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### XXXV. Microscopic vision

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the rearrangement of particles, the viscosity steadily lessens, this entitles us to suppose that with an alternate magnetization the maximum of it does not reach that quantity attained by a static process. And certainly the most recent observations on the exhaustion of iron in transformers seem to corroborate this. From the point of view of the present hypothesis, the exhaustion of iron is nothing but the rearrangement of its particles, in consequence of which the magnetization is performed with a smaller absorption of energy, and therefore calls for a less pronounced magnetic effect.

Physico-chemical Society.  
The University, St. Petersburg, Russia,  
June 15, 1896.

XXXV. *Microscopic Vision*. By G. JOHNSTONE STONEY,  
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PART I.—FUNDAMENTAL PRINCIPLES †.

1. **V**ISION, whether by the naked eye or with the assistance of optical instruments, may be studied in many ways; since a correct investigation may start from any one of the innumerable possible resolutions of the disturbance which exists throughout the æther in front of and close to the object. But two only of these will be here considered, viz.: that most obvious resolution in which the æther in front of the object is regarded as traversed by undulations of *hemispherical waves* emanating from each physical point of the surface of the object, and that other equally general but less obvious resolution of the disturbance in this portion of the æther into undulations of *uniform plane waves* transmitted forwards in all or some directions from the whole extent of the objective field.

2. The first of these modes of resolution—that into spherical waves—is the foundation of Airy's method of studying the images formed by telescopes, in which the image is regarded as arising from the overlapping and interference of the spurious disks with attendant rings which in the image take the place of points on the object. This method has on this account been sometimes called the Spurious Disk Theory. The second mode of resolution—that into plane waves—is the foundation of Abbe's method of studying the images formed

\* Communicated by the Author.

† Part II. deals with the application of these principles to the microscope as at present made.

by microscopes ; and has usually been called the Diffraction Theory, because it gives a *special* prominence to the fact that when we pass beyond the meagre hypotheses of geometrical optics, we find that diffracted light\* is "the machinery by which good definition is brought about." It was undoubtedly desirable to emphasise this fact, because an error prevailed and is not yet extinct that diffracted light intervenes only to impair the image ; and it can scarcely be made any objection to the name that it runs counter to this error. As, however, both processes are only methods of investigation, it would perhaps be desirable to avoid calling either of them a theory. On this account, and to avoid cavilling about mere names, the two methods of investigation are in the present memoir distinguished as the Airy and the Abbe modes of procedure †.

3. In a recent paper by Lord Rayleigh the generality of Abbe's method seems not to have been appreciated (see Phil. Mag. for last August, p. 167); and the main object of the present communication is to offer a fuller account of this generality than the writer has elsewhere given (see "On the Foundation of the Diffraction Theory"; 'English Mechanic' for December 13, 1895, p. 380), and to trace its consequences.

4. Two terms have been used above in the first paragraph which need to be defined.

(a) By a physical point is to be understood an element of the volume of the object (if the object be translucent), or of its superficial layer (if it be opaque), which element of volume is small enough to justify us in substituting for it in our investigation a mathematical point regarded as a centre of an undulation of hemispherical waves. The physical point is small enough for this use of it, if its linear dimensions are in any considerable degree less than  $\lambda/4$ , where  $\lambda$  is the wavelength of the light employed. To give definite form to our conceptions we may suppose its dimensions to be comparable with  $\lambda/10$ . This is a convenient size ; since if an opening of this size were made in a thin opaque screen, and if a pencil of light were incident from any direction upon it, the hole is small enough to ensure that the light which gets through shall spread on the other side of the screen in the form of

\* Light which advances in other directions than those prescribed by geometrical optics is called diffracted light.

† Lord Rayleigh suggests the name Spectrum Theory for the method of investigation which proceeds by resolving the light into plane waves ; but will perhaps not press this name on the acceptance of scientific men when he finds that the limitation which the name implies has no existence.

hemispherical waves; and at the same time the opening is a large one when compared with the transversal\* of the light waves, since molecular considerations indicate that this transversal (or rather these transversals, since there are two, an electric and a magnetic one at right angles to one another) must be regarded as of a length which is from the thousandth to the ten thousandth of a wave-length. Hence the directions of transversals will not be affected in passing through the opening. On this account, if the incident light be a beam of plane waves, whether polarized or not, the intensity of the light will differ on the various parts of the hemispherical waves which spread beyond the screen, being a maximum in the direction of the prolongation of the normal to the incident waves. This must be taken into account in attempts to apply Airy's method of investigation to microscopic vision, since until this is sufficiently done the investigation is too imperfect for us to be justified in relying on its results except so far as they can be confirmed by Abbe's method or some other which does not involve the above consideration. A further and more serious imperfection is introduced when Airy's method is applied only to the light between the objective and the image, and not also to the light between the object and the objective. An inquiry conducted in this way begins too late, after the more important of the events that affect the image have occurred. Nevertheless it seems to be the only one which has as yet been made by Airy's method; see, for example, the investigation on p. 176 of Lord Rayleigh's paper. We shall learn in the second part of this memoir what it is that in this case is being ascertained.

(b) The other term in paragraph 1 that requires definition is *the objective field*. By this term is to be understood the whole of the object and its surroundings of which an image is formed by the telescope or microscope, or in the eye of the observer. Accordingly the objective field at and surrounding the object corresponds to 'the field of view' at and surrounding the image of it which is formed in the eye, or at the focus of an optical instrument.

\* The word transversal is here and elsewhere used for the transversal of the displacement under the dynamical wave theory of light.

The dynamical wave theory is that used throughout this memoir, except where otherwise stated; since, in the present state of our knowledge, it is more easily handled than the electromagnetic wave theory, and since, except in special cases (as for example the distribution of intensity over a spherical wave), it furnishes the same results. Besides, the dynamical theory usually carries us as far as we can go, for, in the special cases where the electromagnetic theory may yield a different result, it seldom happens that we yet know that result.

5. The following important optical theorem may now be enunciated, which in its generality compares with Fourier's Theorem, of which it is, in fact, in ultimate analysis, an extension.

PROPOSITION 1.

*However complex the contents of the objective field, and whether it or parts of it be self-luminous or illuminated in any way, however special, the light which emanates from it may be resolved into undulations each of which consists of uniform plane waves; on the hypothesis that each point of the object emits continuously the same light: an hypothesis the sufficiency of which will appear in Part II. of this memoir.*

By an undulation is meant a succession or train of waves, and by a uniform wave is meant one which is at each instant alike in every part of each wave surface.

6. To prove this theorem we proceed very much in the same way as in dealing with Fourier's Theorem. We begin by positing repetitions of the objective field. For this purpose let a plane be drawn through some point of the objective field, and preferably perpendicular to the line of sight. This plane may be called the Objective Plane. Let a square be drawn in this plane which may be of any size, provided that it shall include within it the projection upon the plane, from the point of view of the observer, of the contents of the objective field: in other words, the square is to be large enough for the whole of the objective field—the whole of what the observer can see—to fall within that square, and preferably well within it. Divide the whole plane up into squares of this size by two systems of equidistant parallel lines, and imagine an exact repetition of the contents of the objective field to occupy the position relatively to each of these except the first, which is the same as the position actually occupied by the contents of the real objective field in reference to the first square. Next suppose light to be emitted from every point of each of these replicas, *which is at each instant similar in every respect—i. e. the same in direction, intensity, phase, and position of transversal—as is the light from the corresponding point of the original objective field at that instant.*

Under these circumstances a point  $p$  in the original objective field, along with the corresponding points  $p'$   $p''$  &c. in the replicas of the objective field, form a system of points equally spaced over a plane which is parallel to the objective plane. Now it is known, from the theory of diffraction gratings (see the figure on p. 340), that such a system of

points equally spaced in a plane, and all emitting light which at each instant is exactly similar, will produce a disturbed condition of the æther which is resolvable into plane waves advancing in certain definite\* directions. The same is true of each other point of the original objective plane with its replicas. Hence, and since by the principle of the superposition of small motions the total disturbance in the æther caused by the whole contents of the objective field and of all its replicas is the resultant produced by a simple geometrical summation of the disturbances which would be produced by the several points of the original field and their replicas, it follows that in ultimate analysis the total disturbance is resolvable into the undulations of plane waves into which its

\* The luminous effects produced in these definite directions are maxima, and they are accompanied by luminous effects produced in other directions also; but it is legitimate to leave these out of account. We are in fact investigating the disturbance within a layer of limited thickness, the layer between the objective plane and the plane in which the front of the objective lies; and the luminous energy expended on any effects within that layer, other than those producing the plane wave dealt with in the text, can be made relatively as small as we please by increasing the spacing between the replicas.

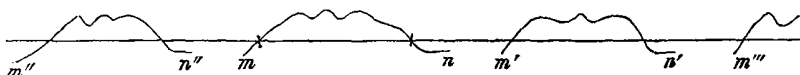
This will perhaps be made clearer by considering the analogue in a Fourier's expansion. If the first  $n$  terms of a Fourier's expansion of any function be added together, they furnish an approximation to that function which is nearer the larger  $n$  is, and which can be made as close an approximation as we please by increasing  $n$ . Now the sources of similar light  $p, p', p'', \&c.$ , furnish a number of fans of undulations of plane waves, each fan analogous to a limited number of terms of a Fourier's expansion, this limited number being proportional for each fan to  $\delta/\lambda$ , where  $\delta$  is the spacing of  $p, p', p'', \&c.$ , and  $\lambda$  is the wave-length. They are therefore susceptible of indefinite increase by increasing  $\delta$ . Moreover, the fans which have the smaller number of terms become rapidly the fainter: see the figure on p. 340, in which the closer the ruling the smaller will be the number of terms of the corresponding fan.

The outcome of these considerations is that the ætherial disturbance in front of the objective plane may be such that to resolve the whole of it with absolute accuracy into undulations of plane waves would require that these undulations shall spread in all (corresponding to  $\delta$  being indefinitely large) instead of some (corresponding to  $\delta$  being finite) directions. But, practically, a very moderate value for  $\delta$  is sufficient; since the approximation is carried far enough when the outstanding luminous effects are too faint either to be seen by the eye or to affect a photographic plate sensibly.

Even if the closest of the replicas were much closer in than we have supposed, they would not sensibly interfere with the vision of the original object. Two or more diatoms seen together within the same field of view do not sensibly interfere with the most satisfactory vision of each of them, nor would they if they all emitted light from their corresponding points which was strictly the same at each instant in phase, direction of transversal, and intensity. Each would still be as fully seen as our eyes are capable of seeing, notwithstanding the presence of the others.

constituent disturbances are resolvable. The number of these undulations may be reduced wherever any of them travel in the same direction, since any number of undulations of plane waves of wave-length  $\lambda$  travelling in the same direction may be combined into a single undulation of plane waves travelling in that direction. Hence the total disturbance is resolvable into undulations of uniform plane waves, only one of which for each value of  $\lambda$  travels in each direction.

7. This valuable optical theorem bears a remarkably close analogy to Fourier's Theorem for the expansion of an immense class of functions. Thus by Fourier's Theorem a portion of



curve  $mn$  along with equidistant repetitions of the same to the left and right may be expanded in the form

$$y = A_0 + A_1 \cos \frac{2\pi x}{a} + A_2 \cos 2 \frac{2\pi x}{a} + \dots$$

$$+ B_1 \sin \frac{2\pi x}{a} + B_2 \sin 2 \frac{2\pi x}{a} + \dots$$

in which the values of the constants  $A_0, A_1, A_2$ , &c.,  $B_1, B_2$ , &c., depend on what direction has been selected for the line over which the repetitions are to be disposed, and on what interval has been chosen for  $a$  ( $a$  being  $mm'$ , the spacing of the curves from one another). So in our optical theorem, the plane waves into which the light emitted by a point  $p$  in the objective field is to be resolved will depend on what plane has been chosen for the objective plane, and on the intervals at which  $p, p', p''$ , &c., are to occur in that plane, as well as on whether the lines joining them lie (as we have placed them above) at right angles to one another, or in other available positions. However, just as in a Fourier's expansion the original curve is always correctly represented whatever assumption we may have made as regards the orientation of the axis of  $x$  and the length of the line  $a$ , and it is only the situation of its replicas which is affected by this choice; so under our theorem the light in front of the objective field is always adequately resolved whatever selection we may have made as regards the optional matters (provided the conditions laid down in the footnote on p. 336 are observed), and it is only where its replicas are to be regarded as situated that is affected by that choice. Moreover, when once we have made

our choice as regards these optional matters, the plane waves emanating from the whole field into which the light emitted by the point  $p$  is to be resolved under our theorem, become as definite and unique as do the coefficients of a Fourier's series when once we have decided on the direction of the line  $mm'$  and have selected a value for  $a$ . This, however, still leaves a considerable latitude under our theorem, as to what the undulations of plane waves shall be, since the objective field may be variously chosen, and the only conditions which limit the positions to be selected for the replicas are that they and the original objective field be equally spaced relatively to the objective plane, and that the nearest of the replicas shall lie far enough outside the objective field to ensure that whether sources of light exist in them or not shall not sensibly interfere with what is seen by the observer. They are to him stars below his horizon, whose positions or even existence in no perceptible degree affect the distinctness with which he sees the stars that are above his horizon.

8. *Principles of Reversal*.—A further insight into what it is that occurs may be gained by a simple expedient. Picture a portion of the objective plane, of limited but large size—large enough to have the original objective field near its middle, and a great many of its replicas disposed round it. If all of these emit light that is exactly similarly circumstanced, then, as already explained, it appears that they, acting together, will produce undulations of very nearly uniform plane waves which will become more and more disentangled from one another the farther out they go. It is in fact when thus disentangled that their consisting of almost quite plane waves becomes most obvious. The approximation to accurately uniform plane waves can, of course, be carried as far as we please by increasing the number of replicas engaged in emitting the light.

Let now all the ætherial motions be suddenly reversed, and let at the same time the objective field and its replicas be got out of the way. The distant undulations which were before advancing outwards will now travel inwards without ceasing to be uniform plane waves, and will by simple geometrical superposition, according as they overlap one another, reproduce at each step of their inward journey exactly the same disturbed state of the æther as had prevailed at the same stations on the outward journey, except that the directions of all motions are reversed. Hence plane waves converging inwards would by their superposition produce precisely the same disturbance in the æther, except only with reversed motions, as that which on the outward journey prevailed close in front



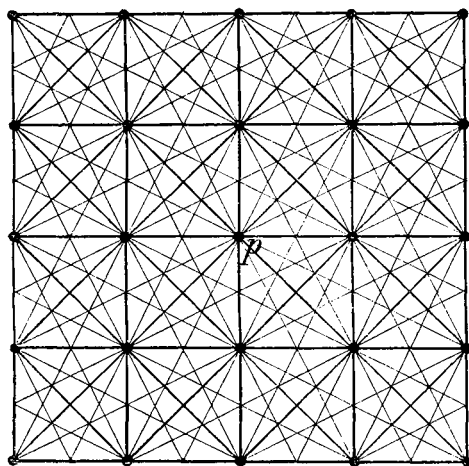
of the object in the objective field ; and, finally, if the travelling backwards is continued long enough for the undulations to reach the positions that had been occupied by the original object, they will there produce an image of it the most perfect which the light that had been emitted by the object is capable of producing. This image thus becomes a standard of perfection which may be approached but cannot be exceeded by the images formed by any optical contrivance from the same light.

From the way in which the standard image is formed it is manifest that it is an image of the same size and general shape as the object or group of objects represented by it. The further excellence of this image depends upon the amount of detail upon the object which it is competent to reproduce ; and this varies, as we shall find presently, with the wavelength of the light employed, and with the way in which that light has been supplied to the object. The standard image may be regarded either as viewed from beyond, or as being transformed into a picture by being thrown on a screen able to scatter whatever light falls on it. The screen should not be flat, but with such prominences and depressions as will enable it to catch the light everywhere exactly where the image is formed. Such a picture is entitled to the name of the standard picture, since it has on it all that part of the detail on the real object which the light is capable of showing\*.

9. *Theorem 2. The Standard Image.*—Let us consider somewhat more closely how the standard image is formed. It is formed by the coalescence and mutual interference of uniform plane waves. Now when we consider how these same undulations originated when starting on their outward journey and remember that the condition of the æther is the same on their return, except as regards the direction of the motions ; when we further remember that the point  $p$

\* Another way of conceiving the standard image which is for some purposes more convenient, is to imagine the retreat of the luminous undulations to be carried farther backwards (the condenser of the microscope and any other obstruction being of course removed) ; and then, at a given instant, to conceive the ætherial motions to be again reversed. The undulations will thereupon travel forwards (*i. e.* in the direction in which the light originally moved), will re-form the standard image when they reach the position that had been occupied by the object, and will thence proceed to the objective of the microscope in precisely the same state as was the light that was transmitted to it by the real object. *It thus appears that the source of light, the condenser, and the object may be all removed, and that the standard image emitting its light forward may be substituted for them.*

and its replicas emitted portions of light which at the instant of starting were exactly alike, and that the undulations which result from this state of things may be thrown into groups of undulations, each of which is the same as would have been emitted by one or other of the uniform rulings of equidistant lines represented in the accompanying figure, as well as the



vast number of others that would arise from sufficiently extending the figure ; when we further bear in mind that every equal element of any one of the lines in each such ruling emits the same amount of light, which is in the same state as that emitted by *p* except as regards intensity : when all these things are taken into account we find that the entire of the standard image may be regarded as built up of such luminous rulings superposed upon and interfering with one another—each of these rulings being due to the convergence and mutual interference of two or more undulations of the uniform plane waves which (since the reversal) have been travelling inwards, and each ruling accordingly being uniform and extending across and even beyond the whole range of the objective field.

This is our second theorem. It may be enunciated as follows:—

#### PROPOSITION 2.

*The standard image may be regarded as resulting from the superposition and mutual interference of uniform luminous rulings of equidistant parallel bright lines extending over the whole field of view ; each ruling being produced by the*

convergence upon it, after the reversal, of two or more of the undulations of uniform plane waves into which the light emitted by the object may be resolved.

10. Of course other resolutions than the two hitherto considered—that into spherical waves thrown off from the several points of the surface of an object, and that into plane waves thrown off from the surface as a whole—are possible: and in fact, if a resolution of the disturbance in the æther between the object and the objective of a microscope is made into plane waves, these will become curved while passing through and after emerging from the objective; and it is as curved waves that they reach and produce the microscopic image. They, in fact, become convex waves that are nearly spherical. The centres of these nearly spherical waves are obviously the points of the focal plane (or rather, focal surface, for it is slightly curved) of parallel light incident on the objective. This focal plane lies between the objective and the microscopic image, and in all the cases that need to be considered it lies near the objective, and therefore sufficiently far from the microscopic image to render the curvature of the waves where they reach that image but slight.

11. *Magnification*.—Let us now return to the standard image. It is of the same size as the object. If we could by any contrivance increase the wave-lengths of the light that forms it—if, for instance, we could make the wave-lengths a thousand times larger, making them the same fractions of a millimetre which actual light-waves are of a micron—we should in this way enlarge the image 1000 times, since the interference of the longer waves coming in the same directions as before would produce rulings all of which would be 1000 times coarser than before. *This enlarged image would obviously contain precisely the same amount of detail as the standard image.*

This method of enlarging an image is only practicable on a small scale, since we can but slightly increase wave-lengths (as when we place the object in a highly refracting medium and its image in the air); but what is very much the same result may be brought about in another way, viz., by diminishing the inclination of the beams of plane waves to one another, without altering the lengths of the waves; since the ruling which results from the interference of two such beams may be made coarser either by lengthening the waves of which each beam consists, or by diminishing the inclination of the beams to one another.

12. *Useful work done by the objective*.—The useful part of what is accomplished by the objective of a microscope is that

it diminishes the inclination of these beams to one another. This brings about two desired results: it enlarges the image, and it makes it possible for its constituent beams, after they have passed the focal image, to be collected by the eyepiece and transmitted through so small an opening as the pupil of the eye, instead of diverging over the great extent to which they were spreading when they left the object.

13. *Useless work done by the objective*.—But the objective cannot accomplish this useful work without at the same time producing other effects which are undesirable. Thus, it transforms the beams of plane waves into convex beams, as explained in § 10. This somewhat distorts the image. The image is still more distorted in the direction of the line of sight, whereby any elevation on the object is shown as unduly prominent in the image\*. Neither of these distortions, however, would cause the amount of detail in the microscopic image to fall short of that in the standard image.

That which above all produces this defect, and produces it however well the spherical and chromatic aberrations of the objective may have been corrected, is that the angular aperture of the objective falls short of  $180^\circ$ . With the best immersion-lenses the angular aperture is about  $120^\circ$  or  $130^\circ$ , so that little more than half the light would be caught by the objective, if the light were emitted equally in all directions. One part of the light thus excluded is that which in the standard image brings out the finest part of the detail which that image can reach, since it is the light which produces the finest of the rulings that form the standard image.

There is another imperfection consequent upon this exclusion of part of the light emitted by the object, viz., the intrusion into the microscopic image of intercostal markings, false resolutions, a general haze of light, and so on—additions to the image and other alterations of it which have nothing to correspond to them either in the object under examination or in its standard image. The following is perhaps the easiest way of understanding how they arise.

14. *The Visual Substitute*.—In order to study microscopic vision, or vision of any kind, with full effect, it is well to begin with the consideration that what we seem to see with the naked eye is never the natural object itself, nor is it an enlargement of it when we examine it through a microscope or telescope. What we see is, in fact, only a visual *substitute* for the real object in the first case, and for an enlargement of the same when we use an instrument; and the study of

\* This distortion may be traced by an elementary investigation in geometrical optics.

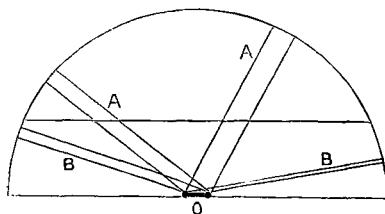
vision, whether microscopic, telescopic, or with the naked eye, is in fact the study of what this visual substitute is and how it stands related to the real object, *i. e.*, what alteration the real object would have to undergo to be transformed into its visual substitute, which is what seems to us to be the object presented to us.

The real object, O, sends forward the light which enters the eye, and, in addition, other light which does not enter the eye, whereas its visual substitute, S, is to be defined as that other object from which would emanate the light which enters the eye *and it only*. It is evident that objects O and S will seem to us exactly alike, but that whereas we receive the whole of the light which S is competent to dispense, we receive only a part of that emitted by O. Similarly, when we use a microscope or telescope, what we seem to see is a visual object, C, which would emit exactly the light which the eye takes in, and it only; and this is in all cases less than the light which an enlargement of the object would emit, and may differ from it in other respects also. It is, accordingly, to the study of what these visual substitutes are that we should apply ourselves.

But as this is a branch of optics which is as yet almost wholly unexplored\*, we must, for the present, be content with the inferences we can draw from such general considerations as the following:—

15. *Proposition 3*.—The objective of a microscope has an angular aperture which is necessarily less than  $180^\circ$ . Hence the image formed by it is formed by a part only of the light emitted by the object.

Imagine a hemisphere in front of the object, of so large a size that the whole object may be treated as though it were



\* In one simple case investigated by the writer the visual substitute for a thin line of light proved to be a double line with a narrow interval and with very thin appendage-lines on either side. Here we have some of the phenomena presented by microscopes—a spurious resolution into a double line, and appendage-lines which are of the same nature as intercostal markings. See abstract of communication to British Association, at p. 583 of the 'Report' for 1894.

at its centre. The luminous beams\* of plane waves, each emanating from the whole front of the surface of the object, spread over this hemisphere, and the only case we need at present consider is where the pupil of the eye (in naked-eye observations) or the front lens of the objective (when we use a microscope) takes in only the beams A, viz., those beams of parallel waves thrown off from the surface of the object which are directed towards the middle sector of the hemisphere, and fails to admit the beams B, which are directed towards the marginal parts of the hemisphere. The excluded beams are partly Ba, those which, if reversed, form the finer of the rulings that go to build up the standard image. The rest of these beams, viz. Bb, are the more oblique members of those fans of beams which produce the coarser rulings—the whole of the standard image being made up of these finer and coarser rulings (see § 9), whereas the image seen by the observer is made up by the beams A alone—by those which the front lens of the objective can catch.

Let us now define  $-B$  to be the same light as  $+B$ , except that all the phases in  $-B$  are at each instant the reverse of what they are in  $+B$ . In other words we get the light  $-B$  by adding  $\pi$  to all phases in the light B; hence if the light  $+B$  and the light  $-B$  are both present, they exactly cancel one another.

Now the whole light emitted by the object is  $A+B$ ; and it is this light which forms the standard image. Hence, if we add the light  $-B$  to the standard image, and can find what modification of that image is thereby effected, we thus arrive at the best image which the light A can form: an image which the image actually formed on a large scale by the objective may approach in perfection, but cannot exceed. We may appropriately name it Standard Image No. 2.

In order to arrive at standard image No. 2, we may add the portions of light  $-Ba$  and  $-Bb$  in succession to standard image No. 1, as these together make up the whole of the light  $-B$ . The addition of  $-Ba$  simply obliterates the finer of the rulings out of which the standard image is constructed.

\* It is convenient to use the word undulation where the waves extend to an infinite or to an indefinite distance in their plane, and to employ the terms beam and pencil where we intend the lateral extent of the waves to be regarded as limited.

Practically luminous beams of plane waves emanating from the objective field, which is, of course, of limited extent, may be used instead of the undulations of the theory, which emanate from the entire objective plane; since the waves of a beam, unless very narrow, do not differ sensibly from the waves of the undulation, except close up to its bounding cylinder.

The chief (though not quite the only) effect of this is simply to render the image incapable of exhibiting some very fine detail upon the object which before it was able to reach. But the addition of  $-Bb$  has a worse effect. It adds to the image an entirely new set of fine rulings which do not represent any of the features which exist upon the object, and by this light such false effects as intercostal markings, spurious resolutions, a general haze of light, &c., are apt to be, and often are, produced. Hence we may enunciate Proposition 3 as follows:—

### PROPOSITION 3.

*When, of the light emitted by the object, only part is employed to form the microscopic image, then features may intrude themselves into the microscopic image which are not present in the standard image, and which do not represent anything upon the object.*

16. *Proposition 4. False Colouration.*—Another deceptive effect which is to be referred to the limited apertures of objectives is the appearance given to uncoloured objects of being coloured. Only the general principle to be kept in view will be stated here, as a fuller treatment of this phenomenon can be more conveniently made in connexion with individual instances which will be dealt with in Part II. of this memoir.

The whole light of wave-length  $\lambda$  which is sent forward by the object may be divided into  $A_\lambda$  which is admitted to the objective, and  $B_\lambda$  which is excluded. A similar partition into these two portions is to be made of the whole light of each wave-length, but the proportion in which the whole light is divided between them in general varies from one wave-length to another. Hence, if the illumination is by white light and the object uncoloured, there may be a preponderance of light of some colours in  $A$  as compared with others, and an equal deficiency of these same colours in  $B$ . In such cases the image seen in the microscope, since it is exclusively formed by the light  $A$ , has not got the colours mixed in the same proportions as they are in white light, and accordingly appears coloured. Hence

### PROPOSITION 4.

*Under the same circumstances as in Proposition 3, the partition of the light between the portions received by and excluded from the objective, will in general be different for different wave-lengths; and when the difference is marked a colourless object will appear to be coloured in the microscope.*

17. *Proposition 5. The Condenser.*—The standard image admits of being either better or worse. It manifestly admits of being improved by forming it out of light of shorter wave-length ; and this may be accomplished either by exchanging the colour of the illumination employed for a colour of higher refrangibility, or by mounting the object in a highly refracting medium.

But the degree in which the standard image correctly represents the object usually depends even more upon the condenser. In fact, the disturbance of the æther in front of the object is determined partly by the features of the object and partly by the condition of the light which illuminates it. This is evident because if the reversal spoken of in § 8 were to take place without removing the object, the light in retracing its steps would first reproduce the disturbed state that had existed in front of the object ; would next form the standard image upon the surface of the object ; and would then pass through the object and form beyond it whatever disturbed state of the æther had existed between the condenser and the object. Hence, that the standard image may represent the features of the object *unmixed with other appearances not belonging to the object*, it is essential that the light provided by the condenser shall be as nearly uniform and featureless as possible where it reaches the part of the object which is being scrutinised. Hence the importance of thin sections, and of a very well-corrected condenser.

The management of stops, and their function, can be better treated of in Part II. of this memoir, when we can enter into details. For the present we content ourselves with a very general proposition, viz.:—

#### PROPOSITION 5.

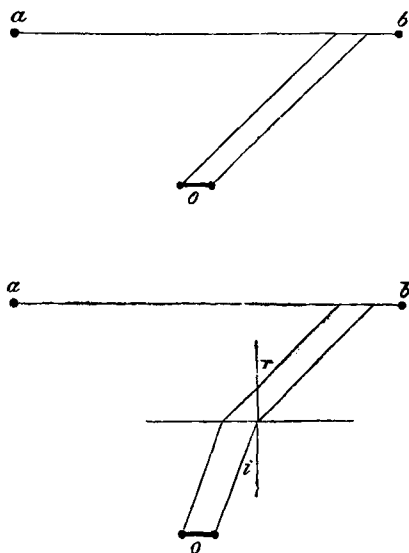
*The standard image is the outcome, partly of the features upon the object, and partly of the state of the light by which the object is illuminated. It may be improved by increasing the degree in which the first of these factors, and by decreasing the degree in which the second, contributes to produce, to modify, or to efface detail in the image.*

18. *Proposition 6.*—When an object is mounted in a more refractive medium than that in front of the objective, standard image No. 1, which depends on the wave-length of the light as it quits the object, is thereby improved ; but standard image No. 2 is not enabled to grasp any finer detail upon the object than it would have grasped if the object had been in a medium of the same refractive index as that in front of the objective.



That is, none of the luminous rulings which form the useful part of standard image No. 2—none of those that represent any feature of the object, excluding those which produce false effects like intercostal markings—are made any finer by mounting the object in a medium of extra high refractive index. But nevertheless an important effect is produced, viz. that the finer of the rulings are made relatively brighter than they were before, so that, *cæteris paribus*, the detail which they portray becomes more conspicuous.

This is evident from the accompanying diagram, in which  $ab$  is the front of the objective and  $o$  the object. Both figures



represent the course of one of the more oblique beams of parallel waves from the whole surface of the object, the first figure representing what occurs when the object is mounted in a medium of the same refractive index as the cover-glass and immersion oil, and the second figure representing what occurs when the object is mounted in an optically denser medium. *Cæteris paribus*, the ratio of the brightness of the beam that reaches the objective in the two cases is as  $\cos i / \cos r$ , which

$$= \left( \frac{1 - \frac{n}{n'} \sin^2 r}{1 - \sin^2 r} \right)^{\frac{1}{2}},$$

where  $n$  and  $n'$  are the refractive indices in the two media.

This is a fraction which the more deviates from unity the greater  $r$  is, *i.e.* the more oblique the beam. Hence, the more oblique beams, which bring out the finer detail, are more increased in brightness than the less inclined, which deal with the larger features of the object. Hence

#### PROPOSITION 6.

*Mounting the object in a medium of extra high refractive index will, cæteris paribus, increase the conspicuousness of the finer detail to be seen upon it.*

Of course other factors, some of which may be even more potent, have to be taken into consideration, such as the ratio of the index of refraction of the object to that of the medium in which it is mounted; for the farther this ratio is from unity, the more conspicuous do *all* the features of the object become.

19. *Proposition 7. Optical Contact.*—Another proposition which is of use in interpreting the phenomena presented by the microscope is a consequence of the condition of the æther in the rare medium when light is totally reflected from a surface separating a dense and a rare medium. What then occurs has been investigated by Sir George Stokes, in his masterly paper “On the Formation of the Central Spot of Newton’s Rings beyond the Critical Angle” (vol. ii. of Stokes’s Collected Papers, p. 56). It is therefore only necessary here to enunciate the result of that investigation in the form in which it explains optical events which the microscopist has occasion to make use of.

Normally, when a microscopic object is “mounted dry,” *i.e.* is situated in an *air-space* between the slip and the cover-glass, no rays from it can, while traversing the cover-glass, be more inclined to the vertical than the “critical angle.” Now immersion objectives are specially designed to admit rays that have passed upwards through the cover-glass in more inclined directions. Accordingly, when an object that is mounted dry is examined by an immersion objective, what *normally* happens is that only a part of the aperture of the objective is made use of. The event is, however, different if the microscopic object is excessively close to the cover-glass, owing to the phenomenon investigated by Sir George Stokes.

It follows from Sir George Stokes’s investigation that when a plane separates an optically dense from a rare medium, then there is a very thin layer of the rare medium of which the optical properties are peculiar. In cases of total reflexion, the

æther within this layer is brought into a disturbed condition. The disturbance in reality penetrates further down, but fades out by a law so rapid that it is only sensible within a very short distance (which depends on the wave-length) of the plane separating the media. The layer of this small thickness, within which the disturbance is sensible, may be called the *Stokes's Layer*.

It further follows from the investigation, that if light emanates from a point within the Stokes's layer, it will be able to pass up through the dense medium at angles that exceed the critical angle. It is easy to verify this experimentally. Take a glass prism—one of the pendants of a glass chandelier is sufficient. Hold it at the distance of distinct vision from the eye, and turn it till the light of the sky is seen like a silvery sheen, totally reflected from the inside of one of its faces. Then, without moving the prism or the eye, press a piece of chalk against the outside of that face. A small portion of the chalk can thereby be brought so close to the glass that the intervening chink is less than the thickness of the Stokes's layer. This small portion of the chalk will then be seen through the face of the prism, while the rest of the chalk and the hand that holds it, which are beyond the Stokes's layer, are quite unseen. The light from the chalk, by which it is seen, has obviously passed through the glass at an angle which is beyond the critical angle. Similarly :

PROPOSITION 7.

*If a microscopic object, mounted dry, is so close to the cover-glass that the chink of air between it and the cover-glass is less than the thickness of the Stokes's layer, then light from it can pass up through the cover-glass and the oil above it, at angles both within and beyond the critical angle, and may accordingly reach any part of the front of an objective whose NA is more than 1.*

20. With the help of these seven propositions, supplementing the more familiar laws of optics, nearly everything in microscopic vision may be explained, and useful rules can be deduced for the manipulation of the instrument. The next part of this memoir will deal with applications of this kind.

[To be continued.]