



XLVII. On the resistance of a fluid to a plane kept moving uniformly in a direction inclined to it at a small angle

Lord Kelvin

To cite this article: Lord Kelvin (1894) XLVII. On the resistance of a fluid to a plane kept moving uniformly in a direction inclined to it at a small angle , Philosophical Magazine Series 5, 38:233, 409-413, DOI: [10.1080/14786449408620651](https://doi.org/10.1080/14786449408620651)

To link to this article: <http://dx.doi.org/10.1080/14786449408620651>



Published online: 08 May 2009.



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The strip began to break down under the action of the alternating current at 900 volts ; at 3000 volts it was emitting small local arcs, some crimson, others violet. After the application for some time of this high voltage the number and brilliancy of the arcs diminished,—the insulation had apparently improved. At 5500 volts there were bright discharges similar to the first. These were not of the nature of long sparks, but of small local arcs.

Rod No. 6 gave way at 6500 volts and exhibited the same apparent improvement in insulation. The arcs shot out in miniature flames, like very small blowpipe blasts ; locally, as in the case of the strip.

With rod No. 9 sparking commenced at 1500 volts, having the appearance of small beads of crimson and violet light burning themselves out at fixed points. When these specimens had become cool their insulation was again tested.

Dielectric.	Resistance.
Strip	$= \infty$.
No. 6	$= 70,750$ megohms.
No. 9	$= 2020$ megohms.

The strip thus appears to have recovered entirely. No. 6 has fallen to a quarter of its first value, and No. 9 has greatly improved, under this trying ordeal.

Rod No. 15 was compounded of iron filings, Sr, &c., with a view to obtaining a brilliant discharge. The salts brought the insulation down rather low ; there was a great deal of heat generated, and little else. Such a rod should probably be made simply of metallic powders intermixed with the dielectric.

XLVII. *On the Resistance of a Fluid to a Plane kept moving uniformly in a direction inclined to it at a small angle.* By Lord KELVIN*.

§ 1. **L**ET q be the velocity ; i its inclination to the plane ; and u , v its components in and perpendicular to the plane. We have

$$u = q \cos i, \quad v = q \sin i.$$

§ 2. Suppose now the moving body to be not an ideal infinitely thin plane, but a disk of finite thickness very small in comparison with its least diameter, and having its edges everywhere smoothly rounded. If the fluid is inviscid and incompressible, and the boundary containing it perfectly

* Communicated by the Author.

unyielding, the motion produced in the fluid from rest, by any motion given to the disk, is determinately the unique motion of which the energy is less than that of any other motion possible to the fluid with the given motion of the disk. We suppose the disk to be *very* thin, and therefore the profile-curvature at every point of its edge to be *very* great: there is no limit to the thinness at which the proposition could cease to be true; so it still holds in the ideal case of an infinitely thin disk, when the fluid and its boundary fulfil the ideal conditions of the enunciation.

§ 3. But in nature every fluid has some degree of viscous resistance to change of shape; and any viscosity however small (even with ideally perfect incompressibility of the fluid and unyieldingness of the boundary) would prevent the infinitely great velocities at the edge of the disk which the unique minimum-energy solution gives when the disk is infinitely thin; and would originate so great a disturbance in the motion of the fluid that the resistance to the motion of the disk would probably be very nearly the same whatever the actual value of the viscosity, if not too great in comparison with the velocity of the disk multiplied by the least radius of curvature of the boundary of its area. No approach, however, has hitherto been made towards a complete mathematical solution of any case of this problem, or indeed of the motion of a body of any shape through a viscous fluid, except when, as in Stokes's original solutions for the globe and circular cylinder, the motion is so slow that its configuration is the same as it would be if it were infinitely slow, and when therefore* the velocity of the fluid at every point is equal to, and in the same direction as, the infinitesimal static displacement of an elastic solid when a rigid body imbedded in it is held in a position infinitesimally displaced from its position of equilibrium, in the manner translationally and rotationally corresponding to the translational and rotational velocity given to the rigid body in the fluid.

§ 4. It has occurred to me, guided by the teaching of William Froude regarding the continued communication of momentum to a fluid by the application of force to keep a solid moving with uniform translational velocity through it, that an approximate determination of the resistance, which is the subject of the present communication, may probably be

* The equations for the *steady* infinitely slow motion of a viscous fluid are identical with those for the equilibrium of an elastic solid. See 'Mathematical and Physical Papers' (Sir W. Thomson), vol. iii., art. cxix. §§ 17. 18.

found by the following method, with result expressed in § 9, which I venture to give as a *guess*, and not as a satisfactory mathematical investigation.

§ 5. Considering a disk of finite thickness, however small, moving in an inviscid incompressible liquid within an unyielding boundary, and, for a moment, thinking only of the u -component of the motion, according to the notation of § 1, let E and E' denote the front and the rear parts of the edge, respectively. Imagine now instead of the real motion of the unvarying solid disk through the fluid, that the disk grows all over E , by rigidification and accretion of the fluid in front of it, and melts away from E' by liquefaction of the solid. In an infinitesimal time δt , the extent of the accretion in front of E will be $u\delta t$. Now if the v component of the motion of the disk is maintained without diminution during this accretion, a force, F , equal to $(I' - I)/\delta t$, must be applied from without, perpendicular to the disk; I denoting the impulsive force which would be required to give the v -component velocity to the unaugmented disk, and I' that required to give the same velocity to the augmented disk. The point of application of the force $(I' - I)/\delta t$ must be that of the resultant of impulses I' and $-I$, applied at the hydraulic centres of inertia* of the augmented disk and the unaugmented disk respectively.

§ 6. Sudden cessation of the rigidity by liquefaction of any portion, (finite or infinitely small) of matter of the disk at E' requires no instantaneous application of force, to prevent change of the v -motion of the residual solid. The continued gradual liquefaction which we are supposing performed, leaves a Helmholtz "vortex sheet" of finite slip growing out in the liquid, behind E' , the evolutions and contortions of which are not easily followed in imagination. This sheet is in the form of a pocket of which the lip remains always attached to the solid disk. The space enclosed between it and the disk is filled by the liquid which was solid. It grows always longer and longer by gain of liquid from the melting solid at E' in front of it, and probably also by its rear extending farther and farther, far away in the wake of the disk.

§ 7. Suppose now that, after having been performed during a certain time T , the ideal processes of §§ 5, 6 are discontinued, and the resulting solid disk, equal and similar to the original disk, but carried in the u -direction through a space

* I call the "hydraulic centre of inertia" of a massless rigid disk immersed in liquid the point at which it must be struck perpendicularly by an impulse, to give it a simple translational motion.

equal to vT , is left with simply its v -motion through the fluid maintained. The pocket of liquefied solid will be left farther and farther behind the disk. Its mouth, still always stopped by the solid, will shrink from its original area which was the whole of E' ; and will become always smaller and smaller, but not infinitely small in any finite time. The neck of the pocket in the wake of the disk will become narrower and narrower, and the whole pocket will be drawn out longer and longer behind; but, through all time, the fluid which was solid will remain separated by a surface of finite slip, or Helmholtz "vortex sheet," from the surrounding fluid, except over the ever diminishing area of the disk, which stops the mouth of the pocket. The motion of the fluid is irrotational outside the pocket, and rotational within it. To keep the solid disk moving with its v -motion constant, and with no other motion whether rotational or translational, it is necessary to apply force to it. But this force becomes less and less, and approximates to zero, as the vortex-trail becomes finer and finer; and the motion of the fluid in the neighbourhood of the disk approximates more and more nearly to perfect agreement with the unique irrotational motion due to v -motion of the solid through the fluid.

§ 8. So far we have, in §§ 5, 6, 7, been on sure ground, and every statement is rigorously true, not only for a "disk" of any shape of boundary and of any thickness however small, but also for a solid of any shape, dealt with according to § 5, provided only that the fluid is inviscid and incompressible, and its boundary unyielding. My hypothesis, or "guess" (§ 4), which forms the subject of the present paper, is that default from infinitely perfect fulfilment of all these three conditions would, for an infinitely thin disk kept moving with uniform translational motion (u, v , § 1), require the continued application to it of force determined in magnitude and position by § 5; *provided v be very small in comparison with u .*

§ 9. The result is worked out with great ease for the case of a rectangular disk of which the length, l , is very great in comparison with the breadth, a . For this case, by the well-known hydrokinetics of an ellipsoid or elliptic cylinder moving translationally in an inviscid incompressible fluid of unit density, we have

$$I = \frac{1}{4}\pi a^2 l v;$$

and, still using the notation of § 5,

$$I' = \frac{1}{4}\pi(a + u\delta t)^2 l v.$$

Hence

$$F = \frac{1}{2}\pi a l u v;$$

and the distance of the point of application of this force from the middle line of the rectangle is

$$\frac{1}{4}a.$$

Comparison of this hypothetical result, with observation, in respect both to the magnitude of the force and its point of application, will, I hope, form the subject of a future communication.

Eastern Telegraph Company's Cable-ship 'Eleetra,'
Athens to Genoa, Sept. 3 ... 6, 1894.

XLVIII. "*Densities in the Earth's Crust.*"

By Rev. J. J. BLAKE, M.A., F.G.S.*

MR. OSMUND FISHER having been unfortunately called upon to categorically assent to or answer my criticisms of a portion of his work on the 'Physics of the Earth's Crust,' has attempted the latter alternative in the April number of this magazine. I had hoped that he would have adopted the former on more careful consideration of the subject; but, as it is not so, I hope I may be excused if I point out still more clearly the gist of my objections to his method.

In several places Mr. Fisher does not seem to recognize that it is not his conclusions but his mathematics that are discussed, for if the mathematics are wrong there are no conclusions to discuss. Thus he begins by stating that if I had more fully mastered his "results," I should not have stated that "the argument ... seems to depend on the greater density of the superficial layer in continental than in oceanic areas," which he observes is the exact opposite of his "conclusion;" but, as ocean water is certainly of less density than rock, this is not a conclusion at all, but a datum—the mathematical problem being this:—Given that the attraction of a sphere on a particle at any point on its surface is constant for all such points— but that the superficial layer in one part is less dense than in another—find the relations between the densities and thicknesses of the underlying layers.

Again, he says it would be absurd to assume any other law of attraction than the Newtonian for the case of nature,—very likely; but the objection is that, if the method were correct, the same results might be deduced even from an absurd law. It is for this reason that I call the functions he speaks of ($f(\theta)$ &c.) "unknown."

Again, he answers my objection that in his solution "it is

* Communicated by the Author.