(*Paper No. 3146.*)

"The Counter-balancing of Locomotives."

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The considerable lack of uniformity apparent in the provision made for the counter-balancing of the inertia forces in locomotive-engines does not seem to be fully accounted for by the variety of the conditions, and may, to some extent, be due to an imperfect appreciation of some of the effects of these forces. In the following Paper the Author investigates principally the special effect due to the changing obliquity of the main connecting-rods, and endeavours to show that this effect is greater than has been generally recognized, and should be taken into account in determining the most advantageous position and magnitude of the counter-balance weights.

It is generally conceded that all the revolving masses should be entirely balanced, each on its own axle, and the principles involved in the determination of the necessary counter-balance weights and their positions are well understood. The discussion of this portion of the subject is therefore omitted. There is some uncertainty, however, as to the best means to be adopted in order to mitigate the displacing effect of the changing velocity of the reciprocating parts. The counter-balancing mass required for this purpose is often less than that requisite to balance the revolving weights, and in determining the one weight to be applied to each wheel in order to counter-balance both these effects, the influence of the revolving weights will predominate, so that, unless a careful estimation is made, the amount by which the effect of the reciprocating action has been taken into account will not be known with precision. This will often explain apparent differences between engines which are really similarly balanced.

The effect of the changing inclination of the main connecting-rods on the magnitude of the inertia forces is sufficiently great to require special consideration, and yet, although formulas have been given to express this effect, and curves have been drawn to represent it,[1] no systematic method of taking account of it has been proposed. It is not unlikely, however, that many engineers have appreciated the facts and successfully dealt with them in practice. On account of the superfluous and objectionable vertical action due to employing revolving counter-balance weights for the horizontally reciprocating masses, it is generally thought unwise to balance more than a fraction of them, about one-half to two-thirds; and because such an undefined fraction is to be provided against, it is often thought unnecessary to inquire into the effect of the changing obliquity of the connecting-rod. It is generally assumed that a small modification of the fraction balanced would cover the supposed small influence of obliquity. It will appear from the following, that whilst the influence of the changing inclination of the main connecting-rods on the alternate forward and backward plunging action due to the changing velocity of the reciprocating parts is practically zero, it has a material effect on the tendency to twist the engine about a vertical axis. So much is this the case that if, for example, it is proposed to balance, by rotating weights, two-thirds of the displacing tendency which would be due to infinitely long connecting-rods, the result would be that two-thirds of the plunging action would be balanced, but less than one-half of the twisting action. But the detrimental effect of leaving a portion of the plunging action unbalanced will probably be small, and at the same time the vertical effect of the revolving masses required for the whole or partial balancing will be objectionably great; whilst, on the other hand, the detrimental effect due to leaving a portion of the twisting action unbalanced will probably be great, and the vertical effect of the requisite counter-balancing weights will not be very serious.

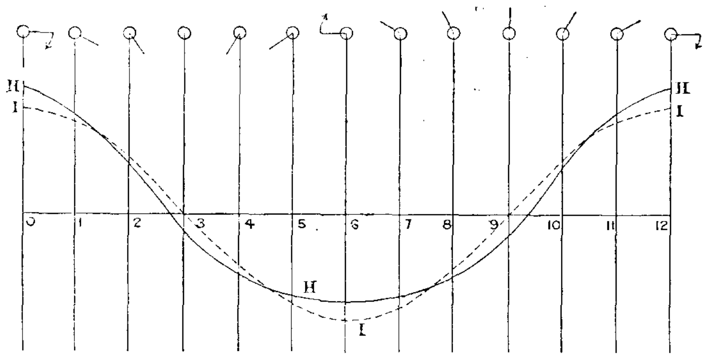
A graphical method will be adopted to determine and exhibit the varying magnitude of the inertia forces. The method is described in the Appendix, *Note 1*, and the result is shown in *Fig. 1*, in which the ordinates of the curve H H H represent the magnitude of the displacing force due to the varying velocity of the reciprocating mass of one engine, the length of the connecting-rod being five and a half times the length of the crank. Ordinates above the base-line represent a forward force, and those below this line a backward force. The position of the ordinate corresponds to the angular position of the engine-crank, the initial position

[1] "The Counter-balancing of Locomotive Engines," by E. L. Hill, Stud. Inst. C.E., Minutes of Proceedings Inst. C.E., vol. civ. p. 265.

being on the line of stroke, pointing forward. The dotted curve I I I shows what the force would have been if the connecting-rod had been infinitely long.

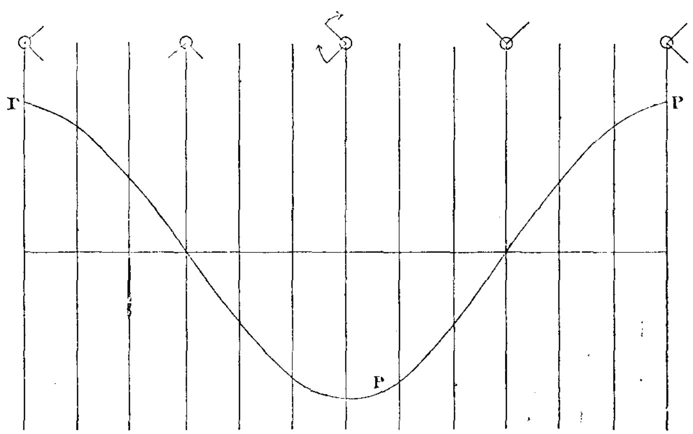
There being two engines, and two sets of reciprocating parts at

Fig. 1.



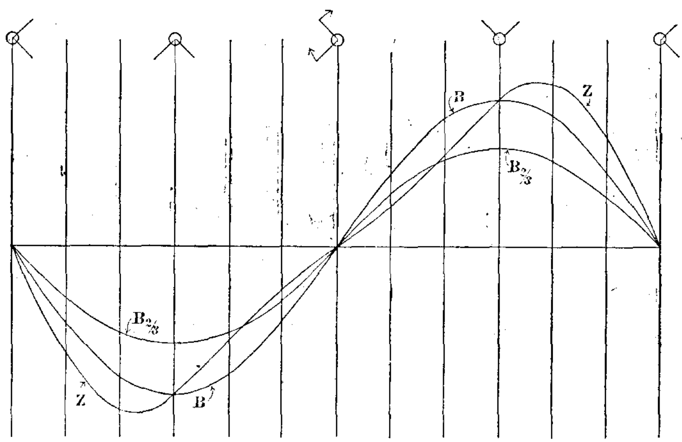
a transverse distance apart, connected to cranks which are at right angles to one another, there will be two kinds of tendency to displace the locomotive. Sometimes the displacing forces of both

Fig. 2.



engines will be in the same direction and sometimes in opposite directions. In the former case there will be a tendency to a bodily displacement of the whole engine in the direction of the line of rails, alternately forwards and backwards. This is called the plunging action. In the latter case there will be a tendency to turn the engine around a vertical axis alternately in either direction. This is called the twisting action. The magnitude of the plunging action is represented by the curve P P P (*Fig. 2*), which is obtained by algebraically adding together two ordinates of the curve H H H (*Fig. 1*), which are at a distance apart corresponding to a right angle, the sum of the two ordinates being set as a new ordinate midway between them. This curve is practically the same as would have been obtained by adding two similarly situated ordinates of the curve I I I (*Fig. 1*), and shows that the shortness of the connecting-rods does not affect the magnitude of the plunging action. (A proof of this is given in the Appendix

Fig. 3.



Note 2.) It would therefore be possible to exactly counter-balance the plunging tendency by means of revolving weights, if the accompanying disadvantages, due to the vertical forces which are incurred, did not preclude it. The magnitude of the tendency to twist is represented by the curve Z Z (*Fig. 3*), which is obtained by taking the algebraical difference of two ordinates of the curve H H H, which are at a distance apart corresponding to a right angle. In this case also the ordinate of the curve Z Z is midway between the two subtracted ordinates. The curve B B (*Fig. 3*) shows what the tendency to twist would have been if the connecting-rods had been infinitely long. From a comparison of the two curves Z Z and B B it will be seen that not only is the maximum twist increased by the shortness of the connecting-rods, but,

what is perhaps more important, the angular intervals between the maximum ordinates are unequal. (Appendix, *Note 3.*)

The curve B B shows what the form of the curve should be in order that it may be exactly counter-balanced by a revolving weight, and the vertical intercept between Z Z and B B shows the magnitude of the twisting moment which would remain when balanced as completely as possible. With a ratio of connecting-rod to crank equal to $5\frac{1}{2}$ to 1, the maximum portion unbalanced is rather more than one-fourth of the greatest tendency to twist with infinitely long connecting-rods. If the tendency to twist were balanced as completely as possible, as represented by the curve B B (*Fig. 3*), there would be alternately a deficiency and an excess of balance-weight, tending to twist the engine in opposite directions alternately, twice in each revolution. Any one of the closed areas between the two curves Z Z and B B would in such a case represent the energy available to twist the engine one way, and, if it were free to obey that tendency, would also represent on another scale the angular momentum which would be generated. If the mass of the balance-weights were made two-thirds of that required to completely balance the tendency to twist, the deficiency in the balancing of the tendency to twist would be as represented in *Fig. 3* by the vertical intercept between the curve Z Z and the curve $B\frac{2}{3}$ $B\frac{2}{3}$. In this case the greatest unbalanced portion amounts to more than one-half of what it would be with infinite connecting-rods if entirely unbalanced. Here also each of the closed areas between Z Z and $B\frac{2}{3}$ $B\frac{2}{3}$ represents the energy available for twisting, and the angular momentum which may be set up, and it amounts to more than $2\frac{3}{4}$ times that of the former case; but this does not express the entire difference between the two cases.

When the twist is as fully balanced as possible, the residuum has only a period corresponding to a quarter of a revolution in which to take effect, after which an opposite tendency comes into operation, whereas when the twist is only partially balanced the reversal practically occurs only once in each half revolution. The friction between the wheels and the rails will probably be sufficient to prevent the unbalanced portion of the twist from causing an actual side-slip, but the rolling of the wheels will permit a moderate twisting moment, if continued long enough, to turn the engine sufficiently to cause a reversal of the pressures between the flanges of the wheels and the rails. The operation may be compared with that of removing a belt from a pulley. A considerable side pressure on the belt will have little or no effect when the pulley and belt are at rest, whilst a moderate side-pressure will produce the displacement, if continued long enough, when the pulley and belt are in motion. The distance travelled by the belt while the side-pressure is being exerted will be of greater consequence than the magnitude of the side-pressure applied to displace it.

In the case of the locomotive-engine, the magnitude of the unbalanced twist is important because the large mass of the engine has to be set in angular motion, but the length of movement available for it to take effect is not less important. There is, therefore, a special merit in balancing the twist as fully as possible, as shown by the curve B B, as compared with a partial balance such as is shown by the curve $B\frac{2}{3}$ $B\frac{2}{3}$. In addition to the tendency to introduce side-pressure between the flanges of the wheels and the rails, which, if excessive, may cause the wheels to mount and derail the engine, the unbalanced portion of the twist will produce superfluous pressure between the axles and axle-boxes and between the boxes and horn-plates, causing retardation of the train, as well as wear.

The plunging action has a less deleterious effect. When there is a full pressure of steam on the engine, with a corresponding pull on the draw-bar, only the excess of the backward plunging force above the propelling force will cause the engine to be displaced relatively to the train it is drawing, and the forward plunge will only create an extra propelling force. On this account, therefore, it will be unnecessary to balance more than the excess of the plunging action above the tractive force. When the engine is running free, without a train, the unbalanced portion of the plunging force will be more able to take effect, but nothing serious or particularly unpleasant will result even then. If revolving weights are employed in such a way as to balance, by the horizontal components of their centrifugal forces, any portion of the plunging action, the vertical components of those forces, which at other portions of the revolution will be of equal magnitude to the horizontal components, will tend to lift the wheels and axle from the rails. Even when the total lifting tendency is not sufficient to overcome the downward pressure due to the portion of the weight of the locomotive imposed on that pair of wheels, it will lessen the adhesion between the wheels and the rails, and will give a momentary opportunity for the steam-pressure to cause the wheels to slip round relatively to the rails. Such an effect will be most likely to occur at full power, and will result in a loss of speed at a time when presumably all possible speed is wanted. The slipping will also cause wear of the rails and tyres. In the latter it will take place repeatedly at the same part of the circumference, and the wheel will become of non-uniform radius, which will introduce dynamic forces of serious magnitude at speeds which correspond to the natural period of vibration of the masses of the wheels and axle pressed on by the springs which intervene between the axle-boxes and the frame of the engine.

It thus appears that, on the one hand, whilst the plunging action itself is not very serious, the attempt to balance it in whole or in part may cause great harm. On the other hand, it is very desirable to eliminate as much as possible of the twisting action, and if this is done by placing balance-weights on the two wheels, exactly on opposite radii, there will be no total tendency for those balance-weights to lift the axle and let the wheels slip. In such a case the two radii will be at right angles to that which bisects the angle between the two cranks. Such positions will enable the twist to be balanced, as far as it can be, with the least possible amount of weight on each wheel. In general, it will be advisable to balance some portion of the plunge as well as the twist, in which case the balance-weights would not be placed on opposite radii. (Appendix, *Note 4.*)

The tendency of each wheel to lift separately, under the action of its own balance-weight, has to be considered, as well as the combined lifting effect of both balance-weights. The individual lifting will not necessarily tend to cause slipping, but may cause what is called pounding, which occurs when the wheel rises off the rail and returns to it with the energy of a blow. To prevent this action, the centrifugal force of the one balance-weight must not exceed the pressure imposed by the weight of the locomotive on the one wheel. On account of the rolling of the engine, due to imperfections in the fairness of the permanent way, the pressure on a wheel will fluctuate considerably, and will give an opportunity for lifting and pounding. Excessive pounding may cause deformation or fracture of a wheel, a rail, or a bridge over which the engine passes.

No hard-and-fast rule can be laid down with respect to balancing, but there appears to be good reason for paying more attention to the balance of the twist, as compared with the plunge, than has been generally recommended, by spreading the angle between the central radii of the two balance-weights. (Appendix, *Note 4.*) With inside-cylinder locomotives it would appear to be advisable to balance as much as possible of the twist, and as much of the plunge, up to one-half, as is possible without introducing too great a liability to pound. In outside-cylinder locomotives, the plunging action is not more than when the cylinders are within the frame, but the twisting action is much greater, and the balance of all that is possible of the twist may involve such a quantity of balance-weight as to incur an undue danger of pounding.

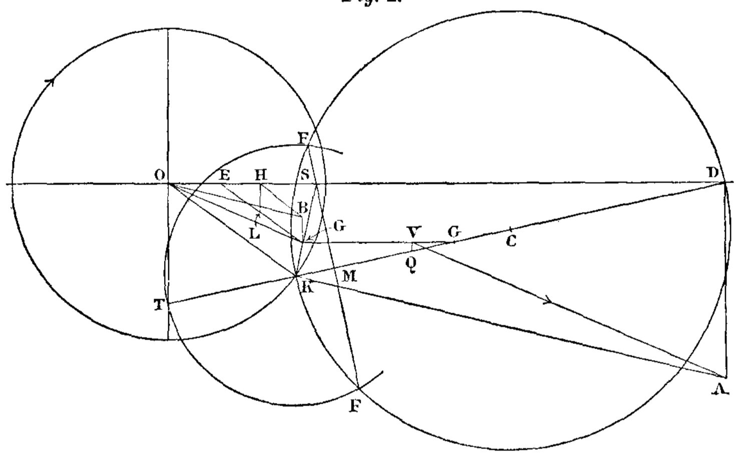
The motion of the main connecting-rods is a compound of a swinging and a sliding reciprocating movement. One end rotates with the crank-pin, and the other slides with the piston. It is customary to take account of these masses in determining the required counter-balancing weights by supposing a portion of the connecting-rod to rotate with the velocity of the crank-pins and the remainder to reciprocate with the changing velocity of the pistons; this is known to be only an approximation. This method is justified by mechanical principles if the whole mass is properly divided, the division which gives the most approximate result being such that the rotating portion is to the reciprocating portion in the inverse ratio of the distance of the centre of gravity of the connecting-rod from the centres of the crank-pin and of the guide-block pin respectively. (Appendix, *Note 5.*)

The Paper is accompanied by five tracings, from which the Figures in the text and the Appendix have been prepared.

APPENDIX.

Note 1.—In *Fig. 4*, let O K and K D represent respectively the crank and connecting-rod of an engine. With C, the middle point of the connecting-rod, as centre and C K as radius, describe a circle. Also with K as centre and radius K T, describe a second circle cutting the first in the points F F′ (T is the point where the line of the connecting-rod cuts a line drawn through O at right angles to the line of stroke). Join F F′ cutting the line of stroke in the point S, then S O will represent the acceleration of the piston on the same scale that K O represents the radial acceleration of the point K.

Fig. 4.



This is known as Klein's construction. If it is carried out for several positions of the crank, and the lengths S O thus found are plotted as ordinates from a base-line, the length of which is equal to the circumference of the crank-pin circle, the position of the ordinate corresponding to the angular position of the crank, the curve H H H, *Fig. 1*, will be derived. Lengths S O to the right of O are set upwards from the base-line, and those to the left are set downwards. The former correspond to a forward force on the engine, and the latter to a backward force.

By the principle of the instantaneous centre, or otherwise, it can be shown that

$$\frac{\text{velocity of piston}}{\text{velocity of crank-pin}} = \frac{v}{v_o} = \frac{\mathrm{O\,T}}{\mathrm{O\,K}} \quad . \quad . \quad . \quad . \quad (Fig, 4.)$$

$$\therefore \frac{v}{v_o} = \frac{\sin \mathrm{O\,K\,T}}{\sin \mathrm{O\,T\,K}} = \frac{\sin(\theta + \phi)}{\cos \phi},$$

θ and ϕ being the angles which the crank and connecting-rod respectively make with the line of stroke;

thus $$v = v_o (\sin \theta + \cos \theta \tan \phi).$$

The connecting-rod being n times the length of the crank—

$$\sin \phi = \frac{1}{n} \sin \theta,$$

from which $\tan \phi$ may be written in terms of θ and substituted in the foregoing expression for the velocity; but ϕ is never a large angle, so that $\tan \phi = \sin \phi$ approximately, and

$$v = v_o \left(\sin \theta + \frac{1}{n} \sin \theta \cos \theta\right) \text{ approximately,}$$

$$= v_o \left(\sin \theta + \frac{1}{2n} \sin 2\theta\right) \text{ approximately.}$$

Assuming v_o to be constant, then

$$\text{acceleration} = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}, \text{ where } \frac{d\theta}{dt} = \frac{v_o}{a},$$

a being the length of the crank.

$$\therefore \frac{dv}{dt} = \frac{v_o^2}{a}\left(\cos \theta + \frac{1}{n} \cos 2\theta\right) \text{ approximately.}$$

One maximum acceleration occurs when $\theta = o$,

then $$\left(\frac{dv}{dt}\right) \text{maximum} = \frac{v_o^2}{a}\left(1 + \frac{1}{n}\right),$$

when $$\theta = \pi, \ \frac{dv}{dt} = -\frac{v_o^2}{a}\left(1 - \frac{1}{n}\right).$$

These two particular values of the acceleration are exact, because then $\phi = o$ and $\tan \phi = \sin \phi$. When $n = \infty$, acceleration $= \frac{v_o^2}{a} \cos \theta$.

On the scale of *Fig. 1*, this will be represented by the distance of K from the line O T in *Fig. 4*, and will, if plotted, give the dotted curve I I I, *Fig. 1.* The vertical intercepts between the curves H H H and I I I represent the effect of the changing obliquity of the connecting-rod.

It will be observed that the curves cross one another at points corresponding to the four positions of the crank when it is inclined at half a right angle to the line of stroke, for then the second term in the foregoing expression, namely $\frac{\cos 2\theta}{n}$, becomes zero.

Note 2.—Consider the two engines of a locomotive with cranks at right angles. Suppose the angle which the leading crank makes with the line of stroke is θ_1, θ_2 being used for the other engine, and ψ be the angle which the radius midway between the cranks makes with the line of stroke, then—

$$\theta_1 = \psi + 45^\circ \text{ and } \theta_2 = \psi - 45^\circ.$$

The algebraical sum of the accelerations of the two engines which will be proportional to the plunging force exerted on the locomotive will be—

$$\frac{dv_1}{dt} + \frac{dv_2}{dt} = \frac{v_o^2}{a}\left\{\begin{array}{l}(\cos \psi - \sin \psi)\frac{1}{\sqrt{2}} - \frac{\sin 2\psi}{n} \\ + (\cos \psi + \sin \psi)\frac{1}{\sqrt{2}} + \frac{\sin 2\psi}{n}\end{array}\right\} = \frac{v_o^2}{a}\sqrt{2}\cos \psi,$$

which is independent of the length of the connecting-rod.

Note 3.—The tendency to twist the locomotive about a vertical axis will be proportional to the algebraical difference of the two accelerations—

$$\frac{d\,v_1}{d\,t} - \frac{d\,v_2}{d\,t} = -\frac{v_0^2}{a}\sqrt{2}\left(\sin\psi + \frac{\sqrt{2}}{n}\sin 2\,\psi\right).$$

The maximum value of this will occur when—

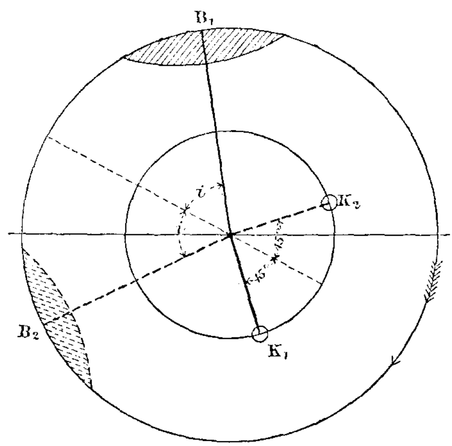
$$\cos\psi + \frac{2\sqrt{2}}{n}\cos 2\,\psi = o,$$

i.e., when—

$$\cos\psi = -\frac{n\sqrt{2}}{16}\left\{1 \pm \sqrt{1 + \frac{64}{n^2}}\right\}.$$

If $n = 5\frac{1}{2}$, $\psi = 90° - 24°$ and $270° + 24°$, so that the angular intervals between

Fig. 5.



the maximum values are respectively $180° + 48°$ and $180° - 48°$, and the maximum values of $\frac{d\,v_1}{d\,t} - \frac{d\,v_2}{d\,t}$ are $\pm \frac{v_0^2}{a}\sqrt{2} \times 1 \cdot 1$, *i.e.*, 10 per cent. greater than the maximum with infinite rods.

Note 4.—In *Fig. 5*, let K_1 be the leading crank and B_1 the balance-weight on the adjacent wheel, K_2 and B_2 referring to the crank and balance-weight on the other side of the locomotive. Also let W be the weight of one set of reciprocating parts and B the magnitude of each balance-weight.

Also let—

$2\,c$ = the transverse distance apart of the lines of stroke of the two pistons,
$2\,d$ = the transverse distance apart of the two balance-weights,
b = the radius measured to the centre of gravity of the balance-weights.

Also let—

p = the fraction of the plunging action which is balanced,
and t = the fraction of the twisting action due to infinite rods which is balanced,
i being the angle as shown in *Fig. 5*.

Then to balance the required fraction p of the plunge—

$$B\,b\,\cos i = p\,W\,a\,\cos 45^\circ,$$

and to balance the required fraction t of the twist—

$$B\,b\,d\,\sin i = t\,W\,a\,c\,\sin 45^\circ,$$

from which taken together $\qquad \tan i = \frac{t}{p}\,\frac{c}{d}.$

If $\qquad t = p,\ \tan i = \frac{c}{d}.$

This angle is the same as would be required for the weights necessary to balance those rotating masses of which the mean radius is in the line of the crank.

Also $\qquad B = \frac{1}{\sqrt{2}}\,\frac{W\,a}{b}\sqrt{p^2 + t^2\,\frac{c^2}{d^2}}$

if $t = p$ then $\qquad B = p\,\frac{1}{\sqrt{2}}\,\frac{W\,a}{b}\sqrt{1 + \frac{c^2}{d^2}}.$

Thus, if the same fraction of the plunging and twisting tendency to displacement due to infinite rods is balanced, the position of the balance-weights will be independent of the amount of that fraction, and the magnitude of the balance-weights will be proportional to that fraction. The combined effect of both balance-weights in lessening the pressure of the two wheels on the rails and permitting them to slip will be proportional to p.

As an example, a comparison may be made of the magnitude and position of the balance-weights which would be requisite

(1) When $p = \frac{1}{2}$ and $t = 1$.
(2) When $p = \frac{2}{3}$ and $t = \frac{2}{3}$.

$$\frac{B_1}{B_2} = \frac{3\sqrt{\frac{1}{4} + \frac{c^2}{d^2}}}{2\sqrt{1 + \frac{c^2}{d^2}}},\ \tan i_1 = 2\,\frac{c}{d},\ \tan i_2 = \frac{c}{d}.$$

Suppose $c = \frac{1}{2}\,d$, which is nearly the case in an inside-cylinder locomotive,

then $\qquad \frac{B_1}{B_2} = \frac{3\sqrt{10}}{10} = 0{\cdot}95,\ i_1 = 45^\circ,\ i_2 = 27^\circ.$

The counterbalancing will be more satisfactory by arrangement (1) than by (2), whilst 5 per cent. of balance-weight may be omitted, and the liability of the wheels to slip in case (1) is only three-fourths of that due to case (2).

If $c = d$, as is approximately the case in an outside-cylinder locomotive,

$$\frac{B_1}{B_2} = \frac{3\sqrt{10}}{8} = 1{\cdot}186,\ i_1 = 64^\circ,\ i_2 = 45^\circ.$$

To satisfy condition (1) 18·6 per cent. more balance-weight will be required than in case (2), with a consequent proportional increase in the tendency of one wheel to lift and pound; but the angular positions of the weights will be such that the combined tendency to lift both wheels and permit them to slip will be only three-fourths of that due to arrangement (2).

Note 5.—Referring to *Fig. 4*, the acceleration of the point K, considered as a point in the crank-pin, must be the same as its acceleration considered as a point in the swinging and reciprocating connecting-rod. According to the former, its acceleration is $\frac{v_o^2}{a}$, which is represented by K O, and according to the latter it will be made up of three parts, a radial acceleration in the direction K D, a tangential acceleration at right angles to K D, and the acceleration of the point D itself along the line D O. These three parts are represented respectively by K M, M S, and S O.

The acceleration of any other point in the rod, such as G, will be determined by drawing G G′ parallel to the line of stroke to cut K S in G′, then G′ O will be the acceleration of the point G, because the radial and tangential accelerations of G relatively to D will be less than that of K in the ratio of D G to D K, whilst that derived from the acceleration of D will be S O as before. If G is the centre of gravity of the connecting-rod, then G′ O will represent the direction and magnitude of the force necessary to give the connecting-rod the requisite change of velocity.

If Q is the centre of percussion of the connecting-rod relatively to D, and through Q a line be drawn parallel to K S, to meet the line drawn through G, parallel to D O, the point U, where they meet, must be in the line of action of the resultant inertia force of the connecting-rod, the magnitude and direction of that force being represented by O G′.[1] The displacing effect of this force will be transmitted to the frame of the engine by the action of a force at D, and another at K: neglecting friction, the former force can act only in a direction at right angles to the line of stroke, and so the direction of the force at K will be determined, being K A.

If O B is drawn parallel to K A, and G′ B parallel to A D, then B G′ will represent the magnitude of the force which is exerted at D, and O B that exerted at K.

The latter force will be transmitted to the frame of the engine at O, by means of the crank and crank-shaft. Draw B H and G′ E parallel to the crank, and H L parallel to B G′.

The force which is represented by O B may be resolved into two components, one O H in the direction of the line of stroke, and the other H B in the direction of the line of the crank. If, for the purpose of counteracting a portion of the displacing forces due to the connecting-rod, a rotating weight is provided whose centrifugal force is such as would be represented by G′ E, then a portion of it, G′ L, would entirely balance the radial force H B, and the other portion L E would, by its horizontal component, balance the part E H of the inertia force which acts along the line of stroke, the vertical component L H being equal and opposite to the force B G′ which is applied to D.

Now $$\frac{\text{E G}'}{\text{O K}} = \frac{\text{G}'\text{ S}}{\text{K S}} = \frac{\text{G D}}{\text{K D}}.$$

Thus if the fraction $\frac{\text{G D}}{\text{K D}}$ of the whole mass of the connecting-rod were imagined to be fixed to the crank-pin, and balanced by an equivalent rotating mass, the remaining displacing effect of the connecting-rod would consist of a force O E acting along the line of stroke, and a couple B G′ with an arm O D.

Also $$\frac{\text{O E}}{\text{O S}} = \frac{\text{K G}'}{\text{K S}} = \frac{\text{K G}}{\text{K D}}.$$

[1] The Author is indebted to Professor Dunkerley, of the Royal Naval College, Greenwich, for this extension of Klein's construction.

Thus the exact displacing effect on the frame of the locomotive engine due to the connecting-rod is to cause a pressure at the guide-block at right angles to the line of stroke, (this part of the action is called (*a*),) and a force on the crank-shaft bearings which can be resolved into three other parts, one called (*b*) acting along the radius of the crank-arm, and two parts (*c*) and (*d*) which act along the line of stroke. The parts (*a*) (*b*) and (*c*) can be so far counteracted by a constant revolving balance-weight as to leave only a tilting couple, which will tend to cause the engine to alternately rear and buck, but of magnitude quite insufficient to produce any appreciable effect. The magnitude of that revolving balance-weight will be that which would be requisite to balance the fraction $\frac{DG}{DK}$ of the whole mass of the connecting-rod, if it were attached to the crank-pin to revolve with it. The other part of the action (*d*) is just that which would be due to supposing that the remainder of the mass of the connecting-rod, viz., $\frac{KG}{DK}$, were attached to the guide-block so as to move with the velocity of the piston. Thus the common assumption that the effect of the connecting-rod is the same as if a part revolved with the crank-pin and the remainder reciprocated with the piston, though not true, will lead to the correct determination of the necessary counter-balance weights.