem. Philosophy, p. 115), and was happily applied by DULONG and PETIT. Suppose the total "rate of cooling" of the same bar to be ascertained in air, first, when it is naked, and, secondly, when covered with paper, and let the ratio of the first case to the second be as $1 : p$. Next, by comparing after the manner of LESLIE's canister-experiments the "emissive power" of the same two surfaces, iron and paper, let it be as $1 : q$. Let the required ratio of the heat lost by convection to that lost by radiation be as $1 : x$ in the first case; then, of course, it will be in the proportion of $1 : qx$ in the second. But as the heat dissipated in each case is the sum of the effects due to convection (which is always $= 1$), and that due to radiation, we have

$$1 : p = 1 + x : 1 + qx$$

and

$$x = \frac{p-1}{q-p}$$

92. I have given in Table VII. the ratios of cooling, at different temperatures, for Cases I. and II., that is, for the same bar covered with paper and naked iron; and though the ratios vary somewhat,* yet they agree pretty nearly within the

* Since this was written, I have observed that a like diminution of the ratios of cooling from glass and silver up to a certain point, and afterwards an increase, was noticed by DULONG and PETIT, in their admirable Memoir on the Law of Cooling, page 102.—*Mem. Acad. Sci. Par.*

safe limits of observation. In fact, if we compare the average ratio from 10° to 100° Cent., and again from 100° to 200° Cent., we shall find them to be almost identical. They give for the value of p , the number 1.6023. This represents the proportion in which the papered bar dissipates its heat more rapidly than the naked bar.

93. For the direct radiating or emissive power of the two surfaces, I had recourse to the kind aid of Mr BALFOUR STEWART, not having had recently conveniences for making the experiment myself. He used the thermo-electric pile, and he found the experiment to be attended with considerably greater difficulty than is commonly attributed to it. I believe that Mr STEWART is not yet satisfied as to the reliability of his methods of observation; but the four best series of experiments made in February and March 1864, gave the emissive power of paper compared to iron as 5.8 to 1.* The value of q is therefore 5.8.

94. Hence by the previous investigation—

The value of x , the heat dissipated by radiation from naked iron (the dissipation by convection being always = 1) is $x = \frac{.6023}{5.8 - .60} = 0.116$. In the case of the paper surface, x is 5.8 times greater, or = 0.673. In other words, of the heat dissipated from the bar in Case I., nearly $\frac{9}{10}$ ths are lost by convection, and $\frac{1}{10}$ th by radiation. In the paper-covered bar (Case II.), only $\frac{6}{10}$ ths are lost by convection, and $\frac{4}{10}$ ths by radiation.†

95. From this it appears that the principal agent in the dissipation of heat in these experiments is Convection and not Radiation; nay, that the effect of the latter is comparatively almost insensible, when naked metallic bars are used. This of itself tends to explain the systematic deviation of the statical curve of temperature (Art. 64) from the logarithmic law. The experiments of DULONG and PETIT show that the dissipation of heat due to Convection increases not as the excess of temperature simply, but as its $\frac{5}{4}$ th power nearly (more exactly 1.233). This accords so far with what has been said of the variation in the rate of cooling in Art. 84; but it gives no adequate explanation of the inflections of the curves of Plate V. at higher temperatures. Were it not for the unquestionable precision of DULONG's admirable experiments, in which the law of cooling due to the contact of air was verified as high as 260° Cent., one might have not unreasonably supposed that the energy of convection was *relatively* less at higher temperatures.

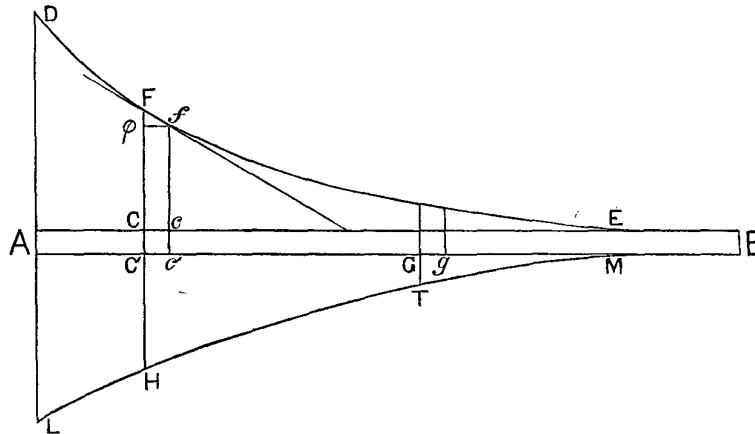
* This corresponds nearly to the relative emissive power of glass and polished silver used by DULONG.

† For in Case I. the whole heat lost from a point having a given temperature being represented by the number 1.116, that due to Convection is 1, that due to Radiation is .116. In Case II. the total loss is 1.673, whereof 1 is due to Convection, and .673 to Radiation.

§ IV.—*The “ Statical Curve of Cooling.” Recapitulation and Application of the Method of Deducing the Conductivity.*

96. It will be convenient here to recapitulate, from Arts. 5, &c. of the former part of this paper, the use which is to be made of the data obtained from the two fundamental experiments described in the previous sections, namely, the determination of the Curve of Statical Temperatures (Table II. Art. 61), and the rate or velocity of Cooling of the bar at any temperature (Table VII. Art. 89).

97. Let A B be the bar, kept hot at the extremity A, and left to assume a per-



manent temperature at its various points under natural causes. Let the upper curve, or DFE, represent by its ordinates (as FC) these temperatures. All the heat that enters the bar at A, and is propagated along it, has to be accounted for. Since the bar is so long that at the end B the heat has become insensible, the entire heat entering the bar at A has been dissipated from its surface in various proportions, according to its temperature, between the point A and some remote point E where the elevation of temperature is practically insensible. In like manner, if we take any point C in the bar, the heat transmitted from the hotter end, by conduction across the transverse section of the bar at C, is dissipated by the cooling of the bar between C and E. To know the quantity of heat passing across this transverse section, we have therefore to ascertain the aggregate loss of heat from the surface of the bar to the right hand of C.

98. To do this, we must construct what I call the *Statical Curve of Cooling*, which is represented in the same figure by the curve LHM, beneath the bar AB. The ordinate C'H represents the heat lost by the bar *per minute*, from the portion CC', whose temperature is represented by CF in the upper curve. This loss or C'H, is found from Table VII., by entering it with the thermometer reading CF, which again is known from Table II. in terms of the position of the point C in the length of the bar. Thus all the ordinates of the Statical Curve of Cooling, LHM, can

be constructed. Dividing the length of the bar into sections, in the three experimental cases so often referred to, the ordinates of the curve of statical cooling, or values of $-\frac{dv}{dx}$, appropriate to every point of the bar, will be found as in the following Table:—

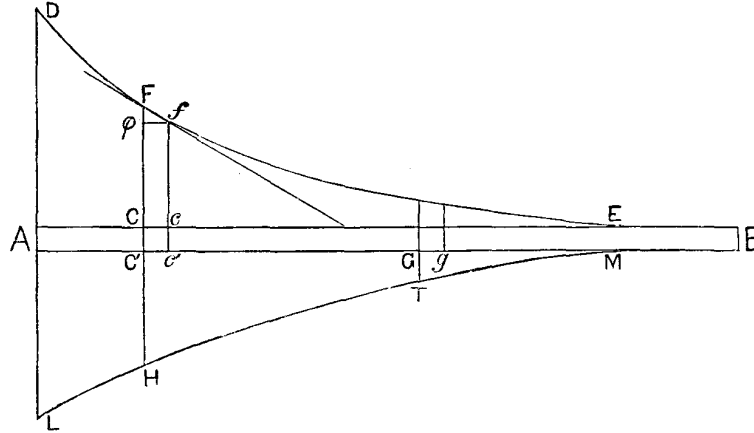
TABLE VIII.—SHOWING THE RATE OF COOLING PROPER TO EACH POINT OF THE LENGTH OF THE BAR (OR ORDINATES OF THE *Statical Curve of Cooling*), CONTAINING ALSO THE VALUES OF $-\frac{dv}{dx}$.

Distance from Origin in Feet and Inches.	CASE I.		CASE II.		CASE III.	
	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$
Ft. In.	°	°	°	°	°	°
" 0	4.71	420	8.30	512	6.75	512
" 1	4.32	362	7.06	423	5.92	432
" 2	3.97	314	6.05	351	5.19	366
" 3	3.64	272	5.20	292	4.54	310
" 4	3.32	237	4.41	245	4.00	264
" 5	3.01	206.8	3.73	206.5	3.44	226
" 6	2.70	181.0	3.14	175.2	2.93	193.6
" 7.5	2.20	148.8	2.48	137.6	2.33	154.6
" 9	1.80	123.0	1.98	109.0	1.85	124.1
I. 0	1.245	85.35	1.28	69.9	1.185	81.6
6	0.620	43.70	0.58	35.2	0.55	38.0
II. 0	0.342	24.47	0.282	16.3	0.258	19.64
III. 0	0.114	8.15	0.070	4.47	0.070	5.52
IV. 0	0.043	3.34	0.021	1.36	0.0185	1.69
VI. 0	0.008	0.57				

N.B.—The values of $-\frac{dv}{dx}$ are computed by the method explained in Art. 71. Only at some of the lower temperatures a mixed method of calculation and projection has been used.

99. It is evident that, if we can effect the quadrature of successive sections of the statical curve of cooling, continued until it vanishes in the direction of the cool end of the bar, we shall have got the "flux of heat" across the section of the bar at which the quadrature commences. The measure of the heat expressed by the area of the curve in question will have for unit the amount of heat required to raise unit of volume (1 cubic foot) of iron by 1° Cent. The shaded curve, in the lower part of Plate III., shows the Statical Curve of Cooling proper to Case I. The ordinates of the curve are related to those of the curve of statical temperature immediately above it, by the relation of $-\frac{dv}{dt}$ to v , shown in the secondary curve of cooling in the upper figure of Plate V.

100. The flux of heat is greatest in the hottest part of the bar, because the temperature of the bar varies most rapidly there, and the heat is more rapidly drawn towards the cold end. To give exact expression to the tendency of the



heat to traverse the section of the bar at C, we will take Cc to represent the thickness of a plate, bounded by imaginary parallel surfaces, situated transversely within the bar through which the flow of heat is to be considered. This is to be compared with the flow of heat across any other plate, Gg, of equal thickness, in a different part of the bar. Then, according to FOURIER, the flow of heat across Cc will be proportional to the small decrement of temperature $F\phi$, by which the side of the plate nearest to A is hotter than the farther side, and to the Conductivity jointly. The value of this decrement, $F\phi$, is evidently nothing else than the differential coefficient $\frac{dv}{dx}$, which has been given in the last Table, as derived from the equations to the curve of statical temperature in Art. 71.

101. Hence (in conformity with Arts. 7, 31, and 35 of the first part of this paper),

$$\text{Flux of heat, or area CFE} = -\frac{dv}{dx} \times \text{conductivity},$$

or

$$\text{Conductivity} = \frac{\text{Area CFE}}{-\frac{dv}{dx}}.$$

§ V.—*The Method of this paper applied, under the usual assumptions made in the Theory of Conduction, as a first approximation to the determination of Conductivity.*

102. The area of the statical curve of cooling to the right hand of any ordinate is therefore to be found. It will be convenient, for this purpose, to show what the nature of this curve would be were the *usual assumptions* of the mathematical theory of Heat adopted. These assumptions are (1.) That the superficial

loss of heat follows NEWTON'S law, or that the loss of heat in unit of time varies simply as the excess of temperature; (2.) That the same law holds for the internal communication of heat, or that the quantity of heat conducted is proportional simply to the difference of temperature of two adjacent elementary portions of a bar.

103. From the *first* assumption it follows, of course, that the temperature of a cooling body of small dimensions varies in a decreasing geometrical progression with the time. The dynamical Curve of Cooling on this assumption is a logarithmic curve, t and v being the variables.

104. From the *second* assumption, taken along with the first, we learn from a well-known analysis, that what we have called the Curve of Statical Temperature is also a logarithmic, x and v being the variables.

105. Now the Statical Curve of Cooling ($\frac{dv}{dt}$, in terms of x) must, on these assumptions, be also logarithmic; for its ordinates—the velocities of cooling—are everywhere proportional to the temperature. Hence also the subtangent to these two last curves* is the same. Let it be M . Then by a property of the logarithmic curve (M being the modulus) the area of the curve bounded by an ordinate y , and carried to infinity, is My . Also the flux of heat corresponding to the position of the ordinate y is (Art. 99) $= My$, y being, as we have seen, $= -\frac{dv}{dt}$. But, by Art. 102, $-\frac{dv}{dt}$ is everywhere *assumed* (for the present) to be proportional to v , or $-\frac{dv}{dt} = pv$. Also since the dynamical curve of cooling is a logarithmic (103), let its modulus be m . Then, by the property of the curve, $-\frac{dv}{dt} = \frac{v}{m}$. Hence, comparing the last two equations $p = \frac{1}{m}$. And,

$$F = \text{Flux of heat} = My = -M \frac{dv}{dt} = M \frac{v}{m}$$

and (by Art. 101).

$$\text{Conductivity} = \frac{F}{\frac{dv}{dx}} = \frac{M \cdot v}{m \cdot \frac{dv}{dx}}$$

But the curve of statical temperature being also assumed to be logarithmic (104);

and consequently

$$-\frac{dv}{dx} = \frac{v}{M};$$

we finally get

$$\text{Conductivity} = \frac{Mv}{m \cdot \frac{v}{M}} = \frac{M^2}{m}.$$

106. A first approximation to the conductivity of the bar may therefore be found by *dividing the square of the modulus or subtangent of the statical curve of*

* Namely, the Curve of Statical Temperature and the Statical Curve of Cooling, being the two curves shown in the wood-cut of last page.

Temperature (assumed to be logarithmic) *by the modulus of the Dynamical Curve of Cooling.*

107. Thus, to illustrate this by a numerical example, were we to attempt to reduce the statical curves of temperature of Table II. to logarithmics after the manner of BIOT, we should probably find the following approximate values of the subtangent M:—

	CASE I.	CASE II.	CASE III.
M	0·9 foot	0·7 foot	0·8 foot

And from Table V. of the Dynamical Curves of Cooling, the subtangents might be nearly

<i>m</i>	60 min.	40 min.	50 min.
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whence

$\frac{M^2}{m}$	·0135	·0122	·0128
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which, it is seen, give nearly approaching values of the conductivity.

§ VI. *Final Determinations of the Conductivity of Iron at various Temperatures.*

108. The results given in the last section are in the highest degree rude, and are introduced merely to illustrate the general form of the method. The curves of Temperature and Cooling are neither of them sensibly logarithmic, and therefore we have found the necessity of dividing them into small portions, and taking their elements from point to point. Therefore, in continuation of what has been said in Art. 99, we must proceed to the quadrature of the Statical Curve of Cooling whose elements are given in Table VIII. This is a curve which though not logarithmic, may, like the other curves we have already discussed, be treated as if it had been, when divided into numerous elements bounded by parallel ordinates. Every one of these segments may have its area estimated by the simple formula proper to a logarithmic curve,* and for the infinite branch a similar formula must be adopted.

109. The following Tables contain the determination of the total Flux of Heat (F) across any section of the bar by the summation of the areas of the statical curve of cooling, commencing from the colder end of the bar, where this curve is (like the primary curve of temperatures) apparently asymptotic. In these Tables (corresponding to the three experimental Cases discussed in this Memoir, the chief uncertainty attaches to the two extremities of the curve. There are difficulties inherent in the precise determination of very small excesses of temperature of a bar, whether in a statical or a cooling condition, above the surrounding air, itself not absolutely constant in temperature. These difficulties have been previously referred to. Moreover, when we have to take the ratio of two quantities, both to be experi-

* Namely, area between ordinates y and $y' = M (y' - y)$ where M the subtangent equals

$$\frac{0·4343 \text{ \&c. } (x - x')}{\log y' - \log y}$$

mentally determined, and both in an almost evanescent state (as is the case in the extreme portion of the curve of statical temperature and of the statical curve of cooling), the quotient may be sensibly in error. To this I add, that in Case I. the length of the bar was certainly not quite sufficient to allow the conducted heat to be entirely spent by dissipation. Consequently there is, as it were, a slight *congestion* of heat towards the extremity—very slight indeed, but still sufficient to give to the subtangent there a too large value, and consequently to the decrement of the primary curve of temperature too small a one. Hence the

ratio $\frac{F}{\frac{dv}{dx}}$ is somewhat too great, both in consequence of the numerator being too

large and the denominator too small. But how little any such ambiguity can effect the general evaluation of the flux of heat in the succeeding lines of the Table, either in the case of this or of the two succeeding experiments, will be seen by noticing the minuteness of the areas representing the flux which correspond to the extreme portions of the curves. They are so small, that an error amounting to one-half their amount, would hardly affect by $\frac{1}{300}$ th or $\frac{1}{400}$ th part the measure of the conductivity in the middle and more important part of the Tables.

TABLE IX.—CASE I. $1\frac{1}{4}$ -INCH IRON BAR, NAKED. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

Limits of Abscissæ.		Limits of Ordinates.		M=Sub- tangent.*	Area M(y'-y).	Total Area F.	$-\frac{dv}{dx}$	Conduc- tivity, F. $-\frac{dv}{dx}$	Corre- sponding Actual Temp. Cent. (v+13).
x.	x'.	y.	y'.						
Ft. Inch.	Ft. Inch.								
∞	VI. 0	0	0.008	1.662	0.0133	0
VI. 0	IV. 0	.008	.043	1.189	.0416	0.0549	3.34†	.0164	17
IV. 0	III. 0	.043	.114	1.026	.0728	.1277	8.15†	.0157	22
III. 0	II. 0	.114	.342	.9104	.2075	.3352	24.47	.0137	37
II. 0	I. 6	.342	.620	.8403	.2336	.5688	43.7	.0130	53
I. 6	I. 0	.62	1.245	.7175	.4484	1.0172	85.35	.0119	85
I. 0	0 9	1.245	1.80	.6777	.3762	1.3934	123.0	.0113	110
0 9	„ 7.5	1.80	2.20	.6233	.2493	1.6427	148.8	.0110	127
„ 7.5	„ 6	2.20	2.70	.6100	.3050	1.9477	181.0	.0107	147
„ 6	„ 5	2.70	3.01	.7669	.2378	2.1855	206.8	.0105	163
„ 5	„ 4	3.01	3.32	.8515	.2640	2.4495	237.1	.0103	182
„ 4	„ 3	3.32	3.64	.9047	.2895	2.7390	272.4	.0100	203
„ 3	„ 2	3.64	3.97	.960	.3168	3.0558†	313.7	.0097†	228
„ 2	„ 1	3.97	4.32	.986	.3451	3.4009†	362.5	.0093†	256
„ 1	„ 0	4.32	4.71	.965	.3764	3.7773†	420.0	.0090†	288
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
* From the formula $0.4343 \times \frac{x-x'}{\log y' - \log y}$									
† From curve; the rest from equation.					‡ More or less uncertain.				

TABLE X.—CASE II. $1\frac{1}{4}$ -INCH IRON BAR, COVERED WITH PAPER. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

Limits of Abscissæ.		Limits of Ordinates.		M.*	Area M (y'-y).	Total Area F.	$-\frac{dv}{dx}$	Con- ductivity, F. $-\frac{dv}{dx}$	Corre- sponding Actual Temp. Cent. (v+13).
x	x'	y	y'						
Ft. In.	Ft. In.	°							
∞	IV. "	0	·021	·980	·0206
IV. "	III. "	·021	·070	·8305	·0407	·0613	4·47†	·01372	17
III. "	II. "	·070	·282	·7177	·1521	·2134	16·3	·01310	26
II. "	I. 6	·282	·58	·6935	·2066	·4200	32·5	·01292	37
I. 6	I. "	·58	1·28	·6317	·4422	·8622	69·9	·01234	62
I. "	" 9	1·28	1·98	·5728	·4020	1·2642	109·0	·01160	84
" 9	" 7·5	1·98	2·48	·5551	·2776	1·5418	137·6	·01120	99
" 7·5	" 6	2·48	3·14	·5301	·3499	1·8917	175·2	·01080	119
" 6	" 5	3·14	3·73	·4840	·2855	2·1772	206·5	·01054	134
" 5	" 4	3·73	4·41	·4978	·3385	2·5157	245	·01027	153
" 4	" 3	4·41	5·20	·5055	·3993	2·9150	292	·00998	176
" 3	" 2	5·20	6·05	·5500	·4675	3·3825†	351	·00964†	202
" 2	" 1	6·05	7·06	·5401	·5455	3·9280†	423	·00929†	234
" 1	" 0	7·06	8·30	·5147	·6383	4·5663†	512	·00892†	273
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

* $M = 0.4343 \frac{x-x'}{\log y' - \log y}$ † More or less uncertain.

† The values of $\frac{dv}{dx}$ are all from equations, and they all agree satisfactorily with projection.

TABLE XI.—CASE III. 1-INCH IRON BAR, NAKED. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

x	x'	y	y'	M	Area M (y'-y).	Total Area F.	$-\frac{dv}{dx}$	Con- ductivity, F. $-\frac{dv}{dx}$	Actual Temp. Cent. (v+11).
Ft. In.	Ft. In.	°							
∞	IV. "	0	0·0185	·820	·0152
IV. "	III. "	·0185	·070	·7515	·0387	·0539	5·52	·00977	16
III. "	II. "	·070	·258	·7667	·1441	·1980	19·64	·01008	27
II. "	I. 6	·258	·55	·6605	·1928	·3908	38·0	·01029	41
I. 6	I. "	·55	1·185	·6515	·4137	·8045	81·6	·00986	68
I. "	" 9	1·185	1·85	·5612	·3733	1·1778	124·1	·00949	94
" 9	" 7·5	1·85	2·33	·5419	·2600	1·4378	154·6	·00930	111
" 7·5	" 6	2·33	2·93	·5456	·3274	1·7652	193·6	·00912	132
" 6	" 5	2·93	3·44	·5192	·2648	2·0300	226·0	·00898	149
" 5	" 4	3·44	4·00	·5526	·3094	2·3394	264·4	·00885	170
" 4	" 3	4·00	4·54	·6580	·3553	2·6947	310·5	·00868	193
" 3	" 2	4·54	5·19	·6229	·4048	3·0995*	366	*·00847	221
" 2	" 1	5·19	5·92	·6339	·4627	3·5622*	432	*·00824	254
" 1	" 0	5·92	6·75	·6349	·5270	4·0892*	512	*·00799	293
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

* More or less uncertain.

110. Uncertainty, I have already said, attends the determinations of conductivity for the higher as well as those at the lowest temperatures. In fact, the former are (as will have been seen from the details already given) the results of analogies rather than of direct experiments. The experiments, whether Statical or Dynamical, rarely extended beyond a temperature of 200° , or at most 220° Cent. The results have been here carried out by the analogies afforded by the equations to the curves to nearly 300° . Nevertheless, the continuity of the law of conductivity diminishing with temperature, is consistently brought out by these approximations.

111. In the preceding Tables the conductivity is expressed in terms of the amount of heat as unity, which is required to raise the temperature of one cubic foot of iron, by one degree Cent. It expresses the amount of heat reckoned in such units which would traverse in one minute across an area of one square foot, a plate of iron one foot thick, with the two surfaces maintained at temperatures differing by 1° Cent.

112. If we now project the values of the conductivity of iron found in the last column but one of the three preceding Tables in terms of the thermometric temperatures (Centigrade) in the last columns, we are enabled to trace easily the connected results of the whole inquiry. (See Plate V. fig. 4.)

113. We find that in each case *the conductivity diminishes as the temperature increases*; and *that*, for the next part, in a progressive manner. The variation with temperature is clearly most rapid at the lower temperatures.

114. The two first series agree very closely in their numerical results, with the exception of certain irregularities in the part of the curve where the temperatures are lowest; which have already been in part accounted for (Arts. 55, 65, 109). These two series belong to one and the same bar, though cooling under very different circumstances, owing to the largely increased radiating power conferred upon it by coating it with paper. And the value of the striking coincidence in the numerical results in Tables I. and II. is enhanced by the consideration, that the numbers expressing the conductivity are obtained by taking the ratios of two different columns (7 and 8), which in the two Tables differ most widely, and the result cannot be even guessed at until the ratio is actually taken.

115. The third series (Table X.) leads to numbers very sensibly differing from the two first series, yet following the same general law, the conductivity decreasing with temperature (excepting at the lowest part of the scale, where we find an anomaly corresponding to that noted in an early part of this paper (Art. 65), showing that the lowest portion of the statical curve has not in this instance been satisfactorily determined). The conductivity in Table X. is smaller throughout than in the two former cases. It is believed that this can be satisfactorily accounted for by the different quality of the iron of which this

bar was made, which came from a different manufactory, and was probably inferior in quality.*

116. Tracing an interpolating curve through the projected observations of Cases I. and II., which run nearly parallel and at no great distance, at temperatures superior to 40° and do not diverge even in the higher and more hypothetical part of the diagram, and doing the same separately for Case III., we obtain the following numbers, purely as results of observation:—In the first

column of each division of Table XII., we have the ratio $k = \frac{F}{\frac{dv}{dx}}$, which expresses the conductivity in terms of the heat required to raise a cubic foot of iron by one degree Centigrade. In the two following columns, we have the same reduced to the usual standard of conductivity in French and English measures respectively.†

TABLE. XII.

Temp. Cent.	CASES I. AND II.			CASE III.		
	$k = \frac{F}{\frac{dv}{dx}}$	CONDUCTIVITY.		$k = \frac{F}{\frac{dv}{dx}}$	CONDUCTIVITY.	
		Units : Foot, Minute and Cent. Deg.	Units : Centi- metre, Minute, Cent. Deg.		Units : Foot, Minute, Cent. Deg.	Units : Centi- metre, Minute, Cent. Deg.
0	·01506	·01337	12·42	·01117	·00992	9·21
25	·01391	·01235	11·48	·01062	·00943	8·79
50	·01288	·01144	10·63	·01014	·00904	8·37
75	·01205	·01070	9·94	·00974	·00865	8·04
100	·01140	·01012	9·40	·00940	·00835	7·76
125	·01088	·00966	8·98	·00916	·00813	7·56
150	·01052	·00934	8·68	·00895	·00795	7·38
175	·01018	·00904	8·39	·00877	·00779	7·23
200	·00987	·00876	8·14	·00860	·00764	7·10
225	·00958	·00851	7·90	·00844	·00749	6·96
250	·00930	·00826	7·67	·00826	·00736	6·84
275	·00902	·00801	7·44	·00815	·00724	6·72

117. The coincidence of the results in the second column with the results of the provisional reduction in the case of the $1\frac{1}{4}$ -inch bar, made in 1852, and printed at Art. 33, page 144, of the former part of this paper, is both striking and satisfactory. For it shows, as I there anticipated (Art. 38), that the method is, to a great extent, independent of the ordinary instrumental errors,

* Dr MATTHIÉSSON in his Experiments on the Electric Conductivity of Iron (Phil. Trans., 1863), has found nearly equally wide variations in different specimens.

† If the numbers in the first column of each division of the Table be called A, then $A \times .888$ will express the conductivity in water-measure for the foot, minute, and Cent. degree; and $A \times .825$ gives the numbers in the third column, where the centimetre is substituted for the foot.

and even of the laborious computations which have formed the basis of the present paper.

118. In the preceding Table I have completed the series for lower temperatures, where the observations were less accordant, in the following way:—I have assumed that the most trustworthy part of the observational curves are those between the actual temperatures of 40° or 50° and 150° or 160° , and that within moderate limits, the conductivities (k) may be represented in terms of the temperature (t), by such a formula as

$$k = A + at + bt^2$$

In the case of the $1\frac{1}{4}$ -inch bar, I find for these constants

$$A = \cdot 01506 \quad a = - \cdot 0000488 \quad b = + \cdot 000000122$$

From which the conductivities corresponding to 0° and 25° have been interpolated. In the case of the 1-inch bar the constants are—

$$A \cdot 01117 \quad a = - \cdot 0000235 \quad b = + \cdot 000000058.$$

119. I must here observe, however, that the above form of relation between k and t , which has been applied by Dr MATTHIESSEN, in his extensive and important researches on electric conductivity, does not satisfy the form of our conductive curves, Plate V. fig. 4, except through a limited range. I have reason, however, to think, that down to 0° of temperature it may be sufficiently exact. The “percentage decrement” of the conductivity between 0° and 100° is 24·5 for the larger bar of iron, and 15·9 for the smaller one. As in the case of Dr MATTHIESSEN’S electrical experiments, the “percentage decrement” diminishes with the conducting power, and in almost exactly the same proportion.* The numerical values in either case are, however, considerably smaller for heat than those obtained by Dr MATTHIESSEN for electricity.

120. With this exception, however, there is an agreement in the character of the metals (so far as is yet known) in conducting heat and electricity. (See Art. 2 of this paper.)

§ VII.—*Concluding Remarks and Suggestions.*

121. In Art. (15) of the first part of this paper, I expressed my desire to afford to future experimenters every aid I possibly could to resume and extend my observations (confined, unfortunately, to only one metal—iron), and to furnish them with such advantages as my experience afforded, as well in methods of observation as of reduction.

122. It was especially with this view that I have spent what may perhaps appear an undue amount of labour on the reduction of the experiments considered

* Phil. Trans. 1863, p. 380.

in the present paper. I do not, however, regard this labour as wasted, for the knowledge thus acquired of the nature of the remarkable curves of which it treats will enable a future observer to attack the question in a far more direct manner, and to obtain, with comparatively little trouble, numerical determinations of the conductivity of the metals under ordinary circumstances, and adapted to most purposes of theory or practice.

123. *Suggestions as to Experiments.*—After mature consideration, I do not think that the experimental methods require almost any modification. The independence of the results of any moderate error in the thermometers seems satisfactorily proved (Arts. 38 and 117); and if the object be merely to ascertain the conductivity and “percentage decrement” for a number of metals, it may easily be done without pushing the observations to the high temperatures used in my experiments, which are always a fertile source of difficulty and error. If, for instance, an extreme temperature of 120° or 140° Cent. only was aimed at, shorter bars might be used; the heat would be more manageable and more quickly attained; the thermometers would be more easily made, more easily used, and subject to far smaller corrections; and the dynamical experiments especially, would be freed from an anxious and troublesome source of error, arising from the irregularity of the primitive distribution of the heat in the cooling bar (Arts. 25, 26, and 90).

124. A more exact knowledge of the form of the statical curve of temperature in any case may be obtained by using sources of heat of progressively lower temperature, as explained in Arts. 27 and 58.

125. It is probable that very good results might be obtained by simply using boiling water as a source of heat at the hottest end of the bar, than which nothing can be more manageable. The duration of the statical experiments could thus be much reduced, and the temperature of the air of the apartment rendered more stable. The difficulties referred to in Arts. 65, 109, as to the determination of very small excesses of temperature next the cool end of the bar might thus be in a great measure removed. Indeed, it would be a worthy object of study, in a theoretical point of view, to determine the form of the Statical and Dynamical curves for those low temperatures more accurately than I have done. I cannot but suspect an anomaly in the conduction of heat when the temperature varies with extreme slowness from point to point, which my observations rather indicate than establish.*

126. Another experimental point of interest for the theory would be to estab-

* I may be allowed to state here generally, that this anomaly would apparently assign a too great conducting power to iron at low temperatures than we can readily admit. [The case of the 1-inch bar might rather lead to an opposite conclusion, but I have less confidence in the observations made on it for very small excesses of temperature] Both the statical curve and the curve of cooling deviate more and more from the logarithmic form as the temperature-excesses diminish.

lish, for a few points of a metallic bar, the difference between the superficial and the internal temperature of the bar in any transverse section. This might be done by thermo-electric methods, such as, I think, were used by the late M. LANGBERG of Christiania in his experiments on the conduction of heat in bars. I made some attempts (which were not unpromising) in a different way, by applying to the surface of the bar small portions of fusible alloys or other substances, liquefying at definite temperatures. There did not appear to be much difficulty by gently sliding these proof-pieces along the bar from the cooler towards the hotter part, of ascertaining with considerable precision the co-ordinate of the superficial point, corresponding to the fusing temperature of the alloy or other substance used. The five following substances, in a descending scale, were found to have tolerably definite fusing-points, and to be sufficiently suitable for the experiment :—Tin ; solder (tin 9 parts, lead 5 by weight) ; fusible metal (consisting of bismuth 2 parts, lead 1 part, and tin 1 part by weight) ; naphthalic acid ; and bees-wax. The fusing temperatures of the *three* first were carefully ascertained by direct experiment to be—

Tin,*	.	229°0 Cent.	=444°2 Fahr.	
Solder,	.	181°6 „	=358°9 „	
Fusible metal,	.	94°15 „	=201°4 „	

The fusing points of the others were not ascertained by me.

127. The experiments which I made in this manner were entirely tentative and preliminary. The following is a specimen :—Statical experiment ; 1851, March 14. $1\frac{1}{4}$ -inch bar, naked [see Table I., page 78 of this paper.] “ At 1^h 40^m I tried the following experiment to test the difference of temperature of interior and exterior of bar. Taking small sharp-pointed pieces of tin [and] fusible metal (prepared on purpose, bismuth 2, lead 1, tin 1 by weight), I rubbed them gently on the surface of the iron bar till I found the melting point, keeping them gently in motion so as not to allow the surface to heat beneath them. I fixed these points with very considerable exactness, in the case of the fusible metal (the best observation), to perhaps within $\frac{1}{20}$ th inch. I did not find the position sensibly [to] vary on the centre of the top, and on the centre of the side of the bar, nor even towards the angle of the bar (with the fusible metal). These experiments deserve repetition.

“ 1 ^h 40 ^m Tin melted when rubbed on					} from origin at the edge of the crucible.
the centre of one side of the bar,					
at	0 ft.	0.65 in.			
Fusible metal,	0	10.45 „	„	„	„
Bees-wax,	1	4.7 „	„	„	„

* The melting point of tin seems to be one of the best determined of the higher temperatures. According to CRICHTON, Senior (of Glasgow), it is 442° Fahr. [T. THOMSON]; KUPFFER, 446°; DANIELL, 441°. On the melting point of lead, see Art. 70.

128. *Suggestions as to Reductions.*—Were any one desirous of pursuing the subject of the theory of conduction into its details, I should be disposed to recommend the employment of BIOT'S formula of 5 constants (used to express the elasticity of steam), instead of ROCHE'S, containing 3 constants, which we have here used, see Art. 66. The method of calculation (which is necessarily laborious), is given in REGNAULT'S large treatise on the Theory of the Steam Engine.* For any merely practical purpose, however, this is not required. An experimenter desiring to compare the conductivity and "percentage decrement" of different metals, may reasonably confine his attention between the useful limits of 20° and 120°, or at most 140° Centigrade. For that interval, ROCHE'S formula will suffice. And the chief use of the formula is to obtain readily and accurately the differential co-efficient $\frac{dv}{dx}$ (see Arts. 71 and 78), on the determination of which the value of the conductivity mainly depends.

129. Though I would not recommend the attempt to proceed by graphical methods *alone*, they are an invaluable help, and also serve as a check to the calculations. Where these are not made throughout in duplicate, the use of curves ensures the detection of any material error of the computer. The check by taking first and second differences should also not be disregarded. The curves of cooling may be treated in a similar way.

130. I believe, however, that very fair results might be rapidly and approximately obtained by graphical methods alone. The curves of Statical Temperature and of Cooling being first projected in the usual way, tangents might be drawn mechanically for ordinates successively differing by 10°. The ordinate divided by the subtangent found would give the numerical values of $\frac{dv}{dx}$ and $\frac{dv}{dt}$. They would no doubt be somewhat irregular from the clumsiness of the graphical process; but being projected in terms of x and v respectively, and equalizing curves drawn through them, fair results would be obtained.† The "statical curve of cooling" is then constructed without any calculation whatever; and for evaluating its area up to any limiting ordinate, it might be sufficient that the curvilinear space it encloses should be defined on writing paper and cut out with scissors: the successive portions being weighed, would represent the flux of heat in known

* I ought perhaps to mention the formula which Professor Rankine has applied with success to express the elasticity of steam at all temperatures (Edin. Phil. Journ. 1849, vol. xlvii. p. 28, and Philos. Mag. 1854, vol. viii. p. 530). It is as follows:—

$$\log P = A - \frac{B}{\tau} - \frac{C}{\tau^2}$$

where P is the elasticity of vapour, and τ the temperature reckoned from an absolute zero (-274° cent). In applying the formula to the temperature of a bar, there can be no natural zero from which the lengths are reckoned along the bar; and therefore the constants, instead of three in number, may be reckoned as four; putting v instead of P in the above formula, and, instead of τ , writing $x + D$, D being some fourth constant. (See article 67.)

† This method was used by me in 1852.

units. I have little doubt that the results would come out within one or two hundredths of those obtained by elaborate calculations.

131. I have only to add, that the greater part of the computations in this paper were executed by Mr ALEXANDER PIRIE of St Andrews. Every part of the projections and graphical interpolations was performed by my own hand; and my thanks are especially due to the Messrs JOHNSTON for the unusual care with which they have been reduced in scale, and transferred to copper, as seen in the Plates.

ST ANDREWS, *April* 1865.