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of the mineral as deduced from the third and fourth columns of the preceding table:—

	By calculation.	No. of atoms.	Atomic weight.
Silica	47·24	23	46
Protoxide of iron. . .	18·45	4	18
Protoxide of manganese	9·24	2	9
Alumina	6·94	3	6·75
Lime	14·39	4	14·
Magnesia	2·59	1	2·5
Water	1·15	1	1·125
	100·	38	97·375

The formula to express the composition of babingtonite, as deduced from these calculations, is as follows:—

$MgO\ SiO, + 4CaO\ SiO + 4FeO\ (SiO)_2 + 2MnO\ (SiO)_2 + 3AlO\ (SiO)_2 + HO$;

or it consists of two simple silicates and three bisilicates.

Babingtonite agrees with the amphibole class in possessing a complicated constitution. It contains nearly the same amount of silica, but much less magnesia and a greater amount of manganese. Its specific gravity is also much higher than that of amphibole. My analysis approaches one by Bonsdorff, of a black hornblende from Nordmark and Pargas, the magnesia being replaced by manganese in babingtonite.

XX. *On the Resolution of Equations of the Fifth Degree. By* James Cockle, *Master of Arts, of Trinity College, Cambridge; of the Middle Temple, Special Pleader*.*

1. IN the equation of the fifth degree in y,

$$y^5+p_1y^4+p_2y^3+p_3y^2+p_4y+p_5=0. \quad . \quad . \quad (1.)$$

Let
$$p_3=\frac{p_1}{5}\left(\frac{2p_1^2}{5}+3\,\mathrm{A}\right). \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

and
$$p_4=\frac{p_1}{5^2}\left(\frac{p_1^3}{5}+3p_1\,\mathrm{A}+\mathrm{A}^2\right), \quad . \quad . \quad . \quad (3.)$$

where
$$\mathrm{A}=p_2-\frac{2p_1^2}{5}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

then, if $v-\frac{p_1}{5}$ be substituted for y in (1.), we obtain an equation in v of the form of De Moivre. Denoting by x the root of the general equation of the fifth degree, let

$$y=\Sigma_2(\Lambda'x^{\lambda'})+\Sigma_4(\mathrm{M}'x^{\mu'})+\mathrm{N}'x^{\nu'}+\mathrm{P}'x^{\pi'}+\mathrm{Q}'x^{\kappa'}+\Sigma_2(\mathrm{R}'x^{\rho'}), \quad (5.)$$

the suffixes denoting the number of quantities included under each Σ, and the expression for y consisting, consequently, of

* Communicated by T. S. Davies, Esq., F.R.S., F.S.A.

eleven terms, so that, by the notation of p. 114 of vol. i.* of the Mathematician, (2.) and (3.), respectively, become

$$f^3(10) = 0 \quad . \quad . \quad (6.), \qquad \text{and } f^4(10) = 0 \quad . \quad . \quad . \quad (7.)$$

Now I have shown, at paragraph 6 of that page, how, by means of a quadratic, which we shall here represent by

$$\theta_{10}{}^2(z_{10}) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

and four *base*† quadratics which may, respectively, be denoted by

$$J_9^{(2)}=0 \;.\; (b.), \quad J_8^{(2)}=0 \;.\; (c.), \quad J_7^{(2)}=0 \;.\; (d.), \quad J_6^{(2)}=0 \;.\; (e.)$$

(6.) may be reduced to

$$h_1{}^3 + f^3(4) = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

and also, how, by means of a quadratic,

$$\theta_3{}^2(z_5) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f.)$$

a 'base' quadratic, $\quad J_4^{(2)} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g.)$

and a cubic, $\quad \theta^3(z_3) = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h.)$

(8.) is ultimately reduced to the base cubic,

$$h_1{}^3 + h_2{}^3 = J_2^{(3)} = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad (i.)$$

whence, by eliminating z_2 between this last equation and (7.) (which we may now reduce to $f^4(2) = 0$), we arrive at a final biquadratic,

$$C_1 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (j.)$$

for determining z_1; and z_2, &c. can be found by means of the other equations distinguished by letters; and, consequently, Λ', Λ'', &c. are known.

2. In this investigation, if we suppose that

$$x = Q\,x^q + \psi(x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

then our object is *not* to satisfy (6.) *independently* of Q, so as to have that quantity at our disposal for the purpose of satisfying (7.)‡, but to reduce (6.) to a linear form, and then eliminate between (6.) and (7.).

3. In general, after effecting our fundamental reduction

$$f^a(b) = \Sigma(h^a), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

we may group any two of the quantities h^a together, as in the above instance, and make their sum equal to zero; or should it in any case be necessary, we may increase the number of disposable quantities, and, possibly, obtain a (transcendental?) result, by grouping two or more terms of the right-hand side of (10.) together, and making their sums equal to arbitrary

* The 2nd page of No. iii. (July 1844).

† The word "base" is used to indicate that they degenerate to linear equations.

‡ See Phil. Mag., S. 3. vol. viii. p. 538, and vol. ix. p. 28.

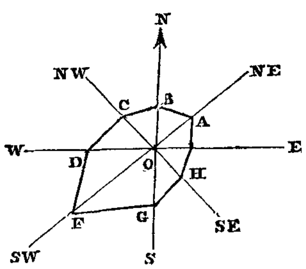
quantities, taking care that the sum of the arbitrary quantities shall vanish; or we may change the right-hand side of (10.) into $\chi(h)$, χ being an arbitrary function.

4. The values of α to be selected in the transformations of quadratics and biquadratics, which I gave at p. 384 of the last volume of this work, are, respectively, 1 and -1.

Cambridge, July 1, 1845.

XXI. *On a certain Method of representing by Diagrams the Results of Observations. By the* Rev. S. Earnshaw, *M.A., Cambridge**.

IN attending the proceedings of the Physical Section of the British Association at the late meeting in this place, I was struck on more than one occasion with a defect in the method which had been employed by some of the experimentalists in representing their results on paper. My remarks refer to those cases in which polar co-ordinates were used, as was done in two very interesting and important papers; one "On the quantity of rain which had fallen with different winds at Toomavara," and the other "On Shooting Stars." In the former paper, the observer (the Rev. Thomas Knox) reduced his observations to a pictorial state, by drawing upon paper from a fixed point several lines to represent as many directions of the wind; and this done, he set off upon each of these lines, measuring from the fixed point, a length proportional to the quantity of rain which had fallen while the wind was in that particular direction, and within certain limits on each side of it. Perhaps this will be plainer by a figure.



From O draw eight lines, making angles of 45° with each other, to represent the directions of the wind according to the letters placed at their extremities. From O set off OB to represent the quantity of rain which fell, not only when the wind was in the north, but also when it was in any direction between the points N.N.E. and N.N.W. On a similar plan set off the other portions OC, OD, ... This was the method employed by Mr. Knox; and a similar principle, I believe, was made use of by M. Gravier† in reducing to paper

* Communicated by the Author.

† A notice of M. Gravier's researches on Shooting Stars will be found among the miscellaneous articles of the present Number.—Ed.