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are instructive, and teach a sense of proportion. The great weakness of boys confronted with a numerical problem is that they cannot see where accuracy is essential and where it is entirely useless.

All the logarithms are of course Napierian. As simpler examples take

$$e^x = 10^{10} x^{10}, \quad e^{x^2} = 100^{100} x^{100}, \quad e^{\sqrt{x}} = 10^{10} x (\log x)^{10},$$

etc., etc. Graphical methods may sometimes be used with advantage.

G. H. HARDY.

**320. [A. 3. g.]** *The solution of the equation  $x^{2n+1} + ax^2 - b = 0$ .*

An extension of the series method of solving an equation proposed by Lagrange (Todhunter, *Theory of Equations*, §§ 295-7) is possible in the case of the equation  $x^{2n+1} + ax^2 - b = 0$ . Let  $\alpha, \beta$  be the two numerically least roots of this equation, then

$$\begin{aligned} x^{2n+1} + ax^2 - b &= (x - \alpha)(x - \beta)\psi(x); \\ \therefore 1 - \frac{b - x^{2n+1}}{ax^2} &= \frac{1}{\alpha} \left(1 - \frac{\alpha}{x}\right) \left(1 - \frac{\beta}{x}\right) \psi(x), \end{aligned}$$

where  $\psi(x)$  is a rational integral algebraical function of  $x$ .

Then, bearing in mind the considerations adduced in Todhunter, § 296, we must get valid results if we take logarithms of both sides and equate the coefficients of the negative powers.

Equating the coefficients of  $\frac{1}{x}$  we get

$$\alpha + \beta = -\frac{b^n}{\alpha^{n+1}} - \frac{(3n+1)3n}{3!} \frac{b^{3n-1}}{\alpha^{3n+2}} - \frac{(5n+2)(5n+1)5n(5n-1)}{5!} \frac{b^{5n-2}}{\alpha^{5n+3}} - \dots$$

Equating the coefficients of  $\frac{1}{x^2}$  we get

$$\frac{\alpha^2 + \beta^2}{2} = \frac{b}{\alpha} + \frac{2n+1}{2!} \frac{b^{2n}}{\alpha^{2n+2}} + \frac{(4n+2)(4n+1)4n}{4!} \frac{b^{4n-1}}{\alpha^{4n+3}} + \dots$$

As these series are frequently very rapidly convergent  $\alpha + \beta$ , and  $\alpha^2 + \beta^2$  can be readily calculated, and  $\alpha$  and  $\beta$  deduced, at all events to 7 figures, by logarithms.

As an example take  $2x^3 + 9x - 1 = 0$ . Here  $\alpha = \frac{9}{2}$ ,  $b = \frac{1}{2}$ ,  $n = 1$ , and we get

$$\begin{aligned} \alpha + \beta &= -\frac{2}{9^2} - 2 \cdot \frac{2^3}{9^5} - 7 \cdot \frac{2^5}{9^8} - 30 \cdot \frac{2^7}{9^{11}} - \dots, \\ \frac{\alpha^2 + \beta^2}{2} &= \frac{1}{9} + \frac{3}{2} \cdot \frac{2^2}{9^4} + 5 \cdot \frac{2^4}{9^7} + 21 \cdot \frac{2^6}{9^{10}} + \dots \end{aligned}$$

Results correct to 5 places of decimals may be obtained from the first three terms. We obtain

$$\alpha + \beta = \cdot 024968\dots,$$

$$\alpha^2 + \beta^2 = \cdot 224085\dots$$

These give  $\alpha = \cdot 322011\dots$ ,  $\beta = -\cdot 346979$ .

As a fact these are correct, the third root being  $-4\cdot 475032$ .

A curious point arises. What is the meaning of the result when there is only one real root? Presumably the series are divergent, but the converse of this statement is clearly untrue. ANON.

**321. [V. 1. a.]** May I add to the suggestions already made for suitable symbols the following:

$$\begin{aligned} \asymp & \text{ for "is approximately equal to."} \\ \gtrsim & \text{ " "is approximately equal to but } > \text{."} \\ \lesssim & \text{ " "is approximately equal to but } < \text{."} \end{aligned}$$

They are associated with the symbols  $>$  and  $<$ , and are easy to write.

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