

a systematic error in observations made with a wedge where the error, arising from the coefficient of absorption, *is cumulative in its character*; but—and this, I submit, is a matter of great importance—this systematic error is of a character which can be removed, as I have pointed out, *not* by any comparison with observations of the same kind by other authorities, and the possible introduction thereby of a fresh source of error, but by a strictly independent and scientific investigation, with some form of photometer such as that devised by Captain Abney.

Ivy House, Clapham Common.

On a Method of Obtaining the Error of a Chronometer by Equal Altitudes of two Stars on Opposite Sides of Meridian. By A. Mostyn Field, Commander R.N.

The principle of this method depends upon the sidereal time of passing the meridian of a place, by an imaginary star, having the mean right ascension of the two stars selected, being compared with the time shown by a sidereal chronometer at that instant; the difference is its error on sidereal time. A mean solar chronometer can be used equally well. The sidereal time required is the mean of the right ascensions of the two selected stars. The chronometer time (either mean or sidereal) at that instant is the mean of the times at which the eastern and western stars had equal altitudes, with the "equation of equal altitudes" applied with its proper sign.

Equation of Equal Altitudes.

The rigorous expression, according to Chauvenet, is:—

$$\sin \alpha = \cot \frac{1}{2} \text{ E.T.} \tan d \tan \delta \cos \alpha - \operatorname{cosec} \frac{1}{2} \text{ E.T.} \tan l \tan \delta,$$

where

$$\alpha = \text{equation of equal altitudes} = \frac{h - h'}{2},$$

$$\frac{1}{2} \text{ E.T.} = \frac{1}{2} \text{ elapsed time} = \frac{h + h'}{2}.$$

d = declination at upper meridian passage.

$d - \delta$ = declination at observation E. of meridian.

$d + \delta$ = declination at observation W. of meridian.

$\delta = \frac{1}{2}$ difference of the declinations at the two times of observation.

h and h' = hour angles from noon at east and west observations respectively.

In the case of equal altitudes of two stars, one east and the other west of meridian, the above formula is strictly accurate whatever may be the difference in the declinations; the half elapsed time being found as follows:—

following investigation of the sign of "equation of equal altitude" holds good also, and is unchanged.

When $\frac{1}{2}$ E. T. exceeds 6^h , the only changes of sign which occur are those in Chauvenet's formula.

In the accompanying figure, drawn on the plane of the horizon,

Let X and X' be positions of eastern star at 1st and 2nd observations,

Y and Y' be positions of western star at 1st and 2nd observations,

T and T' be positions of 1st pt. of *Aries* at 1st and 2nd observations,

and let PX be less than PY.

Bisect angle XPY by hour-circle Pp.

Then, any imaginary star on Pp will have for right ascension the mean of the right ascension of X and Y.

Again, bisect angle X'PY' by hour-circle Pp'.

Then, any imaginary star on Pp' will have for right ascension the mean of the right ascension of X' and Y'.

Since the right ascensions of X and Y do not change *appreciably* in a few hours, we may assume that Pp and Pp' pass over the *same imaginary star*.

Bisect pPp' by the hour-circle Pq.

Then Pq can be shown to bisect XPY'.

That is qPQ or qQ is the "equation of equal altitude."

Now, if C and C' be the chronometer times (mean or sidereal) of observation,

Mid. time by chronometer = $\frac{C' + C}{2}$ = time by chronometer, when the "imaginary star" is on the hour-circle Pq.

$$\begin{aligned} \therefore \text{time by chronometer when imaginary} \} &= \text{time by chronometer when it is on} \\ \text{star is on meridian PQ} \} &\text{hour-circle Pq + arc Qq,} \\ &= \frac{C' + C}{2} + \text{equation of equal altitudes.} \end{aligned}$$

Similarly, when PX is greater than PY,

$$\text{Time by chronometer when imaginary} \} = \frac{C' + C}{2} - \text{equation of equal altitudes.}$$

star is on meridian PQ

Method of Observation.

Select two bright stars, of nearly the same declination, not differing much from the latitude, but differing in R.A. by from 4^h to 8^h .

The mean R.A. is the sidereal time at which the imaginary star referred to above will pass the meridian; therefore the time at which it will be necessary to begin observing will be governed by this, and the observations of one star should be completed shortly before that time, in order to allow an interval to prepare for observing the other.

The sign of the "equation of equal altitudes" remains unaltered, whichever star is first observed.

C C

As a general rule, if the difference in right ascension is less than 6^h , the eastern star should be observed first; if it exceeds 6^h , then the western star; this is in order that the stars may be observed as favourably as possible, with respect to the prime vertical, but it will vary according to the latitude and declination. It will be noticed that if the observations are commenced with the eastern star, then they are, as a whole, taken further from the meridian than in the other case.

If the difference in R.A. exceeds 8^h , then there will probably be an interval between finishing the observations of one star and beginning those of the other, and part of the advantages of the method are lost; the same remark applies if the difference in R.A. is less than 4^h .

Having decided on which star to begin with, observe it continuously in the ordinary way, until the sidereal time is nearly equal to the mean R.A. of the two stars (the error of chronometer on sidereal time should be roughly known), then prepare to observe the other star, commencing at the same altitude as the last observation of the first star, and complete the series, which may be divided into sets, in the usual way.

Owing to the rapid change in "equation of equal altitudes" when there is a large difference in the declinations of the stars, it will be remarked that the "middle times" vary more rapidly than in the case of the Sun; and the rapidity of this change increases as the observations get further away from the sidereal time, at which the "imaginary star" passes the meridian.

If a mean solar chronometer be used, the chronometric interval (corrected for rate) must be turned into a sidereal interval, and the resulting "error of chronometer" will be the error on sidereal time at that particular instant, from which the error on mean time can be readily deduced.

Advantages.

1. The advantages and strength of the equal altitude method are retained, but without the inconvenience of having to wait some hours between the eastern and western observations.
2. There is less time for the chronometer to change its rate.
3. Atmospheric conditions for both eastern and western stars are more likely to be the same than after an interval of some hours.
4. Simplicity of computation; the only logarithms which change being those of \cot and $\operatorname{cosec} \frac{1}{2} E. T.$
5. On account of the shorter interval, there is a greater probability of being able to obtain the second half of the series.

Northampton:

1890 February 7.