



XLIV. On Maxwell's stress theory

V. Bjerknes

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number of thin lenses in contact is the sum of their individual convergences, we immediately have the interesting result that for combinations of thin lenses in contact, without diaphragm separated from the combination, the curvatures of the fields may be added together just as with convergences. We thus have

$$R_1 = 3\mathbf{F} + \Sigma \frac{\mathbf{F}}{\mu}$$

$$R_0 = 2\mathbf{F} + \Sigma \frac{\mathbf{F}}{\mu}$$

$$R_2 = \mathbf{F} + \Sigma \frac{\mathbf{F}}{\mu}$$

where

$$\mathbf{F} = \Sigma F.$$

For such a combination the mean curvature therefore is $2\mathbf{F} + \Sigma \frac{\mathbf{F}}{\mu}$, or $2\mathbf{F}$ if the Petzval condition is satisfied, and under these circumstances R_1 , R_2 , and $R_3 = \mathbf{F}$, $2\mathbf{F}$, and $3\mathbf{F}$ respectively, while the curvature corresponding to the "astigmatic difference" is always $2\mathbf{F}$ independently of the materials of the lenses.

This example will serve to show how easily and directly aberration problems may be solved by physical methods, and the writer proposes to show in another paper that the whole theory of aberrations in refracting systems may be similarly treated with advantage. He ventures to hope, however, that enough has been said in this paper to convince everyone that not only is there no necessity for the abandonment of curvature methods at any stage in optical work, but that there is every advantage in retaining them throughout. It will also be obvious that they lend themselves to combining the study of diffraction and image formation, which should lead to valuable new results.

XLIV. *On Maxwell's Stress Theory.* By V. BJERKNES*.

MAXWELL considered his theory of the stress in the dielectric medium as very important. But, on the other hand, he did not regard it as complete. His own words plainly prove both assertions † :—

"It must be carefully borne in mind that we have made only one step in the theory of the action of the medium.

* Communicated by the Author.

† Maxwell, 'Electricity and Magnetism,' 2nd edition, vol. i. p. 154.

We have supposed it to be in a state of stress, but we have not in any way accounted for this stress, or explained how it is maintained. This step, however, seems to me to be an important one, as it explains, by the action of the consecutive parts of the medium, phenomena which were formerly supposed to be only explicable by direct action at a distance.

"I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point, merely stating etc."

After Maxwell nobody has yet been able to make this *next step*. Some formal improvements have been made, but from the point of view of principle the stress theory has not advanced beyond the point where Maxwell left it. But still we have at least now the advantage of possessing a remarkably complete mechanical image of the electrostatic and the magnetic phenomena, worked out by the late Professor C. A. Bjerknes of Christiania*.

It may therefore be worth while to see how Maxwell's stress theory works in the case of these *electroidic* phenomena, where we are able to account for everything by mechanical considerations.

Let us imagine a man, who sees the experiment with the attractions and repulsions of pulsating bodies, but who is not able to see the small pulsations, nor the liquid which propagates the action from the one body to the other. This observer will then be in exactly the same limited condition as to knowledge of the hydrodynamic phenomena before him as the electrician is relatively to the electrical phenomena. The attractions and repulsions of these bodies will make upon him exactly the same impression as the attraction and repulsion of electrified bodies or of magnetic poles. His measurements will bring him to the result, that they follow the law of Coulomb, only with the sign of the force reversed.

Suppose now this observer to be a Maxwell. He will then suspect, that these actions are not real actions at a distance. He will suspect the presence of a medium, and try to explain the apparent actions at a distance by some stress in this medium. And up to this point he will be perfectly right in his conclusions.

But this will no longer be the case if he follows Maxwell further, in attempting to give an explicit solution of the stress problem. He will then consequently arrive at the expression of the Maxwell stress, only with the sign reversed for each of

* V. Bjerknes, "Vorlesungen über hydrodynamische Fernkräfte nach C. A. Bjerknes' Theorie, Leipzig," 1900-02.

the six stress components. These stress formulæ will enable him to calculate the forces acting on the pulsating bodies. But if he draws the further conclusion from this result, that this stress really exists and really produces the forces, he will be wrong. For the cause of the attractions and repulsions of the pulsating bodies is not the Maxwell stress, but a stress of much simpler nature, namely, the isotropic stress, or the pressure, in the liquid. Still, in the limited state of his knowledge, he will have exactly the same right as the electrician to believe that the Maxwell stress is the real cause of the phenomena.

It is easy to see where the error comes in when Maxwell's developments are applied to these electrodynamic phenomena of hydrodynamics. Maxwell considers his problem as one of pure statics. The stress has therefore only to produce the required mechanical forces. In all points of space where no such force is required, the stress is self-equilibrating. The hydrodynamic phenomena on the other hand are not statical, but kinetic, even if the motions be too small to be observed. The stress or pressure is therefore nowhere self-equilibrating. It has a double duty, first to maintain the hidden motions, which constitute the field, and only secondly to produce the required forces that give rise to the visible motions.

The question whether the Maxwell stress may or may not represent a real stress in the dielectric will therefore be closely related to the question, whether the electric or magnetic phenomena are ultimately of statical or kinetic nature. In the latter case, if they depend upon hidden motions, the stress will have to maintain these motions, and it cannot be self-equilibrating like the Maxwell stress. Now Maxwell himself considers the magnetic phenomena as kinetic, as his application of the Lagrange equations to electromagnetism plainly shows. And if this view be right, his explanation at least of the magnetic actions at a distance will not be legitimate.

In the meanwhile it will therefore be safest to consider the Maxwell stresses as only *fictitious* stresses that might have produced the required forces, and not as the real stresses that do produce them. Other authors have also termed them fictitious stresses, especially Lorentz *, who also considers the stress formulæ only as useful analytical formulæ, but not as representing any physical reality. The reason, however, why he has come to this opinion is quite different from the reason brought forward here. His view is that the stress-

* H. A. Lorentz, "Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern," p. 28. "Elektronentheorie," *Encyklopädie der mathematischen Wissenschaften*, vol. v. 2. p. 163.

problem in itself has no physical meaning. For, according to the doctrine of the immobility of the æther, it is an absurdity to speak of forces acting upon or stresses existing in the æther.

Let us now consider this theory in its application to the electroidic phenomena of hydrodynamics. Just as well as an observer of these phenomena might arrive at Maxwell's views, he might, with his limited knowledge of the things before him, arrive at the views of Lorentz. But, as we see at once, they do not hold. For the hydrodynamic actions at a distance do depend upon a stress, although a much simpler one than that found by Maxwell.

The question to what extent it may be allowable to draw, from the electroidic phenomena of hydrodynamics, positive conclusions as to real electric phenomena may be open to discussion. The intimate nature of the analogy makes it very probable that useful suggestions may be obtained from this analogy. But of course the method must be tried with the greatest caution. The results produced here may perhaps encourage electricians to take up the stress-problem upon a wider base than Maxwell himself was able to do at his first attack upon this, *the* fundamental problem of the theory of electricity. The solution may be simpler than we expect. For the hydrodynamic results plainly show that even the simple isotropic stress may produce actions of the kind, which Maxwell supposed explicable only as the effect of a stress of the anisotropic type.

XLV. *On the Pressure of Gases and the Equation of Virial.*

By LORD RAYLEIGH, O.M., F.R.S.*

IF m be the mass of a particle, V its velocity, p the pressure and v the volume of the body composed of the particles, the virial equation is

$$\frac{1}{2} \sum m V^2 = \frac{3}{2} p v + \frac{1}{2} \sum \rho \phi(\rho), \quad . \quad . \quad . \quad (1)$$

where further ρ denotes the distance between two particles at the moment under consideration, and $\phi(\rho)$ the mutual force, assumed to depend upon ρ only. If the mutual forces can be neglected, either because they are non-existent or for some other reason, (1) coincides with Boyle's law, since the kinetic energy is supposed to represent temperature (T).

According to some experimenters, among whom may be especially mentioned Ramsay and Young, the relation between

* Communicated by the Author.