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XXVI. *On Magnetostriction.* By H. NAGAOKA and
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[Plates I. & II.]

THE object of the present investigation is two-fold : firstly to determine the effect of hydrostatic pressure on the magnetization of iron and nickel and to find whether there exist reciprocal relations between the effects of compression and the volume-change of ferromagnetics by magnetization ; secondly, to examine Kirchhoff's † theory of magnetostriction from measurement of strains produced by magnetization and from the effects of stress on the magnetization of iron and nickel.

Both experiment and theory show that physical changes are mostly reciprocal. In magnetism, this fact is markedly brought out by the mutual relations between twist and magnetization ‡, as well as by the change of length caused by magnetization and the effect of longitudinal pull applied to the magnetized wire. Theoretical exposition of these facts was given by J. J. Thomson §; by applying similar reasoning to the effect of hydrostatic pressure on magnetization, we can show that the change of volume accompanying

* Communicated by the Authors.

† Kirchhoff, *Sitzber. d. k. Acad. d. Wiss. zu Berlin*, p. 137 (1884); *Wied. Ann.* vol. xxiv. p. 52; *Gesammelte Abhandlungen*, Nachtrag, p. 91, Leipzig (1891); see also Pearson's 'History of Elasticity,' vol. ii. p. 105, §§ 1319-1321.

‡ See Wiedemann's *Electricität*, Bd. iii. pp. 767-814 (dritte Auflage).

§ J. J. Thomson, 'Application of Dynamics to Physics and Chemistry' (1888).

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the magnetization must to some extent be reciprocal to the change of magnetization wrought by compression.

Unfortunately our knowledge of the volume-change by magnetization is very scanty and discordant, so that we had to undertake fresh experiments on the specimens of ferromagnetics used in our research. The question regarding the effect of hydrostatic pressure on the magnetization is intimately connected with the thermodynamics of elastic bodies. From this standpoint, the problem was for the first time attacked by Wassmuth*, whose experimental results are in rough agreement with his theory. His experiments were rather of a qualitative nature, no absolute measurement of pressure as well as that of magnetization being undertaken. H. Tomlinson†, in his series of experiments on the effect of stress on the properties of matter, has examined this point, and was led to the following conclusion:—"Fluid pressure does not temporarily affect either the temporary magnetic susceptibility of annealed iron, or the permanent magnetization of hard steel, except, it may be, to a degree which is not comparable with that of the effect of stress in any one direction."

Although experiments on the effect of hydrostatic pressure are very scanty, the effect of one-sided pressure was a subject of investigation by several physicists; the effect of transverse stress on the magnetization of iron was examined by Lord Kelvin‡, and that of longitudinal compression by Ewing§ and Chree||. Unlike all these effects, the change wrought by hydrostatic pressure is of different order of magnitude, as remarked by Tomlinson. Without special arrangements for detecting a minute change in magnetization, we cannot well measure the change produced by all-sided pressure.

In a paper on the effect of magnetic stress in magnetostriction, Mr. E. T. Jones and one of us¶ have pointed out the importance of investigating the relation of magnetization to hydrostatic pressure in deciding the intricate question of magnetostriction. Mr. Jones** has, however, found out that it is unnecessary to take up experiments on hydrostatic pressure, inasmuch as the quantity which is required to settle

* Wassmuth, *Sitzber. d. Akad. d. Wiss. zu Wien*, vol. lxxvi. 2, p. 539 (1882).

† H. Tomlinson, *Proc. Roy. Soc.* vol. xlii. p. 230, art. 49 (1887).

‡ Lord Kelvin, *Phil. Trans.* vol. clii. 1878, p. 64.

§ Ewing, *Phil. Trans.* vol. clxxix. 1888, p. 333.

|| Chree, *Phil. Trans.* vol. clxxxi. A, 1890, p. 329.

¶ Nagaoka and Jones, *Phil. Mag.* May 1896.

** E. T. Jones, *Phil. Trans.* vol. clxxxix. A, 1897, p. 189.

the question can be deduced by means of simple experiments on the effect of longitudinal pull on a ferromagnetic wire.

As our object was not confined merely to the question of magnetic stress, we were enabled, after several fruitless attempts, to establish the fact that the effect of hydrostatic pressure is not immeasurably small, but that there is a remarkable reciprocal relation between the volume-change due to magnetization and the change of magnetization by compression.

In order to settle the question of magnetostriction, we measured the change of length and the effect of longitudinal pull on the magnetization of iron and nickel. From the different combinations of these effects, we can calculate the coefficients k' and k'' introduced by Kirchhoff. We are thus enabled to examine the effect of stress from the strains caused by magnetization and *vice versa*.

The present paper will therefore be divided into the following sections:—

- (1) Measurement of the change of volume and of length by magnetization.
- (2) Measurement of the effects of hydrostatic and transverse pressures and of longitudinal pull on the magnetization of iron and nickel.
- (3) Calculation of the coefficients k' and k'' , and a comparison between theory and experiment.

§ 1. *Measurement of the Change of Volume and of Length by Magnetization.*

Measurement of the Intensity of Magnetization.—We shall hereafter consider the strains produced by magnetization as functions of the magnetizing force and the intensity of magnetization; it will thus be necessary in the first place to determine the magnetizations of the various specimens of the ferromagnetics used in the present experiment. They were of the following dimensions:—

1. Ovoid of Swedish iron.

Length of major axis = 20 cm.; minor axis = 0.986 cm.; volume = 10.18 c.cm.; mass = 82 grm.; demagnetizing factor $N^* = 0.0848$.

2. Cylinder of Lowmoor iron.

Length = 25 cm.; diameter = 0.947; volume = 17.55 c.cm.; mass = 136 grm.; demagnetizing factor $N = 0.053$.

3. Nickel rod of square cross-section.

Length = 26 cm.; side = 0.514 cm.; section = 0.264 sq. cm.; volume = 6.86 c.cm.; mass = 58 grm.; demagnetizing factor $N = 0.020$.

* See du Bois, *Magnetische Kreise* (1894).

The demagnetizing factor for the rod was calculated on the supposition that N was equal to that of a circular cylinder of the same cross-section.

The magnetizing coil was 30 cm. long, and wound in 12 layers; its resistance was 0.63 ohm, and gave the field of 37.97 C.G.S. units at the middle of the coil due to a current of one ampere.

The magnetometer consisted of a small bell-magnet suspended in a thick copper case by a quartz fibre and provided with a plane mirror. It was placed due magnetic east of the coil, and its deflexion read by means of scale and telescope.

The following table gives the magnetization in different fields:—

H.	I (Iron Ovoid).	I (Iron Cylinder).	I (Nickel).
5	660	158	47
10	1020	380	100
20	1220	770	175
30	1270	900	240
40	1310	980	280
50	1340	1030	308
75	1390	1100	358
100	1440	1143	392
125	1480	1180	414
150	1500	1210	432
200	1550	1270	455
250	1600	1315	469
300	1640	1350	477
350	—	1380	482

Change of Volume produced by Magnetization.—Before we proceed to the description of the method employed in the present experiment, it will be worth while to compare the results of several previous investigators on the change of volume produced by magnetization.

It was generally admitted that there is no change of volume by magnetization, but it will be easily seen that most of these experimenters tried to increase the volume of the magnet by unusually increasing the thickness instead of length, thus incurring the risk of increasing the demagnetizing factor. They did not therefore arrive at a field-strength sufficient to produce appreciable change of volume.

Joule* was the first to call attention to the change of volume which may accompany the magnetization of iron. The result was in the negative; but as he gave neither the strength of the magnetizing current nor the intensity of magnetization, it is difficult to compare his result with that

* Joule, Phil. Mag. vol. xxx. p. 76 (1847).

of his successors. It is beyond doubt that the change of volume was very minute, and there was sufficient evidence that the elongation in the direction of magnetization is accompanied by contraction in the direction perpendicular to it.

The elaborate researches of Cantone* on the strain of ferro-magnetic ovoids are not free from the fault above mentioned. The major axis of the ovoid was 16·7 times that of the minor, so that the demagnetizing factor = 0·1134. As his results are given in terms of the magnetizing current and the moment of the magnet, we have thought it advisable to recalculate the result in magnetizing force H ($= H_0 - NI$, where H_0 stands for the magnetizing force in the coil) and I . The following table gives Cantone's determination of the intensity of magnetization :—

H_0 .	H .	I .
13·5	2·0	102
26·7	3·8	202
38·1	4·4	293
51·6	6·8	397
58·5	7·5	450

It will be seen from the above table that on account of the great demagnetizing factor the magnetizing force was less than 8 C.G.S. units although the field in the coil was nearly 60.

Cantone observed no alteration of volume in an iron ovoid even with a magnetizing current of 12 amperes ; but it is quite probable that the intensity of magnetization, as well as that of magnetizing force, was insufficient to produce appreciable change.

On account of the small susceptibility of nickel †, the effect of the great demagnetizing factor in Cantone's nickel ovoid was not so marked as in iron. So far as we are aware, he was the first to notice the diminution of volume in nickel by magnetization. Although his measurements with dilatometer filled with water and with alcohol are widely different, it is beyond doubt that the readings with alcohol are the more reliable for reasons which will be afterwards given. His calculation of Kirchhoff's coefficients k' and k'' based on the measurement of the change of length and of volume in nickel by magnetization throw much light on the theory of magnetostriction.

* Cantone, *Mem. d. R. Accad. dei Lincei*, vol. vi. p. 487 (1890).

† Cantone, *Rendiconti d. R. Accad. d. Lincei*, vol. vi. p. 352 (1890).

Our knowledge of the change of internal volume in iron, steel, and nickel tubes in the magnetizing field has been largely extended by the numerous researches of Dr. C. G. Knott*. It is much to be regretted that the magnetization was not uniform in his experiments, and consequently the change of volume could not be expressed as a function of the magnetization. The discussion of his results is rendered doubly intricate by the influence of the steel or brass cap for fixing the capillary tube to the hollow cylinder. Such inconvenience will disappear if the change of volume of the magnet itself be observed, as is easily possible if sufficient precaution be taken in the arrangement of the measuring apparatus.

These circumstances show that the question regarding the change of volume by magnetization is by no means settled; as almost all theories of magnetostriction make the strain in ferromagnetics depend on the intensity of magnetization and that of magnetizing force, we have examined the alteration of volume as functions of these two quantities.

The change of volume was determined by means of a dilatometer. The specimen to be tested was placed in a glass tube provided with a capillary neck (fig. 1). B shows the upper part of the capillary tube (0.215 mm. radius) with reservoir for filling the dilatometer with liquid. In supporting the ovoid in the tube, care was taken not to let it touch the sealed end of the glass tube. Two circular brass rings (*a*, *a'*) were inserted into the tube, and made to fit tightly against the wall of the dilatometer. A brass plate of the form given at A was soldered to the ring at S. The ends of the ovoid were then placed loosely in the triangular holes. The ovoid was thus supported in the central line of the dilatometer without touching the glass tube. A similar arrangement was employed for supporting the nickel rod within the dilatometer.

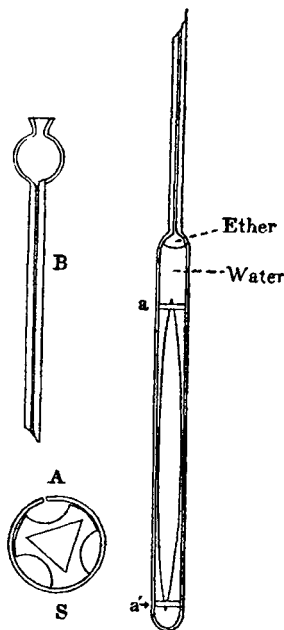
To prevent rusting of iron, the dilatometer was filled with very dilute solution of caustic soda nearly up to the neck. The capillary tube and a small portion of the main tube near the neck contained ether. When the dilatometer was all filled with water or petroleum, the indication of the volume-change was very irregular, as fine drops of the liquid stuck to the wall of the capillary tube and as the liquid was not sufficiently mobile. Cantone† had also similar experience in measuring the volume-change of the nickel ovoid. It would have been easier to fill the dilatometer all with ether, but

* Knott, Proc. Roy. Soc. Edinburgh, vol. xix. p. 249 (1892); vol. xx. pp. 290, 295, 334 (1893-1895); Trans. Roy. Soc. Edinburgh, vol. xxxviii. p. 527 (1896).

† Cantone, *loc. cit.*

there was difficulty in the observation owing to the greater expansion of the liquid due to the heating of the coil. We consider that the present mode of filling the dilatometer provided with fine capillary tube can be successfully applied for other purposes of a similar nature.

Fig. 1.



The ovoid was placed in the middle of the magnetizing coil and the rise or fall of the meniscus in the capillary tube was observed by means of a microscope with micrometer ocular. Although the resistance of the coil was only 0.6 ohm, the heating effect was considerable, so that only an instantaneous observation could be made. The difficulty was to a great extent overcome by passing the current for some time in the coil; the ovoid was then demagnetized by the method of reversals; waiting for some time, the meniscus became stationary, the magnetizing current was then made and the reading taken. The measurement was made in a cellar with gaslight at some distance behind the capillary tube; by this arrangement the meniscus was sharply defined.

The following table gives the determination of the change of volume in iron ovoid and cylinder by magnetization.

Ovoid.

H.	I.	$\frac{\delta v}{v} \times 10^7$.
2	155	0.1
3	340	0.3
4	540	0.4
6	800	0.6
12	1100	0.9
17	1200	1.1
29	1270	1.3
49	1340	1.6
113	1470	1.7
151	1510	1.8
203	1560	2.0
251	1630	1.1

Cylinder.

H.	I.	$\frac{\delta v}{v} \times 10^7$.
5	186	0.1
8	308	0.2
14	598	0.3
23	804	0.5
34	912	0.6
51	998	0.8
85	1199	0.9
102	1143	1.0
155	1220	1.1
207	1280	1.2

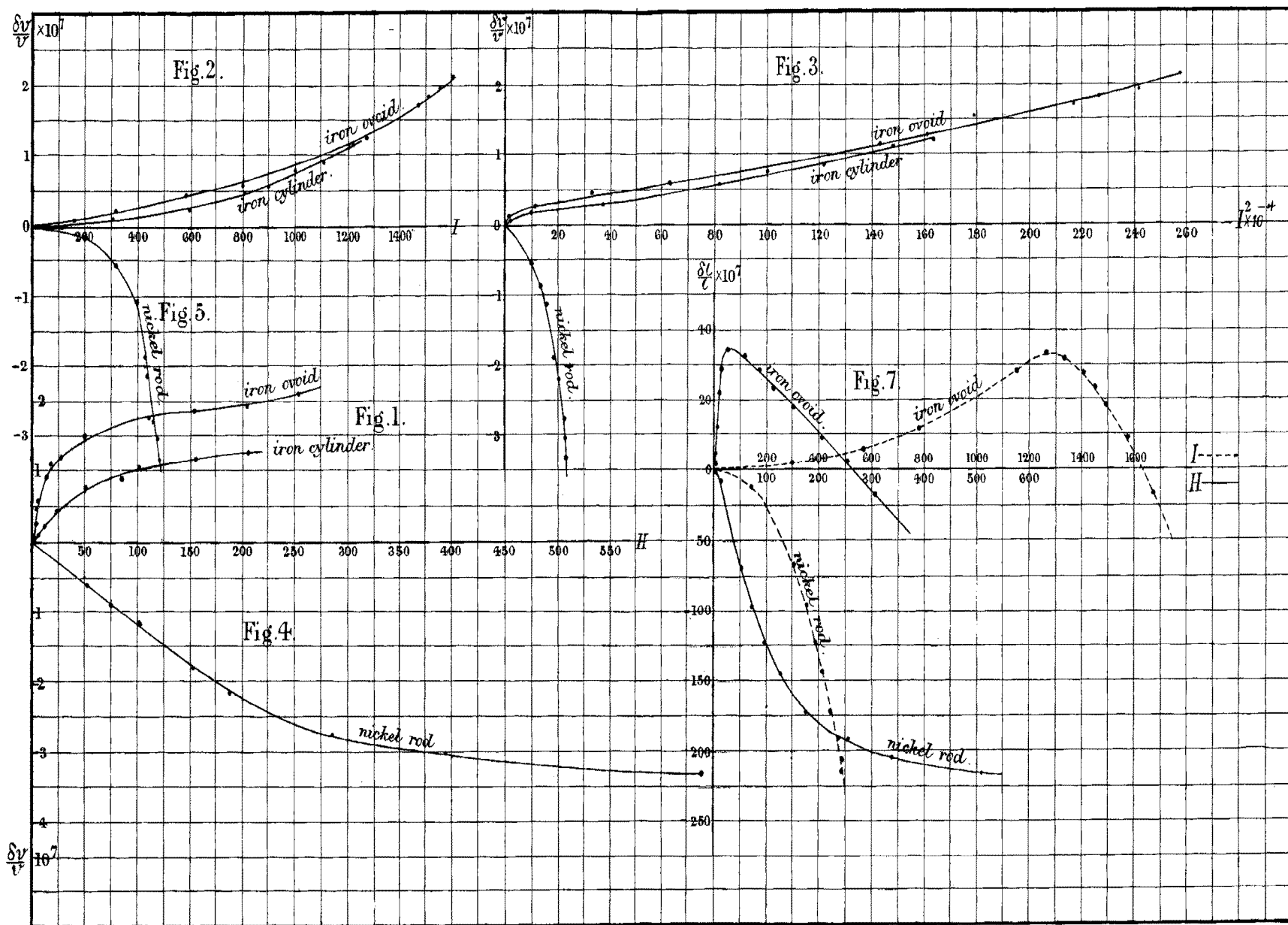
These numbers plotted against H, I, and I^2 are shown in figs. 1, 2, 3 (Pl. I.). Fig. 1 shows that iron increases in volume very rapidly with the magnetizing-force; but it soon reaches the "Wendepunkt," whence to increase asymptotically with further increase of the field-strength. Fig. 2 shows that the increase of volume takes place very slowly with increase of magnetization, but goes on rapidly as the magnetization becomes stronger. It will be seen from fig. 3 that the increase of volume is approximately proportional to the square of the intensity of magnetization.

Bidwell* found from measurement of the change of dimension of iron rings that there is diminution of volume in weak field; and experiments by Dr. Knott on the change of internal volume seem to confirm the result. In the present experiment with ovoid or with cylinder we found no such diminution, but always increase of volume in iron. The behaviour of iron, as regards the change of volume, is in rough agreement with the result obtained by Dr. Knott with tubes of wide bore.

The following table gives the determination of the change of volume in nickel.

H.	I.	$\frac{\delta v}{v} \times 10^7$.
55	320	-0.6
74	360	-0.8
101	396	-1.1
127	416	-1.4
152	432	-1.9
188	450	-2.2
288	476	-2.7
391	484	-3.1
640	490	-3.4

* Bidwell, Proc. Roy. Soc. vol. lvi. p. 94 (1894).



Nickel shows always diminution of volume, and the change is greater than in iron.

From figs. 4, 5, 6 we gather the following facts: in low field the change is very small, it then goes on increasing rapidly until it reaches the "Wendepunkt," whence to increase steadily though at a slower rate.

For feeble magnetization the change of volume is very small; but with the strong, the change is nearly proportional to the square of the intensity of magnetization. Compared with Dr. Knott's measurements, we find that the initial behaviour of nickel as regards the internal and external volume-changes is different, Dr. Knott finding the increase of internal volume. In other respects the quality of the change is similar, but the amount is nearly ten times smaller in the present experiment than the determination of Dr. Knott on the internal volume of the nickel tube; Cantone's determination with nickel ovoid is twice as large as in the present measurement.

Change of Length by Magnetization.—It would be superfluous to give minute details of the measurement of the change of length by magnetization. The apparatus was the same as that used by one of us* some years ago in the measurement of hysteresis accompanying the change of length. It consisted of a single optical lever with arrangement for temperature compensation on the same principle as the gridiron-pendulum. The mirror described in the former paper was, on this occasion, replaced by a small right-angled prism.

The measurements of the length-change in iron and nickel are given in the following table, with the corresponding values of H and I .

Iron Ovoid.

H.	I.	$\frac{\delta l}{l} \times 10^3$.
3	250	1.1
6	780	11.3
8	920	22.7
14	1160	23.3
30	1270	33.1
51	1340	31.6
86	1420	28.0
113	1460	23.8
151	1500	16.6
210	1560	8.3
253	1600	2.5
306	1650	-6.4

Nickel Rod.

H.	I.	$\frac{\delta l}{l} \times 10^3$.
15	143	-11.3
53	315	-69.4
74	355	-95.3
98	394	-124.0
122	414	-142.6
177	444	-172.8
255	474	-190.8
337	483	-207.0
507	485	-216.5

* Nagaoka, Phil. Mag. Jan. 1894; Wied. Ann. vol. liii. p. 487 (1894).

These changes are plotted against H and I in fig. 7 (Pl. I.) It will be seen from these curves that the length-change produced in the ovoid or in the nickel rod is similar to that obtained by one of us, and described in the papers above cited. The determinations are in close agreement with the results of Bidwell and several other investigators. The inspection of these figures for nickel shows a striking resemblance between similar curves for the change of volume in the same metal. The behaviour in iron is different as regards the change of length and that of volume.

§ 2. *Effects of Hydrostatic and Transverse Pressures on the Magnetization of Iron and Nickel.*

The remarkable effect produced by longitudinal pull or compression on the magnetization of ferro-magnetic bodies premised the outcome of a similar result by the application of hydrostatic pressure, as shown by the experiments of Wassmuth*. No such marked influence of compression was observed, but a minute change in the reading of the magnetometer showed that the effect was not immeasurably small. It was only by special arrangement that the nature of the change was clearly made out.

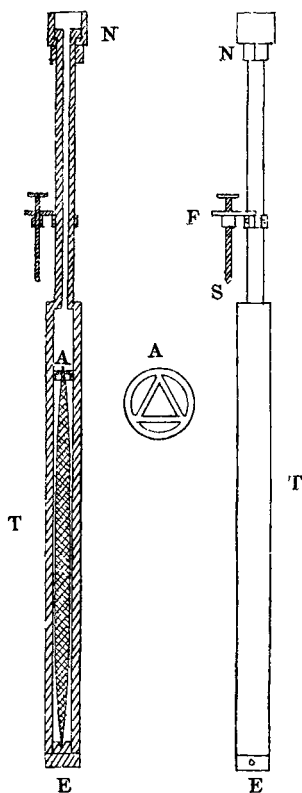
The hydrostatic pressure was given by means of Cailletet's pump used for liquefying gases. The pump was provided with Ducretet manometer indicating pressure up to 300 atmospheres. These indications, on being gauged by measuring the volume of dry air, showed wide difference from the actual pressure, the relation between the change of volume and pressure being taken from Natterer and Amagat's determinations. One end of a seamless copper tube 4·7 metres long and of 3 millim. internal, 7 millim. external diameter was attached to the pump; by pumping water into the tube pressure was communicated to a vessel containing iron or nickel which is to be compressed.

The ovoid or rod, which was to be examined under different pressures, was enclosed in a short brass tube T (fig. 2) (internal diameter 1·1 centim., external diameter 2 centim., length 31 centim.) filled with water. The tube fitted loosely in the magnetizing-coil. The lower end of the ovoid was placed in a conical hole bored in the end screw fitting into the main tube T . To prevent dislocation of the ovoid and to keep it always vertical, the upper end was loosely placed in a triangular hole in the brass plate A in the manner already described in the determination of the change of volume. The neck of the vessel consisted of a smaller brass tube provided

* Wassmuth, *loc. cit.*

with a flange F; by means of a strong brass screw S attached to F the whole vessel can be slowly moved in the solenoid, so that there was no difficulty in placing the magnetized body in proper position for experiment.

Fig. 2.

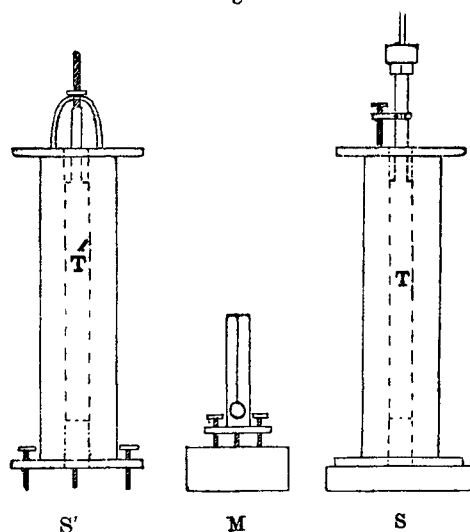


The vessel was connected with the copper tube from the pump by a screw-nut N. Before experiment it was always necessary to apply a pressure of more than 250 atmospheres to find if there was leakage at the screw-joints.

Two such brass tubes T and T', both containing the magnet in the same geometrical form, were placed in the solenoids S and S' (fig. 3), which also were of the same dimensions, as shown in the accompanying figure. The coils were each 40 cm. long, wound in six layers, and gave the field of 17.7 C.G.S. units for a current of one ampere. The magnetometer M was placed midway between the solenoids, due

magnetic east and west. When the current was made, the magnetization of two similar bodies produced nearly the same effect on the magnetometer in opposite sense; thus the deflexion of the magnetometer was so compensated that the magnetometer could be placed very near the solenoids and thereby rendered sensible to a slight change in the condition of the magnetized body. In order to make the compensation exact, the auxiliary solenoid S' was provided with levelling-screws, and the brass tube containing the auxiliary magnet with screw for adjusting the vertical position of the magnet. The magnetizing solenoid was firmly fixed to the solid stone pier, so that there was no risk of being disturbed by the application of strong pressure to the vessel through bent copper tubes; it was also tested by means of a long thread pendulum attached to the solenoid that no appreciable displacement of the solenoid took place during the application or removal of pressure.

Fig. 3.



To make the reading of the magnetometer sensitive, it was necessary to place the magnet in a position of maximum deflexion; this can be effected either by calculation for the ovoid * or experimentally determined for other shapes. The brass vessel was moved slowly up and down to such a position that it was not affected by the small vertical displacement,

* Nagaoka, Wied. *Ann.* vol. lvii. p. 275 (1896).

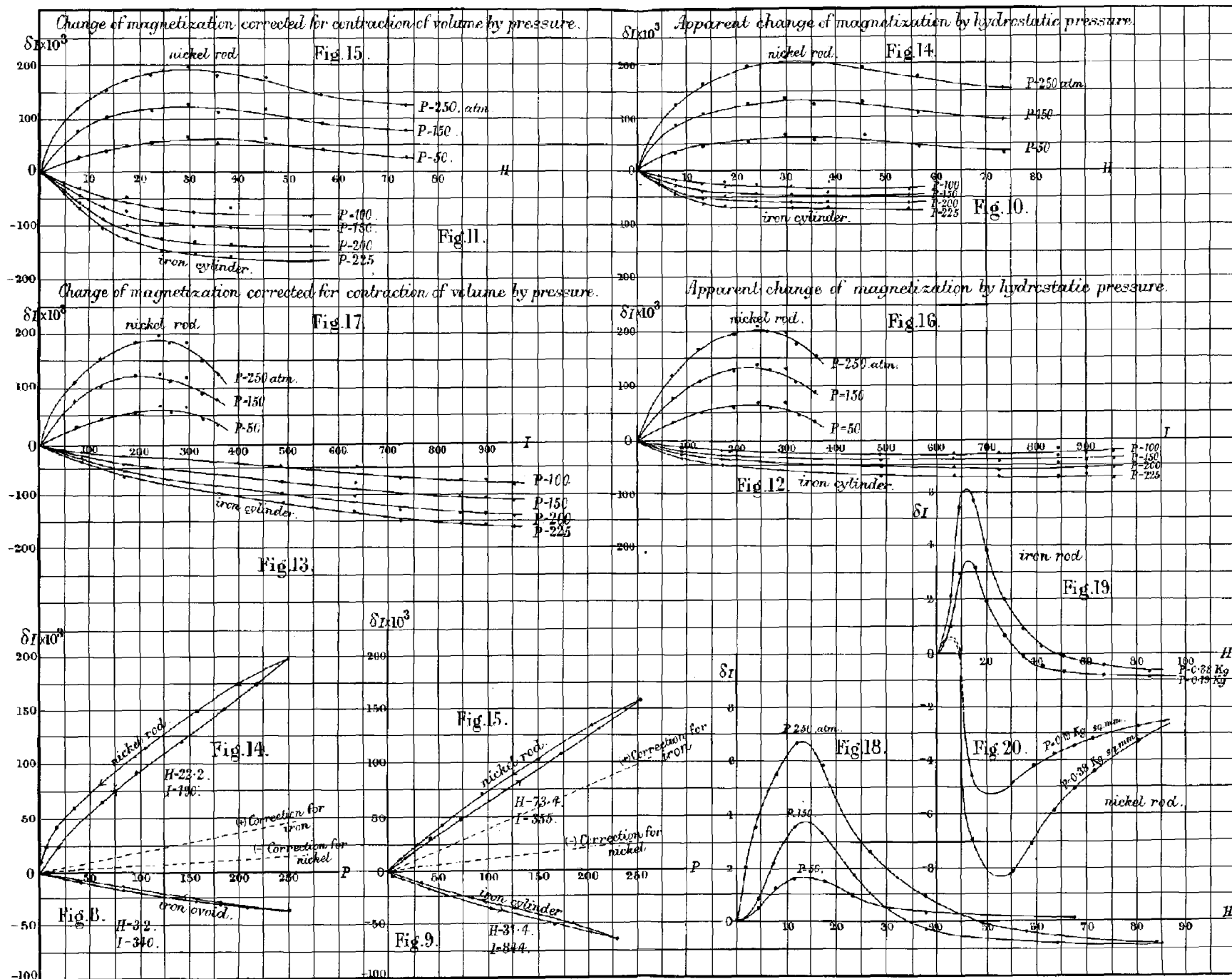
thus giving the position of maximum deflexion. It was necessary to place the magnet in the above position, owing to the slight displacement due to the strain caused by strong pressure. For a small vertical motion, the magnetometer could remain practically unaffected in the position above chosen.

It is clear from the arrangement for compensation that not only is the effect of magnetization on the magnetometer to a great extent compensated, but the effect of temperature rise due to the magnetizing current is also compensated, as the auxiliary magnet is enclosed in a similar brass tube and placed in a coil of the same dimension and resistance. It is to be remarked that the compensation was never exact, for though the ovoids or rods were made of the same material, there was some difference of quality as regards magnetization. Thus the compensation, though exact in certain fields, was not fulfilled throughout the whole range of fields; nevertheless the difference was not very great, and we believe that the influence of the rise of temperature or that of change of position due to the strain caused by pressure would not be so large as to materially deteriorate the experimental results. In spite of this, care was taken to keep the field during the experiment constant by watching the indication of the deciampere balance, by which the current was measured; further, it was generally possible to perfect the compensation for feeble change of current by slightly shifting the auxiliary magnet or the coil.

The horizontal component of the terrestrial magnetic force was slightly affected by thus placing the coils very near the magnetometer, so that it was necessary to measure the period of vibration of the magnetometer magnet by means of a chronograph, and to apply the correction to the observed intensity of magnetization.

The change in the intensity of magnetization due to alteration of volume is evidently nearly equal to $-I\delta v/v$. The diminution of volume will therefore produce increase of magnetization which is generally of the same order of magnitude as the change in magnetization wrought by compression.

Results in Iron.—A few of the observed results with iron ovoid or cylinder are given in figs. 8 and 9 (Pl. II.). The dotted lines indicate the correction due to change of volume by compression, which must be added to the apparent change. The inspection of these figures shows minute diminution of magnetization by the application of hydrostatic pressure: in fact, the apparent change measured in C.G.S. units does not even amount to 0.1 with the pressure of 250 atmospheres. At the above-mentioned pressure the change of intensity for



$H=54$ is less than $\frac{1}{20,000}$ of the intensity of magnetization. During a pressure cycle there is distinct hysteresis, and the curve of the change of magnetization generally forms a single loop. On account of the inconstancy of the field, the measurement with the ovoid could not extend beyond $H=15$; with the iron cylinder $H=54$ was the strongest field in which the cyclic change could safely be observed.

If, from experiments of pressure cycles, the curves of the change in magnetization for constant pressure in different fields be plotted, we obtain fig. 10, when the change of magnetization due to contraction of volume is not taken into account; if the correction be applied, then we obtain fig. 11. These curves show that the range of the change in magnetization due to pressure increases with the field; the increase takes place very rapidly at first, but becomes asymptotic in moderate fields. Plotting these changes against magnetization we obtain figs. 12 and 13.

Comparing these curves with those for the change of volume by magnetization, we find similarity between the two. It is interesting to remark that whereas *increase of magnetization produces increase of volume in iron, the diminution of volume produces diminution of magnetization*. Thus a reciprocal relation between the strain caused by magnetization and the effect of compressional stress on the magnetization of iron is established.

Results in Nickel.—The curves of pressure cycle in nickel are shown in figs. 14 and 15 (Pl. II.). The change of magnetization wrought by compression is exceedingly small, but comparatively greater than those in iron, and the hysteresis during the cycle is more decided. Whereas hydrostatic pressure causes diminution of magnetization in iron, there is *increase* of magnetization in nickel. Similar to other effects of stress as stretching and twisting, we find that the change in iron is opposite to that in nickel.

The curves of the change of magnetization by constant pressure in different fields (figs. 14 and 15) show that there is increase of magnetization in weak fields until it reaches a maximum in moderate fields; it then goes on slowly decreasing. This feature is characteristic of all pressures up to 250 atmospheres. Plotted against magnetization, the general appearance of the curves is the same as that for magnetizing fields (figs. 16 and 17, Pl. II.)

Comparing these curves with those obtained from change of volume by magnetization, we notice that whereas *increase of magnetization produces diminution of volume in nickel, the diminution of volume produces increase of magnetization*.

It will be shown later on that the minuteness of the effect of compression on the magnetization of iron and nickel leads to an important conclusion in the theory of magnetostriction.

Effect of Transverse Stress on the Magnetization of an Iron Tube.—Lord Kelvin*, in his series of experiments on the electrodynamic qualities of metals, investigated the effect of transverse stress on the magnetization of an iron tube by subjecting the inner surface of a gun-barrel to hydrostatic pressure. In our experiment it was of no small importance to try similar experiments with iron, to decide whether the minute change produced by all-sided pressure was also characteristic of the effect of transverse stress produced by pressure on the external surface of an iron or nickel tube.

To the extremities of a hollow iron cylinder (external diameter 0.936 cm., internal diameter 0.400 cm., length 25 cm.) were soldered two thick brass caps in the manner shown in the annexed diagram (fig. 4), and placed in the compressing-vessel above described. By pumping in water to the vessel, the iron tube was subjected to pressure on its external surface alone, and the change of magnetization tested in the manner above described. It was soon noticed that the effect was enormously large and opposite to that of all-sided pressure. By keeping the pressure constant, the difference in magnetization when the tube was in the strained and unstrained state was determined for different fields; the curves of the change in magnetization thus obtained for pressures of 50, 150, and 250 atmospheres are shown in fig. 18 (Pl. II.).

Fig. 4.



The present experiment is just the reverse of Lord Kelvin's, and the inspection of the figures (fig. 18) will show that the result is just the reverse. With increase of the magnetizing force there is increase of magnetization till it reaches a maximum, thence to diminish in stronger fields. As the pressure is increased, the decrease of magnetization after once reaching a critical value is so great that the magnetization in strong fields is less than in the unstrained condition. The result is thus in close agreement with Lord Kelvin's anticipation that the effects of positive pressure will be opposite to the effects of negative pressure.

These experiments show that the application of stress so as to produce no shear affects the magnetization of iron and nickel only very slightly; but the remarkable change in magnetization produced by tensional or compressional stress

* Kelvin, *Phil. Trans.* clii. p. 64 (1878); *Mathematical and Physical Papers*, ii. p. 370 (1884).

applied longitudinally, as well as that due to twisting-wrench, is always accompanied by a shearing strain, a result which will be of no small value in the theory of molecular magnetism.

Effects of Longitudinal Pull on the Magnetization of Iron and Nickel.—This subject has been studied by several investigators, but, so far as we are aware, the change in magnetization by the application of feeble stress is scarcely known. It will be seen from experiments on the strain produced by magnetization that the deformation corresponds to the effect of a feeble stress. As our principal object was a comparison of Kirchhoff's theory of magnetostriction with experiment, we found it necessary to pay special attention to the change in the magnetic qualities of iron and nickel, when the rods of these metals are subjected to small loading, which will strain the ferromagnetic body to an extent comparable with the deformation in the magnetizing field.

As the iron ovoid used in the preceding experiment was unfit for studying the effect of longitudinal pull, an iron rod of 0.273 sq. cm. section made of the same material as the ovoid was used for measuring the change of magnetization in the free and in the feebly stretched condition. The nickel rod used in all the preceding experiments was also examined. The magnetometer was made sensitive by means of the compensation arrangement used in studying the effects of hydrostatic pressure.

The following table gives the change of magnetization in different fields due to longitudinal stresses 0.19 kg. sq. mm. and 0.38 kg. sq. mm.; corresponding to elongations 0.85×10^{-5} and 1.71×10^{-5} resp. in iron; and to elongations 0.90×10^{-5} and 1.8×10^{-5} resp. in nickel.

Iron (fig. 18).

H.	δI	δI
	0.38 kg. sq. mm.	0.19 kg. sq. mm.
6.1	+2.15	+1.01
9.6	+5.46	+2.95
15.5	+5.74	+3.10
20.4	+3.72	+1.85
27.3	+2.00	+0.64
34.6	+0.92	-0.13
42.5	+0.24	-0.56
50.8	-0.11	-0.77
67.6	-0.49	-0.86
84.6	-0.69	-0.94

Nickel (fig. 19).

H.	δI	δI
	0.38 kg. sq. mm.	0.19 kg. sq. mm.
7.9	+0.63	+0.52
14.5	-6.96	-4.62
22.2	-8.35	-5.34
30.2	-8.13	-4.82
38.5	-7.07	-4.16
47.1	-5.85	-3.73
55.3	-5.02	-3.45
63.9	-4.41	-3.21
81.0	-3.27	-2.77
98.3	-2.44	-2.44

The magnetization of iron in the stretched state increases with the magnetizing force till it reaches a maximum in $H=14$ nearly; it then goes on slowly diminishing, and ultimately becomes less than in the free state. In nickel there is decrease of magnetization in the stretched state, except in weak fields, where a slight increase was observed. Corresponding to a critical field in iron, for which the change of magnetization is maximum, there is also a critical field for which the diminution of magnetization in nickel is maximum. It will be seen from the figures that the change of magnetization is not exactly proportional to the amount of longitudinal stress.

It appears from Prof. Ewing's* experiment that the increase of magnetization in iron in weak fields becomes more pronounced with greater loading, but the field at which the magnetization becomes smaller than in the unloaded state recedes towards the weaker side. Although Prof. Ewing did not observe these points in fields greater than 8, with loading which is far greater than that in the present experiment, we can see from the course of curves of magnetization that if the loading be greatly diminished, the above-mentioned field will become correspondingly large. In the present experiment, it occurs in $H=48$ for iron with longitudinal stress 0.38 kg. sq. mm. Thus the general feature of the present investigation agrees with that of Prof. Ewing.

In nickel, there was a slight increase of magnetization in a weak field when the rod was loaded; whether this has any connexion with the Villari effect observed by Heydweiller† is a question which, without special examination, cannot be easily decided.

§3. *Calculation of the Coefficients k' and k'' . Comparison between Theory and Experiment.*

According to Kirchhoff's‡ theory of magnetostriction, the coefficients k , k' , k'' are defined by the equations :

$$\begin{aligned} I_x &= \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_x\} H_x, \\ I_y &= \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_y\} H_y, \\ I_z &= \{k - k'(\lambda_x + \lambda_y + \lambda_z) - k''\lambda_z\} H_z; \end{aligned}$$

where I_x , I_y , I_z are the components of the magnetization, H_x , H_y , H_z those of the magnetizing force, and λ_x , λ_y , λ_z are

* Ewing, Phil. Trans. clxxvi. (ii.) p. 608 (1885).

† Heydweiller, Wied. Ann. lii. p. 462 (1894).

‡ Kirchhoff, *loc. cit.*; see also Pockels, *Arch. d. Math. u. Phys.* (2) xii p. 57 (1893).

the component elongations. The coefficient k is nearly equal to susceptibility as the strain due to magnetization is negligibly small. The determination of the coefficients k' and k'' involves considerable difficulty, because the strains produced by magnetization or the effects of stress on magnetization generally depend on both of these coefficients. In a joint paper with Mr. E. T. Jones, one of us remarked that the easiest method of testing Kirchhoff's theory would be to measure the change of volume of a ferromagnetic ring. The volume change is theoretically equal to

$$\frac{\delta v}{v} = 3 \frac{\delta l}{l} = \frac{3H}{4K(1+3\Theta)} (1-k'H),$$

following Kirchhoff's notation. Unfortunately there is great experimental difficulty, if the test be made by means of a dilatometer, except in the manner introduced by Bidwell of measuring the change in the section of the ring.

Cantone found that the change of length and of volume of an elongated ovoid are given by the following formulæ according to Kirchhoff's theory:—

$$\frac{\delta l}{l} = \frac{H^2}{E(1+2\Theta)} \left\{ \frac{4\pi k^2}{3} (1+\Theta) + \frac{k-k'}{2} - \frac{k''}{2} (1+2\Theta) \right\}, \quad (a)$$

$$\frac{\delta v}{v} = \frac{H^2}{K(1+3\Theta)} \left\{ \pi k^2 + \frac{3(k-k')}{4} - \frac{k''}{4} \right\}, \quad \dots \dots (b)$$

where E is Young's modulus, K the rigidity, and Θ a constant defined by the relation

$$\frac{E(1+2\Theta)}{2(1+3\Theta)} = K.$$

In the above formulæ terms involving the ratio $\left(\frac{\text{minor axis}}{\text{major axis}}\right)^2$ are neglected.

Corresponding expressions for a long prismatic body as wires or rods placed in uniform magnetizing field can be approximately calculated in the following manner:—

Let the field-strength in the coil be denoted by H_0 ; then the potential of the magnetizing force would be

$$\phi = -\frac{H_0}{1+Nk} x = -Hx,$$

where N is the demagnetizing factor, k the susceptibility, and x the direction of magnetization.

Supposing the magnetization to be uniform, the component of the internal force would be

$$A = \frac{1}{2} \left(k' + \frac{k''}{2} \right) \frac{\partial H^2}{\partial x} = 0,$$

$$B = 0, \quad C = 0.$$

The surface-traction on the end-faces, which we consider to be perpendicular to the x -axis, has components

$$\bar{A}_1 = \left(2\pi k^2 + \frac{k - k' - k''}{2} \right) H^2 = \alpha H^2,$$

$$\bar{B}_1 = 0, \quad \bar{C}_1 = 0.$$

On the lateral faces

$$\bar{A}_2 = 0, \quad \bar{B}_2 = \frac{k - k'}{2} H^2 \cos \theta = \beta H^2 \cos \theta,$$

$$\bar{C}_2 = \frac{k - k'}{2} H^2 \sin \theta = \beta H^2 \sin \theta,$$

where θ is the angle made by the normal with the y -axis. Thus the surface-traction

$$\bar{X}_n = \alpha H^2 = -X_x,$$

$$\bar{Y}_n = \beta H^2 \cos \theta = -Y_y \cos \theta,$$

$$\bar{Z}_n = \beta H^2 \sin \theta = -Z_z \sin \theta,$$

and

$$X_y = Y_z = Z_x = 0,$$

which satisfy the equations

$$\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} = 0,$$

$$\frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} = 0,$$

$$\frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y} + \frac{\partial Z_z}{\partial z} = 0.$$

Thus we get the equations

$$-X_x = 2K \left(\frac{\partial u}{\partial x} + \Theta \sigma \right) = \alpha H^2$$

$$-Y_y = 2K \left(\frac{\partial v}{\partial y} + \Theta \sigma \right) = \beta H^2$$

$$-Z_z = 2K \left(\frac{\partial w}{\partial z} + \Theta \sigma \right) = \beta H^2,$$

where

$$\frac{\partial u}{\partial x} = \lambda_x, \quad \frac{\partial v}{\partial y} = \lambda_y, \quad \frac{\partial w}{\partial z} = \lambda_z,$$

and

$$\sigma = \lambda_x + \lambda_y + \lambda_z.$$

These equations give

$$\sigma = \frac{(\alpha + 2\beta)}{2K(1 + 3\Theta)} H^2 = \frac{\gamma}{2K\Theta} H^2,$$

or

$$\sigma = \frac{\delta v}{v} = \frac{H^2}{2K(1 + 3\Theta)} \left\{ 2\pi k^2 + \frac{3}{2} (k - k') - \frac{k''}{2} \right\} \quad (c)$$

Similarly we obtain

$$u = \frac{\alpha - \gamma}{2K} H_x^2, \quad v = \frac{\beta - \gamma}{2K} H_y^2, \quad w = \frac{\beta - \gamma}{2K} H_z^2.$$

The elongation in the direction of magnetization is thus

$$\lambda = \frac{\delta l}{l} = \frac{H^2}{E} \left\{ 2\pi k^2 - \frac{k''}{2} + \frac{k - k'}{2(1 + 2\Theta)} \right\} \quad (d)$$

Supposing that the Poisson ratio is $\frac{1}{2}$, or $\Theta = \frac{1}{2}$, we find for a prismatic body

$$\lambda = \frac{\delta l}{l} = \frac{I^2}{E} \left(2\pi + \frac{k - k'}{4k^2} - \frac{k''}{4k^2} \right),$$

$$\sigma = \frac{\delta v}{v} = \frac{I^2}{E} \left(\pi + \frac{3(k - k')}{4k^2} - \frac{k''}{4k^2} \right).$$

Corresponding formulæ for the ovoid are, according to Cantone,

$$\lambda = \frac{\delta l}{l} = \frac{I^2}{E} \left(\pi + \frac{k - k'}{4k^2} - \frac{k''}{2k^2} \right),$$

$$\sigma = \frac{\delta v}{v} = \frac{I^2}{E} \left(\pi + \frac{3(k - k')}{4k^2} - \frac{k''}{4k^2} \right).$$

Comparing the formulæ (a), (b), (c), (d), we find that the change of volume is the same for the ovoid as for the prismatic body; the difference in the length-change is equal to $\frac{2(1 + 4\Theta)I^2}{3(1 + 2\Theta)E}$, being slightly greater for the prismatic body than for the ovoid.

The formulæ (c) and (d) are never exact, as prismatic bodies cannot be magnetized uniformly, and consequently there must be also internal forces acting. But to the first approximation we can use these formulæ, inasmuch as the strain caused by magnetization can only be roughly measured.

The change in magnetization due to increase of volume by hydrostatic pressure σ is evidently

$$\delta I = -H(k' + \frac{1}{3}k'')\sigma, \quad (e)$$

and the change of susceptibility due to longitudinal stretching λ of a prismatic body

$$\delta k = \left\{ k' \frac{E}{K} - 3(k' + \frac{1}{3}k'') \right\} \lambda. \quad (f)$$

For the determination of the coefficients k' and k'' the combination of the experimental data in any two sets of the experiments already described can be conveniently used. In order to test Kirchhoff's theory we have calculated k' and k'' from experiments on the change of volume and of length by magnetization, and compared them with values deduced from experiments on the change of magnetization produced by compression and by stretching.

For an ovoid we obtain from (a) and (b)

$$(A) \begin{cases} k' = \frac{p(1+2\Theta)-q}{2(1+3\Theta)}, \\ k'' = \frac{3q-p}{2(1+3\Theta)}; \end{cases}$$

and for a prismatic body we obtain from (c) and (d)

$$(B) \begin{cases} k' = k + \frac{E^2}{2KH^2}(\lambda - (1+2\Theta)\sigma), \\ k'' = 4\pi k^2 - \frac{E^2}{2KH^2}(3\lambda - \sigma); \end{cases}$$

where

$$p = -\frac{4K(1+3\Theta)}{H^2}\sigma + 4\pi k^2 + 3k, \\ q = -\frac{2E(1+2\Theta)}{H^2}\lambda + \frac{8\pi k^2}{3}(1+\Theta) + k.$$

From (e) and (f) we find

$$(C) \begin{cases} k' = \left(\delta k - \frac{3\lambda \delta I}{\sigma H} \right) \frac{K}{\lambda E}, \\ k'' = -3 \left(\frac{\delta I}{\sigma H} + k' \right). \end{cases}$$

As these coefficients depend on Young's modulus and rigidity, it was necessary to determine the constants on the specimens of ferromagnetics used in these experiments. Young's modulus was determined in the usual way, from flexure experiments on iron and nickel rods already examined. The modulus of rigidity was found from measurement of

the torsion of the rods by a known twisting couple ; for calculating the rigidity the following formula, due to Saint Venant *, was used :

$$\text{Modulus of rigidity} = \frac{6 \text{ twisting couple}}{0.8435 \times \text{angle of torsion} \times \text{cross-section}^2}.$$

The following are the results :—

	E (Young's modulus).	K (Rigidity).	Bulk modulus.	Θ.
Iron	2.16×10^{12}	0.800×10^{12}	1.88×10^{12}	0.844
Nickel	2.07×10^{12}	0.771×10^{12}	2.16×10^{12}	1.082

The constants for nickel are in fair agreement with Prof. Voigt's † determination, who gives $\bar{E} = 2.03 \times 10^{12}$, $K = 0.782 \times 10^{12}$.

We now possess sufficient experimental data to calculate the coefficients k' and k'' . In finding k' and k'' from experiments on the effects of stress on magnetization we shall combine the measurement of the changes produced by pull and by hydrostatic pressure. Taking into account the minuteness of the effect of pressure compared with that of pull, we can greatly simplify the calculation.

From § 2 we know that $\frac{\delta I}{H\sigma}$ is very small compared with $\frac{\delta k}{\lambda}$; we therefore conclude from (e) that in the combination above mentioned

$$3k' \doteq -k'',$$

whence (f) gives for the change of susceptibility due to longitudinal pull

$$\delta k = \frac{E}{K} k'.$$

We can thus find k' and k'' without further experiment from measurement of the effect of longitudinal pull only.

The difference in the value of k' caused by neglecting $\frac{\delta I}{H\sigma}$ lies within the experimental error as will be immediately shown by actual calculation.

k' and k'' for Nickel.—As the nature of these coefficients is simpler in nickel than in iron, we shall first give the result of calculation for the former metal.

The following table contains the numbers obtained from the experimental curves and used in the calculation of k' and k'' from the strains produced by magnetization.

* Saint Venant, *Torsion des Prismes*, p. 376 (1855).

† Voigt, *Wied. Ann.* vol. xlix. p. 396 (1893).

TABLE I.

H.	k .	$\frac{\partial l}{l} \times 10^7$.	$\frac{\partial v}{v} \times 10^7$.	k' (calculated).	k'' (calculated).	$3k' + k''$.	$3k' + k''$ (calc. from $\frac{\partial v}{v}$).
5	4.20	- 2.5	-0.01	-14930	+46400	+1610	810
10	9.50	- 7.5	-0.03	-11410	35760	1530	1540
20	9.15	-20.5	-0.07	- 7800	24710	1310	1340
30	8.10	-33.8	-0.13	- 5700	18150	1050	1060
40	6.98	-50.0	-0.22	- 4740	15040	820	840
50	6.10	-65.0	-0.34	- 3930	12470	680	690
100	3.90	-124.0	-1.00	- 1850	5910	360	350
150	2.88	-158.0	-1.59	- 1040	3340	220	230
200	2.27	-175.6	-2.04	- 650	2090	140	150
300	1.59	-201.3	-2.47	- 330	1060	70	80
400	1.22	-214.1	-2.70	- 200	630	30	50
500	0.98	-217.6	-2.83	- 130	410	20	30

The coefficients k' and k'' are very large in low fields and diminish rapidly as the field is increased. The values of $3k' + k''$ calculated from the strains caused by magnetization show that it is generally very small compared with k' or k'' and the condition

$$3k' \doteq -k''$$

is nearly fulfilled. Since the change of magnetization due to increase of volume σ is $\delta I = -\left(k' + \frac{k''}{3}\right)\sigma$, we see that if $k' + \frac{k''}{3} > 0$, there is increase of magnetization by compression.

Thus, if we accept Kirchhoff's theory, the smallness of the volume-change by magnetization is necessarily accompanied by the smallness of the effect of hydrostatic pressure, and the strains produced in nickel by magnetization lead to the conclusion that the pressure must increase the magnetization.

Using the experimental results for the change of magnetization by longitudinal pull, we find the following numbers for k' :—

TABLE II.

H.	δI_1 (for 0.38 kg. sq. mm.).	δI_2 (for 0.19 kg. sq. mm.).	k' (from δI_1).	k' (from δI_2).
10	-2.95	-1.25	-6130	- 5200
15	-7.20	-4.84	-9980	-13390
20	-8.27	-5.32	-8610	-11060
30	-8.16	-4.85	-5650	- 6740
40	-6.80	-4.05	-3530	- 4200
50	-5.52	-3.62	-2290	- 3010
70	-3.97	-3.03	-1180	- 1860
90	-2.80	-2.57	- 646	- 1190
100	-2.37	-2.41	- 493	- 1000

The numbers for $k' = -\frac{k''}{3}$ calculated from the stress-effect on the magnetization of nickel is in rough agreement with those deduced from the strains caused by magnetization, the coincidence becoming closer with smaller loading.

Let us now calculate the strain which should be produced by magnetization, according to Kirchhoff's theory, from the effects of stress, and the stress-effect from the strains produced by magnetization.

If we adopt the numbers in Table I., and calculate the

change of magnetization due to longitudinal pull (0.19 kg. sq. mm.), we obtain the following numbers :—

H.	δI (calculated).	δI (experiment).
10	-2.9	-1.0
20	-4.0	-5.3
30	-4.5	-4.9
40	-4.9	-4.2
50	-5.0	-3.7
90	-5.0	-3.3
100	-4.8	-2.5

The critical field given by theory is greater than that found by experiment.

We now use the numbers in Table II., and calculate the strains due to magnetization ; we thus obtain

H.	$\frac{\delta l}{l}$ (calculated from k_2').	$\frac{\delta l}{l}$ (experiment).	$\frac{\delta v}{v}$ (cal.)	$\frac{\delta v}{v}$ (experiment).
10	-3.1×10^{-7}	-7.5×10^{-7}	0.1×10^{-7}	-0.0×10^{-7}
15	-18.9	-11.0	0.1	-0.0
20	-27.7	-20.5	0.3	-0.1
30	-37.5	-33.8	0.5	-0.1
40	-41.3	-50.0	0.6	-0.2
50	-46.0	-65.0	0.7	-0.3
70	-55.3	-93.0	0.9	-0.6
90	-58.1	-115.5	1.0	-0.9
100	-60.2	-124.0	1.1	-1.0

The change of length in nickel, as calculated from the stress-effect, agrees fairly with the observed values, except in strong fields, where the deviation becomes apparent. Of the two sets of k' , the one derived from the effects of smaller stress gives results which are more conformable to experiment, at least in quality. The agreement between theory and experiment would perhaps be closer, could we measure the change in the intensity of magnetization by smaller loading, or, better still, from the effects of small longitudinal compression. Adopting the numbers obtained from the stress-effect, the change of volume by magnetization ought to be very small. The discrepancy between theory and experiment lies in the sign ; theory gives increase of volume instead of diminution as in the actual case. But considering the minuteness of the change and the experimental errors which enter in the determination of k' , we cannot say that the discrepancy is very great.

It must not be forgotten that these coefficients are, strictly speaking, functions of the strain caused by mechanical action on nickel. Taking Prof. Ewing's* experiments on the magnetization of nickel under various loadings, we find the following values of k' on the supposition that

$$3k' + k'' = 0.$$

H.	5.5 kg. sq. mm.	11 kg. sq. mm.	16.5 kg. sq. mm.	22 kg. sq. mm.	27.5 kg. sq. mm.	33 kg. sq. mm.
30	-4250	-3620	-2950	-2350	-1680	-1170
50	-2320	-2140	-1930	-1660	-1260	- 900
100	- 950	- 850	- 810	- 800	- 670	- 520

The above values will probably not be far from those obtained by actual determination. The coefficients k' and k'' are thus functions of the strains of the magnetized body. In nickel, k'' diminishes as the longitudinal pull is increased. In calculating the coefficients k' and k'' from the stress-effect, we have taken care to use such values of δI as are due to very small loading, in order that the result may be comparable to those obtained from the strains produced by magnetization. The diminution of k'' with increased loading is greater in the weak than in strong fields. Applying equation (d) for measuring the length-change produced by magnetization, we notice that $\delta l/l$ diminishes with the coefficient k'' , so that we expect, from the above result, diminution in the contraction of nickel wire with increased longitudinal pull; but as the rate of diminution of k'' becomes less as the field-strength is increased, the lessening of contraction will not be so marked in strong fields as in weak. This theoretical conclusion is borne out by the experiments of Bidwell on the effects of longitudinal stress on the length-change of nickel wire. The change of volume due to magnetization will somewhat diminish for nickel wire under longitudinal pull, but the difference will not be so pronounced as for the length-change. The experimental verification of these conclusions will be attended with considerable difficulty.

k' and k'' for *Iron*.—Making use of the measurement of strains in the ovoid produced by magnetization, we find the following numbers for k' and k'' .

* Phil. Trans. clxxix. A, p. 325 (1888).

TABLE III.

H.	k .	$\frac{\delta l}{l} \times 10^7$.	$\frac{\delta v}{v} \times 10^7$.	k' (calcul.).	k'' (calcul.).	δI (calcul.).	δI (exp.).
5	140	8.0	0.50	+93300	-56300	1.84	1.2
10	101	26.0	0.79	61600	-74400	8.60	5.6
15	78.0	29.4	0.97	36800	-29100	3.94	5.8
20	61.0	31.5	1.11	21100	-19400	3.94	4.1
30	42.3	34.0	1.30	10090	-9230	2.84	1.5
40	32.5	32.8	1.41	5650	-4620	1.72	0.4
50	26.7	31.8	1.50	3660	-2590	1.05	-0.1
75	18.6	28.0	1.62	1590	-685	0.11	-0.6
100	14.5	24.1	1.70	870	-121	-3.47	...
125	11.8	20.4	1.77	530	+58	-5.45	...
150	10.0	16.8	1.84	353	+134	-6.78	...
200	7.8	9.7	1.99	183	+183	-8.57	...
300	5.5	-4.6	2.28	65	+170	-9.94	...

The above table shows that k' and k'' are of the same order of magnitude as for nickel; they are, however, of opposite sign. The approximate relation $k' + \frac{k''}{3} \doteq 0$ does not hold for iron, the quantity $k' + \frac{k''}{3}$ amounting to several thousands in low fields. It therefore appears that the effect of hydrostatic pressure must result in considerable increase of magnetization, which is irreconcilable with the experiments already described. The change in magnetization due to longitudinal pull (0.38 kg. sq. mm.) is calculated in the seventh column; in low fields there is increase of magnetization, which ultimately reaches a maximum in $H=12$ nearly; the magnetization then begins to diminish very slowly until it becomes less than in the unstrained state in $H=90$. This theoretical conclusion agrees with experiment, although the actual numbers are somewhat different, as will be seen in the last column.

If, on the other hand, we make use of the experimental result that the effect of hydrostatic pressure is negligible compared with that of longitudinal pull, we obtain the following values of $k' = -\frac{1}{3} k''$ by easy calculation from experiments on stretching.

TABLE IV.

H.	k' .	$\frac{\delta l}{l} \times 10^7$ (calc.).	$\frac{\delta l}{l} \times 10^7$ (exp.).	$\frac{\delta v}{v} \times 10^7$ (calc.).	$\frac{\delta v}{v} \times 10^7$ (exp.).
10	10000	20.2	25.5	11.4	0.8
15	12640	31.9	29.3	17.2	1.0
20	9730	44.6	31.5	16.7	1.1
30	4120	45.2	33.0	18.2	1.3
40	1420	37.4	32.8	18.8	1.4
50	360	30.0	31.8	20.2	1.5
70	-100	23.2	28.3	21.4	1.6
90	-160	21.7	25.6	24.2	1.7

The numbers found above are widely different from those calculated from the strains due to magnetization, but the general character of the coefficient k' is similar. Using the values of k' in Table IV. we find that the change of length (3rd column) agrees fairly with the experimental determination (4th column). The field of maximum elongation given by calculation coincides pretty well with the actual result. According to calculation, there is always increase of volume with increasing field, but the calculated result is about 15 times greater than the experimental numbers. The theoretical conclusion as regards the change of volume by magnetization agrees only in quality.

The values of the coefficients k' and k'' for iron and nickel agree in sign with Cantone's determinations from the strains produced by magnetization, but are far behind them in actual numbers. Drude * found from the effect of twist on a circularly magnetized iron that $\mu'' = 4\pi k'' = -400000$ in weak fields. In the present case $k'' = -30000$ in $H=10$, if it be calculated from the stress-effect; thus $\mu'' = -380000$, which is very near to Drude's observation.

Summary.

The principal results obtained in the present research are given in the following summary.

1. Magnetization produces minute increase of volume in iron.
2. Diminution of volume by hydrostatic pressure produces minute decrease of magnetization in iron.
3. Magnetization produces minute diminution of volume in nickel.
4. Diminution of volume by hydrostatic pressure produces minute increase of magnetization in nickel.
5. Positive transverse stress produces increase of magnetization in an iron tube, which reaches a maximum in a certain critical field.

Kirchhoff's theory of magnetostriction leads to the following conclusions :

I. Effects of stress deduced from the strains due to magnetization.

- (a) (Theory).—Hydrostatic pressure produces increase of magnetization in iron.
 (Experiment).—Hydrostatic pressure produces decrease of magnetization in iron.

* Drude, *Wied. Ann.* vol. lxiii. p. 9 (1897).

- (b) (Theory and Experiment).—Hydrostatic pressure produces small increase of magnetization in nickel.
- (c) (Theory and Experiment).—By the application of small longitudinal pull, there is increase of magnetization in iron till it reaches a maximum in moderate fields, thence to diminish till the magnetization becomes smaller than in the unstretched condition.
- (d) (Theory and Experiment).—By the application of longitudinal pull, there is decrease of magnetization in nickel till it reaches a minimum in moderate fields, thence to increase gradually but not in such a degree as to reach a value greater than in the unstretched condition.

II. Strains caused by magnetization deduced from the effects of stress.

- (a) (Theory and Experiment).—Magnetization produces increase of volume in iron (the value assigned by theory being about 15 times greater than the observed numbers).
- (b) (Theory).—Magnetization produces small increase of volume in nickel (to a degree which is within the errors of experiment).
(Experiment).—Magnetization produces decrease of volume in nickel.
- (c) (Theory and Experiment).—Magnetization produces increase of length in iron till it reaches a maximum in about $H=30$, thence to diminish gradually with increasing field.
- (d) (Theory and Experiment).—Magnetization produces continuous diminution of length in nickel.

Experiments show that the coefficients k , k' , k'' are all functions of the strain, but Kirchhoff's theory makes the change of magnetization proportional to the strain. Strictly speaking, the present theory is a rough approximation and will perhaps only hold when the strain is infinitely small. We cannot, therefore, expect that such a theory can explain the relations between the strains caused by magnetization and the effects of stress on magnetization in all their qualitative and quantitative details. In the present investigation, we have taken care to measure such effects as will be most conformable to theory. We have thus found out that, excepting the theoretical deduction as to the effect of hydrostatic pressure on the magnetization of iron, there are no serious

discrepancies between theory and experiment. In default of a more perfect theory, it will be of no small interest to see how far the aforesaid theory can explain the correlation of strain and stress in magnetism ; we intend to continue similar investigations on the Wiedemann effect, and see how the mutual relations between the strains due to magnetization and the effects of stress on magnetization can be traced.

Physical Laboratory, Tōkyō,
March 15th, 1898.

XXVII. *On the Photography of Ripples.—Third Paper.*
*By J. H. VINCENT, D.Sc., A.R.C.Sc.**

[Plates III.-V.]

THE present paper deals with some further experiments on wave-motion, which have, as before, been photographically recorded. In the first two papers† the apparatus used was described. The work has been continued with the second form of apparatus, but it has been necessary to re-arrange the whole, as the induction-machine and tuning-forks suffered so much from the damp that the former finally refused to work and the latter required constant attention to prevent rusting. It was therefore decided to set up the apparatus in another portion of the laboratory ; while this was being done the induction-machine was repaired. The room in which the experiments were continued is at the top of the building, but the unsteadiness was overcome by suspending the trough by a rubber cord.

The floor of this room being unsteady, it was necessary to stop the motor and induction-machine some seconds before the spark was allowed to pass ; otherwise the motion of the floor would cause objects attached to the forks to give rise to disturbances on the mercury surface.

Some alterations were made in the apparatus. The first spark-gap was reduced very considerably in width and was about 2 millim. across. With this alteration it was found that much larger apertures of the lens-stop could be used without any ill-effects on the photographs. This spark-gap was short-circuited by a coil, about 8 centim. in diameter, of a dozen turns of rubber-covered wire. The other electrical arrangements were similar to those described in the first paper. Four gallon Leyden jars were used, and the whole electrical circuit was insulated. The first spark-gap was

* Communicated by Professor J. J. Thomson.

† Phil. Mag. June 1897 ; Proc. of Phys. Soc., July 1897 ; Phil. Mag., Feb. 1898.