



LXVIII. On the motion of a small sphere vibrating in a resisting medium

Rev. J. Challis

To cite this article: Rev. J. Challis (1840) LXVIII. On the motion of a small sphere vibrating in a resisting medium, *Philosophical Magazine Series 3*, 17:112, 462-467, DOI: [10.1080/14786444008650214](https://doi.org/10.1080/14786444008650214)

To link to this article: <http://dx.doi.org/10.1080/14786444008650214>



Published online: 01 Jun 2009.



Submit your article to this journal [↗](#)



Article views: 3



View related articles [↗](#)



Citing articles: 1 View citing articles [↗](#)

fects up to this time have been obtained from locomotive engines, in which water is heated in contact with brass tubes. How far this may influence the production of electricity, further experiments must determine. It is certainly somewhat curious to consider the splendid locomotive engines we see daily in the light of enormous electrical machines; but this they undoubtedly are; the steam is analogous to the glass plate of an ordinary machine, the boiler to the rubbers; and a conductor properly exposed to the escaping steam gives out torrents of electricity.

I am, Gentlemen,

Your obedient Servant,

Bentham-Grove, Gateshead,
November 21, 1840.

H. L. PATTINSON.

LXVIII. *On the Motion of a small Sphere vibrating in a resisting Medium.* By the Rev. J. CHALLIS, Plumian Professor of Astronomy in the University of Cambridge*.

IN the London and Edinburgh Philosophical Magazine for September, 1833 (vol. iii. p. 186.), I have given a solution of the problem of the resistance to the motion of a ball-pendulum vibrating in the air, by making use of the principle of the conservation of *vis viva*, and assuming that for slow vibrations the motion of the air surrounding the ball is the same as if the fluid were incompressible. I have given another solution in the Cambridge Philosophical Transactions (vol. v. part ii. p. 200.), by adopting the above assumption without using the principle of the conservation of *vis viva*; and in the latter solution it is not taken for granted, as in the other, that the same considerations apply to fluid motion directed to or from a moving centre, as to motion to or from a fixed centre. The two methods lead to the same result. In 1835, M. Plana published at Turin a Memoir (for a copy of which I am indebted to the kindness of the author) containing a solution of the problem in question, the same in principle as that of Poisson in vol. xi. of the *Mémoires* of the Paris Academy of Sciences†, with the difference of treating separately the motions in a compressible and an incompressible fluid, and so obviating some objections to which Poisson's reasoning appeared liable. M. Plana adverts to my communication in the Philosophical Magazine, and subjoins a translation of it, but is unwilling to admit the correctness of the principle of the method I have employed, apparently for no other reason than that it leads to a result differing from his own.

* Communicated by the Author.

† Poisson's memoir is also inserted in the *Connaissance des Temps* for 1834.

The two methods are, in fact, so dissimilar in principle, and in their results, that if one is right the other must be wrong. But after the lapse of some years I am not able to discover any error either in the principle or the details of the method I have employed in the *Philosophical Magazine*, nor of that in the *Cambridge Philosophical Transactions*. The object of my present communication is to give a *third* solution, which applies expressly to vibrations of the ball in a *compressible* fluid.

It will be proper to begin with proving generally that the same equations apply to the motion of the fluid when directed to or from a moving centre, as when directed to or from a fixed centre.

Considering, first, the motion of the fluid to be in the direction of radii from a fixed centre, conceive two spherical surfaces described about this centre at the distances r and r' differing very little from each other; and let the interior one pass through the point at which we consider the motion. Conceive also a conical surface, having its vertex at the centre of the spherical surfaces and its vertical angle indefinitely small, to intersect with its axis the interior spherical surface at that point. Let m^2 = the small portion of the interior spherical surface included by the conical surface; then $\frac{m^2 r'^2}{r^2}$ = the corresponding portion of the outer surface. It will be assumed that during a very small time δt , the velocity and density of the fluid which passes the area m^2 are uniformly v and ρ ; and, similarly, that the velocity and density of the fluid passing in the same time the corresponding area of the other surface are uniformly v' and ρ' . Then the quantity of fluid which passes m^2 in the time δt is $m^2 \rho v \delta t$; and that which passes the other area in the same time, $\frac{m^2 r'^2}{r^2} \cdot \rho' v' \delta t$.

The increment of matter in the included space is, therefore, $-m^2 \delta t \left(\frac{r'^2 \rho' v'}{v^2} - \rho v \right)$, the velocities v and v' being reckoned positive when directed *from* the centre. The space itself is ultimately $m^2 (r' - r)$. Hence the increment of density $\delta \rho$ is equal to $-\frac{m^2 \delta t (r'^2 \rho' v' - r^2 \rho v)}{m^2 (r' - r)}$.

Consequently,

$$\frac{\delta \rho}{\delta t} + \frac{r'^2 \rho' v' - r^2 \rho v}{r^2 (r' - r)} = 0;$$

and passing from differences to differentials,

$$\frac{d\rho}{dt} + \frac{d \cdot r^2 \rho v}{r^2 dr} = 0; \quad . \quad . \quad . \quad . \quad (1.)$$

where, from the nature of the investigation, the differential coefficients are evidently partial.

Now suppose the motion of the fluid to be directed to or from a moving centre, and let two spherical surfaces separated by a very small interval be described about this centre, the interior one always passing through the point of space at which we consider the motion. On account, therefore, of the motion of the centre, the spherical surfaces will not be stationary. We may, however, conceive a conical surface, described as in the former case, to have its axis always passing through the moving centre and the point of space at which the motion is considered, and to include a given small portion m^2 of the interior spherical surface. The velocity and density of the fluid passing the area m^2 may, as before, be considered uniform during a very small time δt ; as may also, without entailing error, the velocity and density of the fluid passing the portion of the outer surface always included by the conical surface. Hence, using the same letters as in the case of a fixed centre, the quantity of fluid which passes m^2 in the time δt is $m^2 \rho v \delta t$. We have now to ascertain the quantity of fluid which in the same time passes the corresponding area of the exterior surface. Let r and r' be the radii of the two concentric surfaces at the beginning of the interval δt , and let α be the velocity of the centre resolved in the direction of r . Then after an interval τ , less than δt , the radii of the surfaces are $r \pm \alpha \tau$ and $r' \pm \alpha \tau$ ultimately. Hence the area

$$\begin{aligned} \text{of the outer surface corresponding to } m^2 &= m^2 \cdot \left(\frac{r \pm \alpha \tau}{r \pm \alpha \tau} \right)^2 \\ &= \frac{m^2 r'^2}{r^2} \cdot \left(\frac{1 \pm \frac{\alpha \tau}{r'}}{1 \pm \frac{\alpha \tau}{r}} \right)^2 = \frac{m^2 r'^2}{r^2}, \text{ by neglecting terms that} \end{aligned}$$

may be neglected, since by hypothesis r' differs very little from r , and $\alpha \tau$ is very small. This result is independent of τ , and is the same as if the centre had been fixed. The rest of the reasoning would consequently conduct to the equation (1.). Hence from this equation combined with the known

equations p (the pressure) $= a^2 \rho$, and $\frac{dp}{\rho dr} + \left(\frac{dv}{dt} \right) = 0$,
equations applicable to motion directed to or from either a

fixed or a moving centre may be deduced. I will not stop to make the deduction, which presents no difficulty, but at once employ the equations given in the Treatises on Hydrodynamics for motion propagated from a fixed centre. (Propagation *towards* the centre is excluded by the nature of the question.) These equations are (putting $1+s$ for ρ),

$$v = \frac{f'(r-at)}{r} - \frac{f(r-at)}{r^2} \quad (2.), \text{ and } as = \frac{f'(r-at)}{r} \quad (3.), \text{ which}$$

as they contain arbitrary functions, apply immediately to the arbitrary disturbance given to the fluid. In the problem before us they apply, therefore, to the motion given to the fluid by the vibrating sphere *at its surface*. For as the sphere is supposed to be perfectly smooth and consequently to impress motion only in a direction normal to its surface, the motion at the surface is plainly directed to or from a moving centre. The arbitrary condition of the motion is that at a given distance (r), equal to the radius of the sphere from the centre regarded as fixed, and at a given point of the surface of the sphere, the velocity impressed follows either exactly or very approximately the law of a vibrating pendulum. Let the velocity of the centre of the sphere at any time t be $V \sin bt$. Then for any point the radius to which makes an angle θ with the direction of the motion, we shall have the normal velocity u equal to $V \cos \theta \sin bt$. Hence, putting for brevity $u = f(r-at)$, and substituting in the equation (2.), it will be

$$\text{found that } \frac{du}{dt} + \frac{a}{r} u + V ar \cos \theta \sin bt = 0,$$

an equation in which u and t are the only variables, and which is true whatever be t . The integral of this equation is

$$u = C e^{-\frac{at}{r}} - V r^2 \cos \theta \cos \phi \sin (bt - \phi),$$

$\tan \phi$ being put for $\frac{br}{a}$. The term involving C will be insensible for all but very small values of t , on account of the factor $e^{-\frac{at}{r}}$, and may therefore be omitted. Hence, by differentiating and putting $a \tan \phi$ for br ,

$$\frac{du}{dt} = -V ar \cos \theta \sin \phi \cos (bt - \phi).$$

Now the pressure at the point of the sphere we are considering is equal to $a^2 s$, or by equation (3.) $a \cdot \frac{f'(r-at)}{r}$, or $-\frac{1}{r} \cdot \frac{du}{dt}$. Hence this pressure is $V a \cos \theta \sin \phi \cos (bt - \phi)$.

And by integrating in the usual way to obtain the pressure on the whole sphere, it will be found to be $\frac{4\pi V a r^3}{3} \cdot \sin \phi \cos (bt - \phi)$. This is reckoned positive in the direction contrary to that of the motion of the sphere. Hence if σ = the ratio of the specific gravity of the fluid to that of the sphere, the accelerative force of the resistance in the positive direction of the motion is $-\frac{V a \sigma}{r} \cdot \sin \phi \cos (bt - \phi)$. If λ = the distance to which motion is propagated in the fluid in the time of one vibration of the sphere, $b = \frac{2\pi a}{\lambda}$, and consequently, $\tan \phi = \frac{2\pi r}{\lambda}$. This is an exceedingly small quantity. Hence very approximately $\sin \phi = \frac{2\pi r}{\lambda} = \frac{br}{a}$, and the accelerative force of resistance = $-V b \sigma \cos bt$. Again, if x = the distance of the centre of the sphere at the time t from the mean place about which it is oscillating, $\frac{dx}{dt} = V \sin bt$, and $\frac{d^2x}{dt^2} = V b \cos bt$. Hence the accelerative force of the resistance = $-\sigma \cdot \frac{d^2x}{dt^2}$. The length of the pendulum being l and the force of gravity g , the accelerative force of gravity, taking account of the buoyancy of the fluid, is $-\frac{g}{l}x(1-\sigma)$. Hence,

$$\frac{d^2x}{dt^2} = -\frac{g}{l}x(1-\sigma) - \sigma \cdot \frac{d^2x}{dt^2},$$

and consequently

$$\frac{d^2x}{dt^2} = -\frac{g}{l} \cdot \left(\frac{1-\sigma}{1+\sigma} \right) x.$$

This is the result I obtained by my two former methods. As it does not contain a , it is applicable to any resisting medium, supposing the vibrations to be slow. Putting the factor in brackets, under the form $1-n\sigma$, we shall have $n = \frac{2}{1+\sigma}$. For a brass ball of specific gravity 8, vibrating in air, $n = 2$ very nearly; and for the same vibrating in water,

$n = 1.78$. The experiments of Bessel give for these two cases, 1.95 and 1.63.

I do not consider the above solution of value for the numerical results to which it leads, so much as because it serves to establish the principles to be adopted in the treatment of another problem (perhaps the most important that could be proposed in the present state of physical science), the solution of which has hitherto been unattempted, viz. *if a minute spherical atom were subject to the mechanical action of the vibrations of a very elastic medium, like those which take place in air, would it, in addition to a vibratory motion, receive also a permanent motion of translation?* I propose at a future opportunity to state my reasons for considering this an important question, and to advance some ideas respecting the method in which I conceive it may be answered.

Cambridge Observatory, Nov. 16, 1840.

LXIX. *On the Heat of Vapours and on Astronomical Refractions.* By JOHN WILLIAM LUBBOCK, Esq., Treas. R.S., F.R.A.S. and F.L.S., Vice-Chancellor of the University of London, &c.

[Continued from p. 280.]

On the Conditions of the Atmosphere, and on the Calculation of Heights by the Barometer. (Resumed.)

AS the expression which has served to calculate the temperatures evidently represents the state of the atmosphere far within the limits of the applicability of this or any other formula founded upon a state of repose to an atmosphere continually agitated by currents, it must of course serve to eliminate the density and to obtain an expression for the height in terms of the pressures and temperatures at the extremities of any atmospheric column.

If z be the altitude of the place above any fixed point, a the distance of the fixed point from the centre of the earth, g the force of gravity,

$$\frac{dp}{g} = - \frac{g a^2}{(a + z)^2} dz,$$

and putting the expression for g' , at vol. xvi. p. 440,

$$\frac{k(1 + \alpha \theta)(p^\beta - E) dp'}{p'(p'^\beta - E)} = - \frac{g a^2 dz'}{(a + z')^2}.$$

2 H 2