POWER REACTOR NOISE:
from the modelling of noise sources to their effect onto the neutron flux

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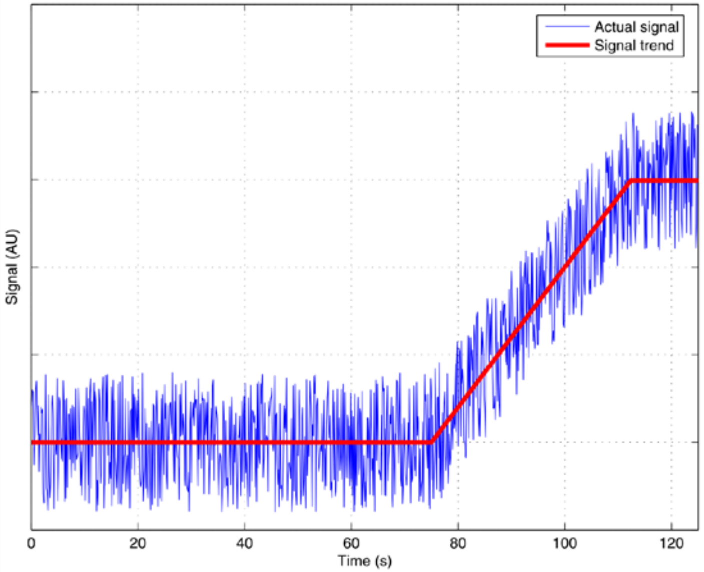
DREAM

Task Force on Deterministic REActor Modelling
at Chalmers University of Technology

Introduction

- Fluctuations always existing in dynamical systems even at steady state-conditions:



$X(\mathbf{r},t)$

Conceptual illustration of the possible time-dependence of a measured signal from a dynamical system

$$X(\mathbf{r},t) = X_0(\mathbf{r},t) + \delta X(\mathbf{r},t)$$

Introduction

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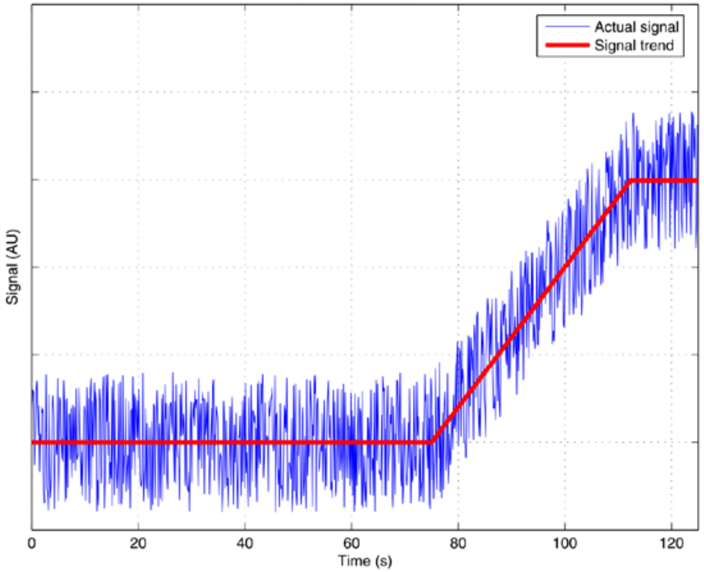
$X(\mathbf{r},t)$



Conceptual illustration of the possible time-dependence of a measured signal from a dynamical system

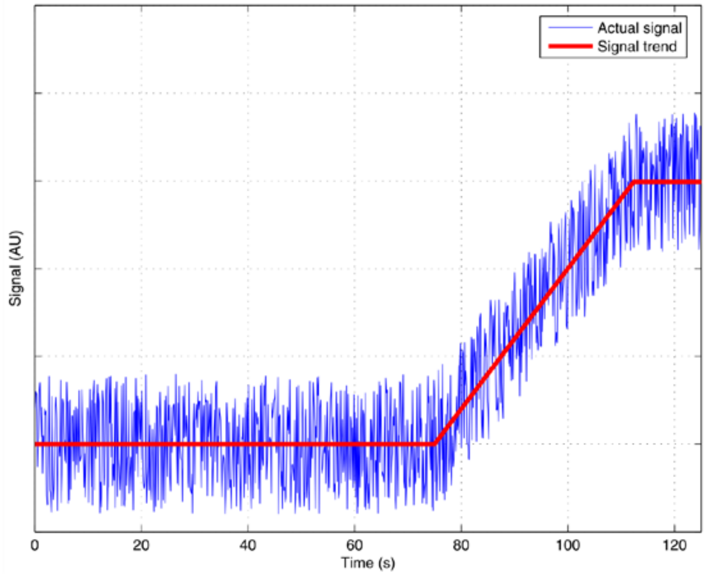
$$X(\mathbf{r},t) = X_0(\mathbf{r},t) + \delta X(\mathbf{r},t)$$

actual signal

Introduction

- Fluctuations always existing in dynamical systems even at steady state-conditions:

$X(\mathbf{r},t)$



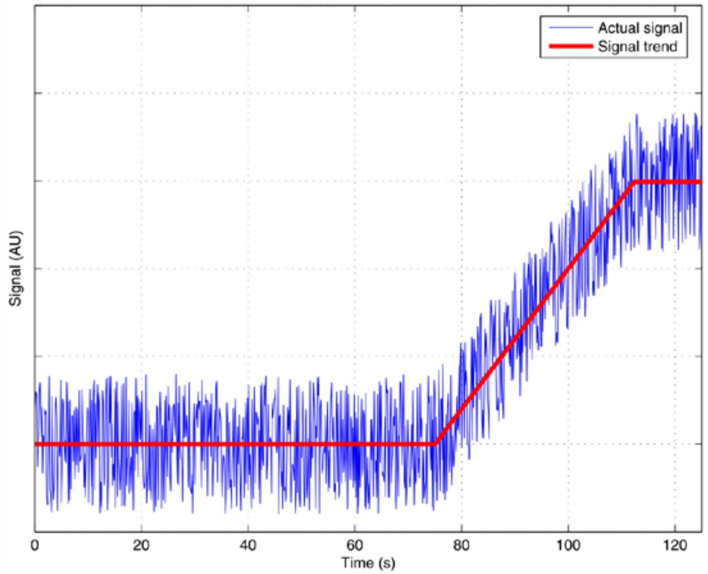
Conceptual illustration of the possible time-dependence of a measured signal from a dynamical system

$$X(\mathbf{r},t) = X_0(\mathbf{r},t) + \delta X(\mathbf{r},t)$$

signal trend or mean

Introduction

- Fluctuations always existing in dynamical systems even at steady state-conditions:



$X(\mathbf{r},t)$

Conceptual illustration of the possible time-dependence of a measured signal from a dynamical system

$$X(\mathbf{r},t) = X_0(\mathbf{r},t) + \delta X(\mathbf{r},t)$$

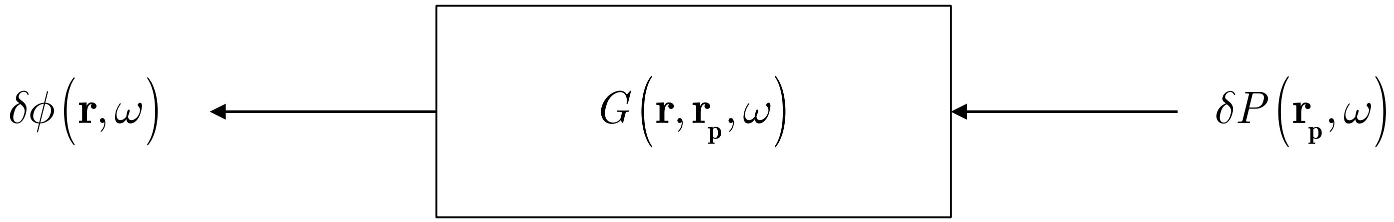
fluctuations
or “noise”

➢ Fluctuations carrying some valuable information about the system dynamics

Introduction

- Fluctuations could be used for “diagnostics”, i.e.:
  - Early detection of anomalies
  - Estimation of dynamical system characteristics

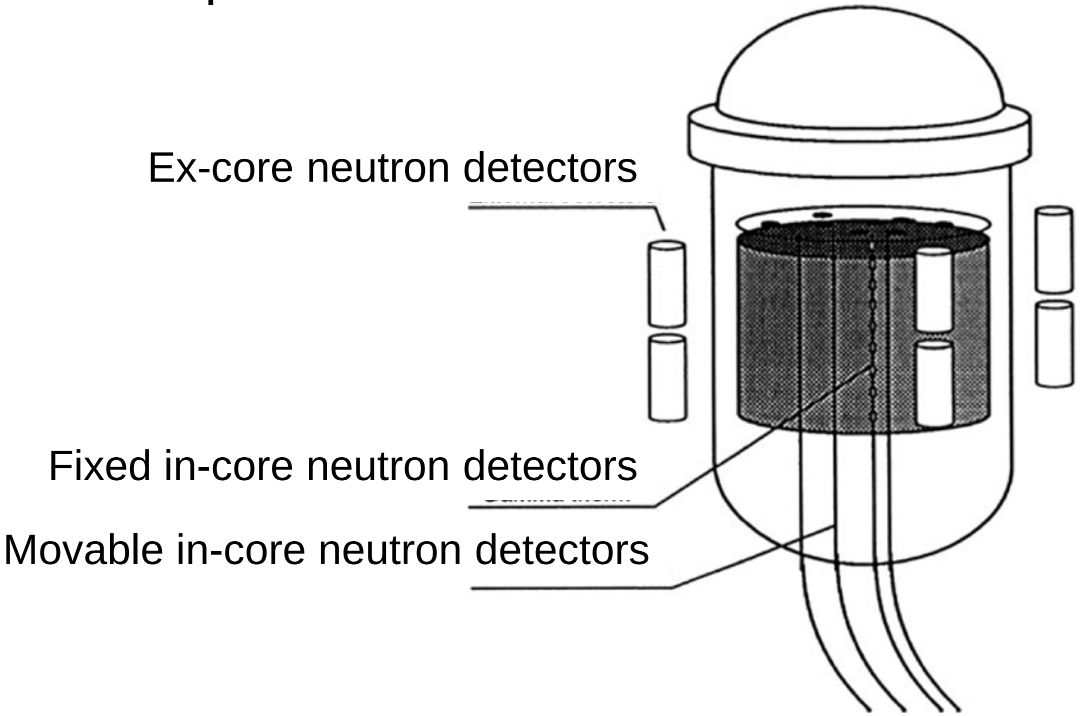
… even if the system is operating at steady-state conditions



System transfer function

Noise diagnostics in nuclear reactors

- Neutron detectors present both in-core and ex-core:



- ➢ Advantage: “sense” perturbations even far away from the perturbations
- ➢ Disadvantage: western-type reactors do not always contain many in-core neutron detectors

Noise diagnostics in nuclear reactors

- Neutron noise diagnostics requires establishing relationships between neutron detectors and possible perturbations

- Could be done using the neutron transport equation (Boltzmann equation)

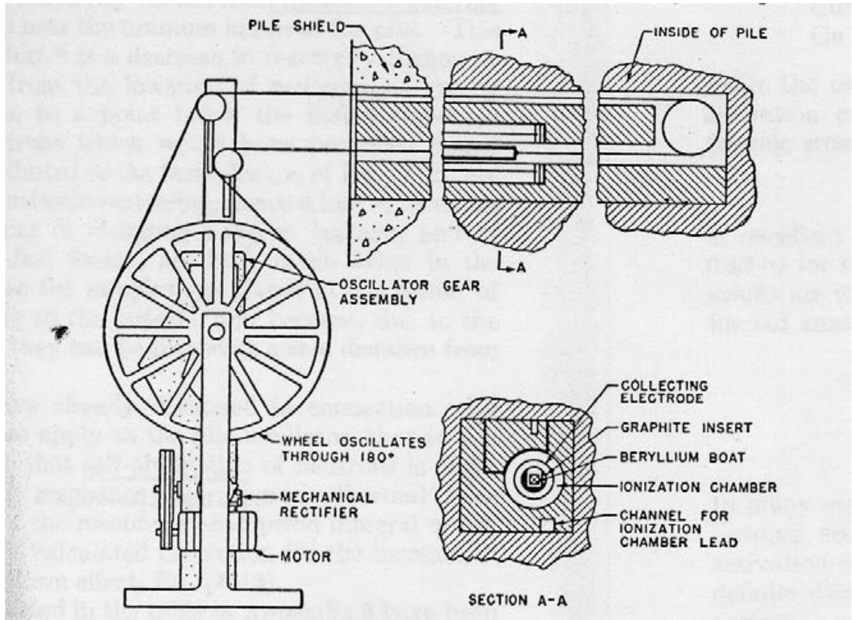
- Simpler formalisms usually used for modelling nuclear reactor cores, such as the multi-group diffusion approximation

Noise diagnostics in nuclear reactors

- Procedure to solve the system of equations for noise applications:
  - Splitting between mean values and fluctuations
  - Linear theory because of the smallness of the fluctuations
  - Assuming stationarity, use of frequency-domain

Early development in noise analysis

- Oscillator experiments in the Clinton Pile at ORNL, USA



➢ Response in neutron flux corresponding to a local (but stationary) excitation of the system deviating from point-kinetics: local component of the neutron noise (1949)

Early development in noise analysis

- Detection of excessive vibrations of control rods in the Oak Ridge Research Reactor and the High Flux Isotope Reactor (1971)
- ➢ Noise analysis was born

- First applications in commercial reactors:
  - Core-barrel vibrations at the Palisades plant, USA (1975)
  - Estimation of in-core coolant velocity in German BWRs (1979)

- Many other practical applications of noise analysis, generally aimed at detecting and localizing anomalies

Modelling of the induced neutron noise

- Induced neutron noise depending on:
  - Reactor transfer function
  - Noise source

➢ Importance of the noise source representation for diagnostic purposes

“Absorber of variable strength” type of noise source

- “Absorber of variable strength” = localized perturbation of which its amplitude varies in time at a fixed position

- Induced neutron noise given by the following balance equation (2-group diffusion theory):

$$\left\{\nabla\cdot\left[\mathbf{D}\left(\mathbf{r}\right)\nabla\right]+\Sigma_{dyn}\left(\mathbf{r},\omega\right)\right\}\times\begin{bmatrix}\delta\phi_1\left(\mathbf{r},\omega\right)\\ \delta\phi_2\left(\mathbf{r},\omega\right)\end{bmatrix}$$

$$=\Phi_r\left(\mathbf{r}\right)\delta\Sigma_r\left(\mathbf{r},\omega\right)+\Phi_a\left(\mathbf{r}\right)\begin{bmatrix}\delta\Sigma_{a,1}\left(\mathbf{r},\omega\right)\\ \delta\Sigma_{a,2}\left(\mathbf{r},\omega\right)\end{bmatrix}+\Phi_f\left(\mathbf{r},\omega\right)\begin{bmatrix}\delta\upsilon\Sigma_{f,1}\left(\mathbf{r},\omega\right)\\ \delta\upsilon\Sigma_{f,2}\left(\mathbf{r},\omega\right)\end{bmatrix}$$

"Absorber of variable strength" type of noise source

- In case of a point-like source:

$$\left[\nabla_{\mathbf{r}} \cdot \left[\mathbf{D}\left(\mathbf{r}\right)\nabla_{\mathbf{r}}\right] + \Sigma_{dyn}\left(\mathbf{r},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{r}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{r}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right) \end{bmatrix}_{g=2}$$

➢ Green's function

“Absorber of variable strength” type of noise source

- General solution to the original problem can be given by convolution integrals

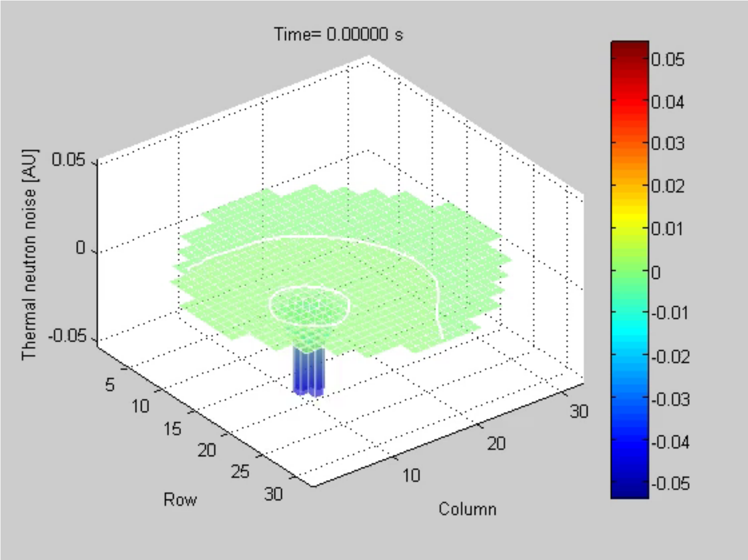
$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

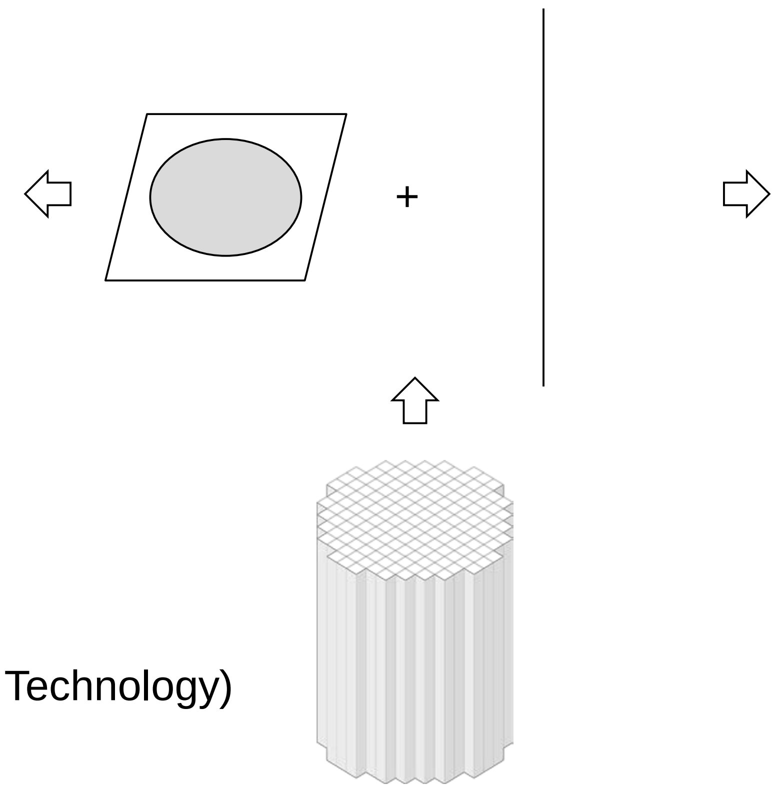
with

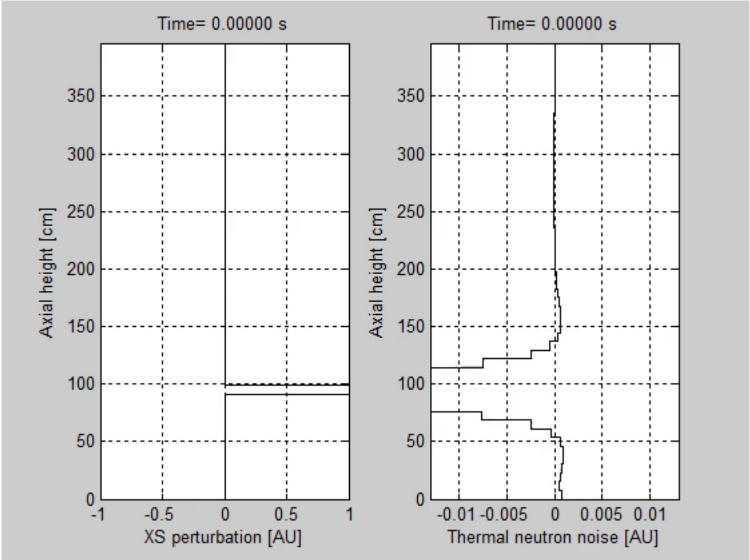
$$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \phi_r(\mathbf{r}') \delta\Sigma_r(\mathbf{r}',\omega) + \phi_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \phi_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\upsilon\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\upsilon\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

“Absorber of variable strength” type of noise source

- Example of a localized “absorber of variable strength” @ 1kHz







“Vibrating absorber” type of noise source

- Lateral movement of the absorber represented as (weak absorber):

$$\delta\Sigma_{a,2}\left(\mathbf{r},t\right)=\gamma\theta\left(z-z_0\right)\left[\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}-\boldsymbol{\varepsilon}\left(t\right)\right)-\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}\right)\right]$$

- A first-order Taylor expansion of the noise source would give for the induced neutron noise (in the frequency-domain):

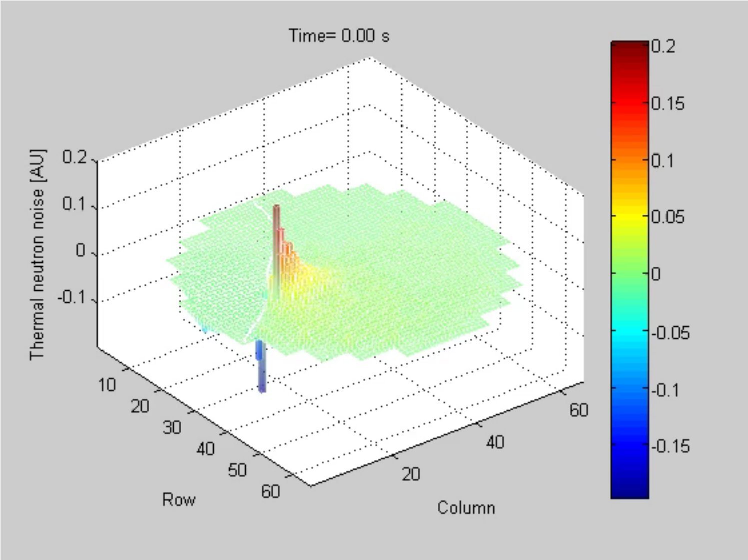
$$\delta\phi_g\left(\mathbf{r},\omega\right)=-\gamma\boldsymbol{\varepsilon}\left(\omega\right)\cdot\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right)$$

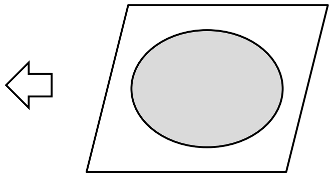
with

$$\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right)=\nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}\left(\mathbf{r},\mathbf{r}_{p,xy},\omega\right)$$

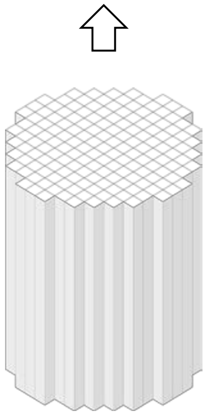
“Vibrating absorber” type of noise source

- Example of a vibrating control rod @ 0.2 Hz





(2-D calculations)



Axially-travelling perturbations

- Noise source represented in the time-domain as:

$$\delta\Sigma_{rem}\left(\mathbf{r},t\right) \equiv \delta\Sigma_{rem}\left(x,y,z,t\right) = \begin{cases} 0,\ \text{if}\ \left(x,y\right) \neq \left(x_0,y_0\right) \\ 0,\ \text{if}\ \left(x,y\right) = \left(x_0,y_0\right)\ \text{and}\ z < z_0 \\ \delta\Sigma_{rem}\left(x_0,y_0,z_0,t-\dfrac{z-z_0}{v}\right),\ \text{if}\ \left(x,y\right) = \left(x_0,y_0\right)\ \text{and}\ z \geq z_0 \end{cases}$$

Axially-travelling perturbations

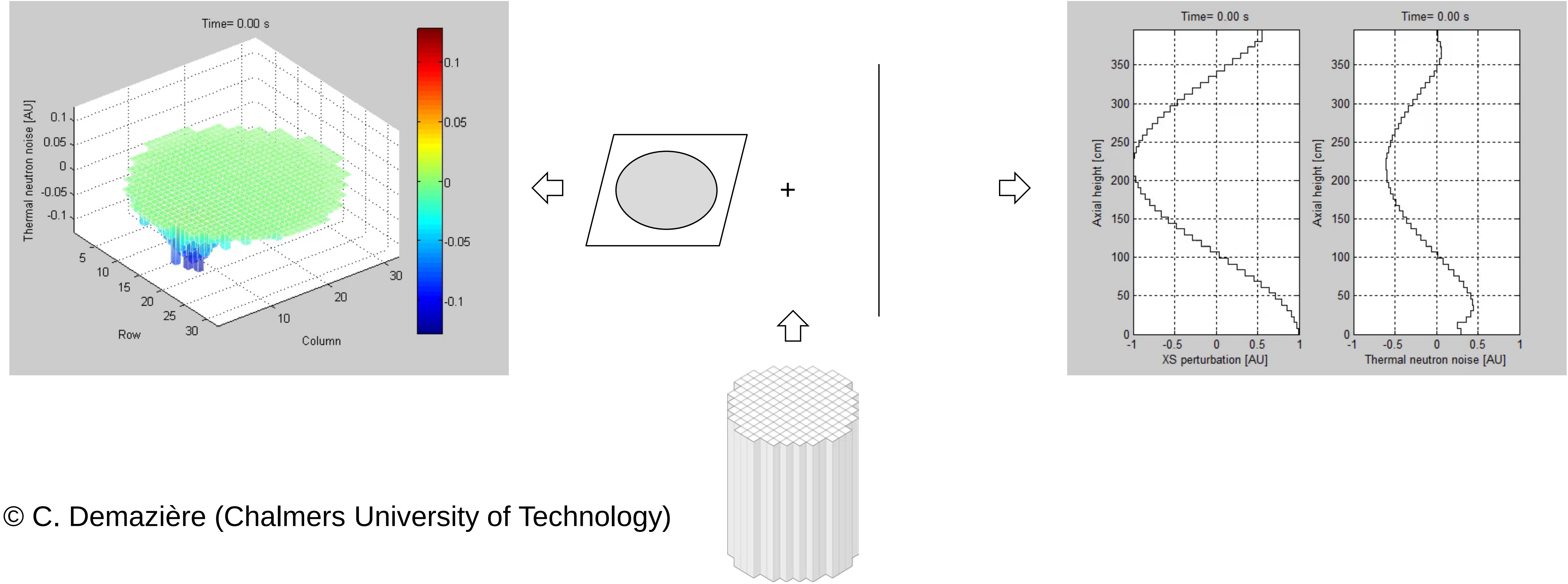
- Noise source represented in the frequency-domain as:

$$\delta\Sigma_{rem}\left(\mathbf{r},\omega\right) \equiv \delta\Sigma_{rem}\left(x,y,z,\omega\right)$$

$$= \begin{cases} 0,\ \text{if } \left(x,y\right) \neq \left(x_0,y_0\right) \\ 0,\ \text{if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z < z_0 \\ \delta\Sigma_{rem}\left(x_0,y_0,z_0,\omega\right)\exp\left[-\dfrac{i\omega\left(z-z_0\right)}{v}\right],\ \text{if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z \geq z_0 \end{cases}$$

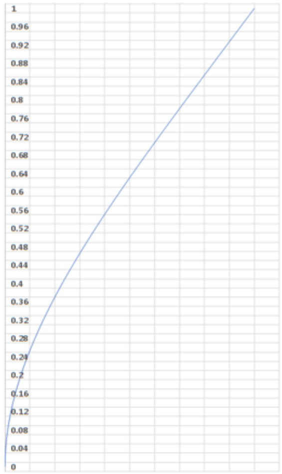
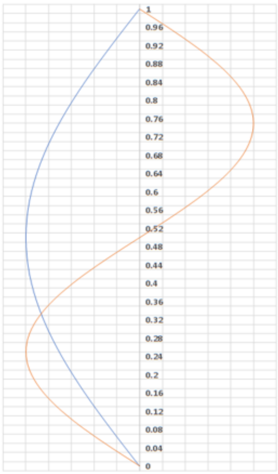
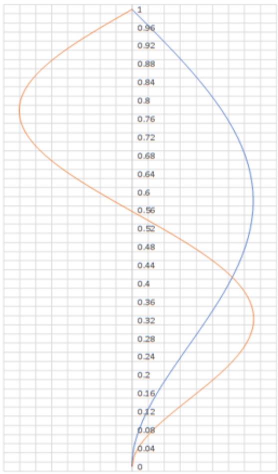
Axially-travelling perturbations

- Example of a travelling perturbation @ 1Hz



Fuel assembly vibrations

- Different possible axial vibration modes for fuel assemblies:

	Cantilevered beam	Simply supported on both sides	Cantilevered beam and simply supported
Axial shape of the displacement $d(z,t)$ in arbitrary units as a function of the relative core elevation z		 first mode in blue, second mode in orange	 first mode in blue, second mode in orange
Oscillation frequency	Ca. 0.6 – 1.2 Hz	Ca. 0.8 – 4 Hz for the first mode Ca. 4 – 10 Hz for the second mode	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode

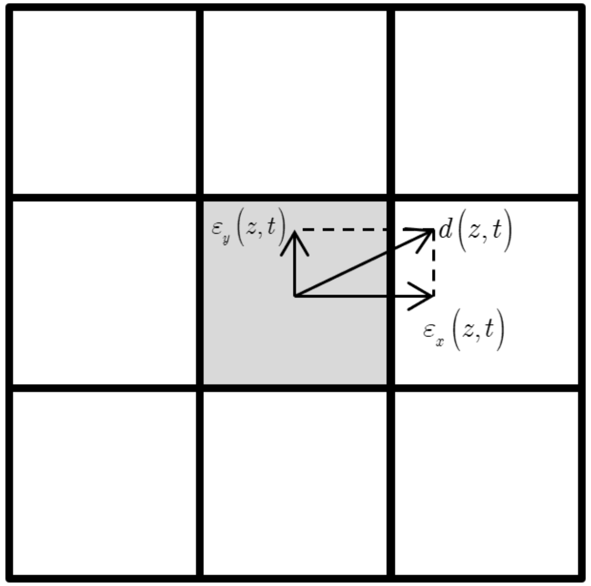
Fuel assembly vibrations

- Fuel assembly vibrations described at the pin level:
  - Can be modelled as “vibrating absorbers”
  - Can be modelled as “absorbers of variable strength” !

- Fuel assembly vibrations at the nodal level can only be modelled as “absorber of variable strength” !

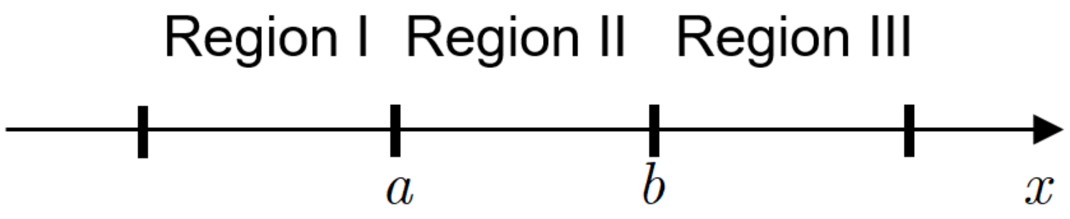
Fuel assembly vibrations

- Lateral vibrations represented as:



Fuel assembly vibrations

- In e.g. the x-direction, one has:



with static cross-section between Regions II and III given as:

$$\Sigma_{\alpha,g}^{x}\left(x\right)=\left[1-\Theta\left(x-b\right)\right]\Sigma_{\alpha,g,II}+\Theta\left(x-b\right)\Sigma_{\alpha,g,III}$$

Fuel assembly vibrations

- For a time-dependent boundary:

$$b\left(z,t\right) = b_0 + \varepsilon_x\left(z,t\right)$$

one obtains after a first-order Taylor expansion in the time-domain:

$$\Sigma_{\alpha,g}^{x}\left(x,z,t\right) = \left[1 - \Theta\left(x - b_0\right)\right]\Sigma_{\alpha,g,II} + \Theta\left(x - b_0\right)\Sigma_{\alpha,g,III} + \varepsilon_x\left(z,t\right)\delta\left(x - b_0\right)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

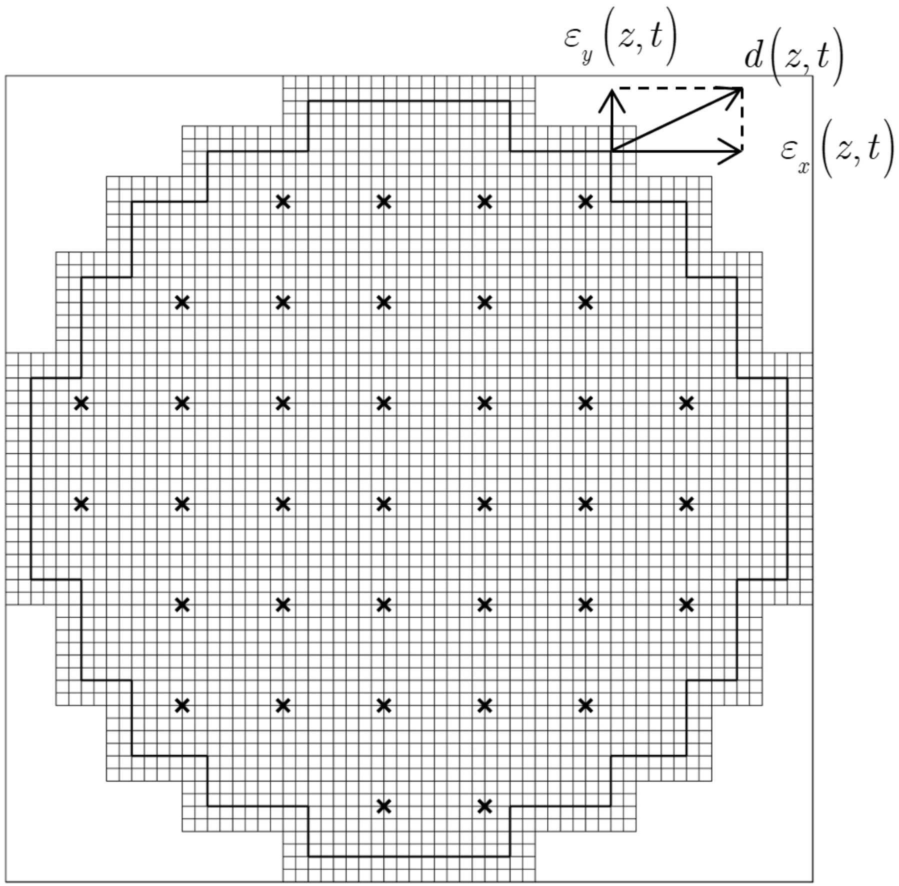
- Noise source in the frequency-domain:

$$\delta\Sigma_{\alpha,g}^{x}\left(x,z,\omega\right) = \varepsilon_x\left(z,\omega\right)\delta\left(x - b_0\right)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

- Point-like source!

Pendular core barrel vibrations

- Core barrel vibrations can be seen as a relative displacement of the active core with respect to the reflector:



Pendular core barrel vibrations

- Same technique as for fuel assembly vibrations can be used:

$$\delta\Sigma^{x}_{\alpha,g}\left(x,z\right)=h\left(z\right)\sum_{n}\delta\left(x-x_{n}\right)\left[\Sigma_{\alpha,g,x_n^-}-\Sigma_{\alpha,g,x_n^+}\right]$$

$$\delta\Sigma^{y}_{\alpha,g}\left(y,z\right)=h\left(z\right)\sum_{m}\delta\left(y-y_{m}\right)\left[\Sigma_{\alpha,g,y_m^-}-\Sigma_{\alpha,g,y_m^+}\right]$$

- Point-like source!

Estimation of the induced neutron noise

- Generically, the induced neutron noise is given as (e.g. in 2-group theory):

$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

with

$$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \phi_r(\mathbf{r}') \delta\Sigma_r(\mathbf{r}',\omega) + \phi_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \phi_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\upsilon\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\upsilon\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

Estimation of the induced neutron noise

… or given as:

$$\delta\phi_g(\mathbf{r},\omega) = -\gamma\boldsymbol{\varepsilon}(\omega)\cdot\boldsymbol{\delta\varphi}_g(\mathbf{r},\omega)$$

with

$$\boldsymbol{\delta\varphi}_g(\mathbf{r},\omega) = \nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}(\mathbf{r},\mathbf{r}_{p,xy},\omega)$$

- In essence, only the Green's function is needed

Estimation of the induced neutron noise

- The Green's function can be estimated:
  - Either deterministically
    - Using diffusion theory

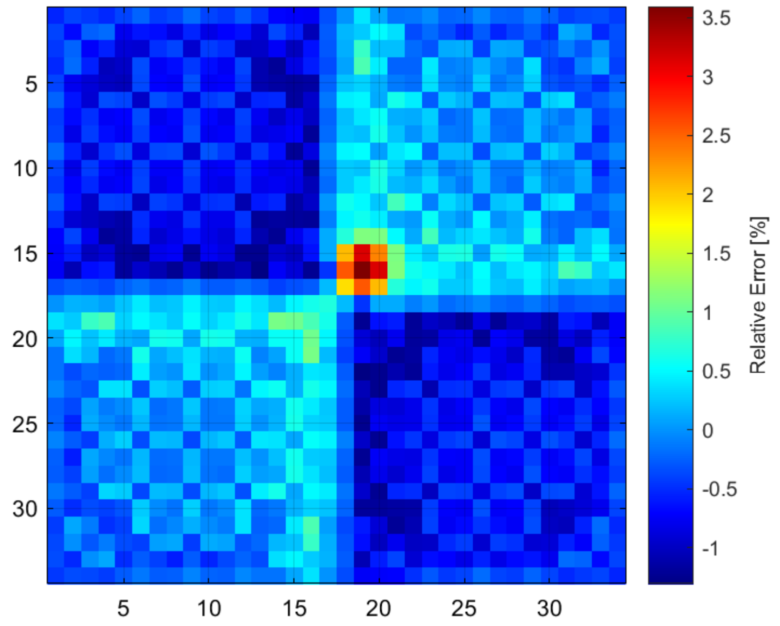
$$\left[\nabla_{\mathbf{r}} \cdot \left[\mathbf{D}\left(\mathbf{r}\right)\nabla_{\mathbf{r}}\right] + \Sigma_{dyn}\left(\mathbf{r},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{r}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{r}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right) \end{bmatrix}_{g=2}$$

    - Using transport theory

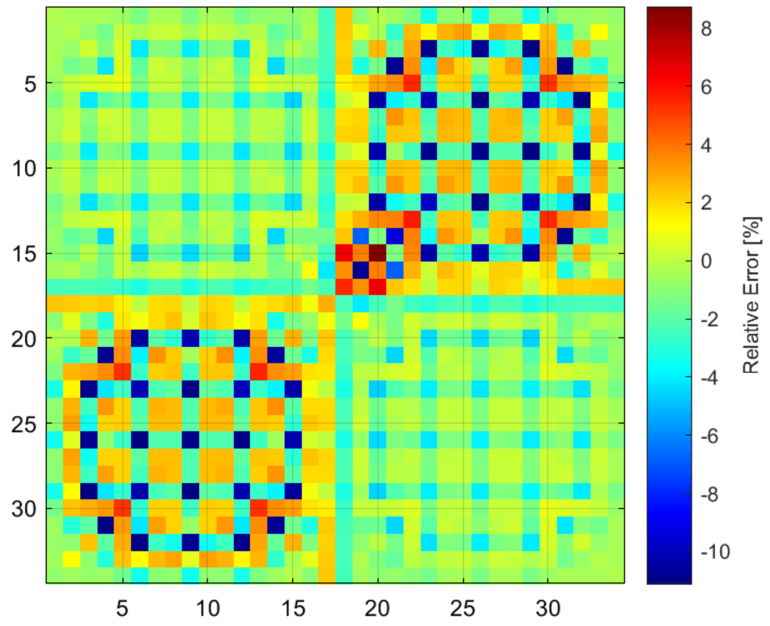
$$\left[\mathbf{\Omega} \cdot \nabla_{\mathbf{r}} + \Sigma_{dyn}\left(\mathbf{r},\mathbf{\Omega},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right)\delta\left(\mathbf{\Omega}-\mathbf{\Omega}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right)\delta\left(\mathbf{\Omega}-\mathbf{\Omega}'\right) \end{bmatrix}_{g=2}$$

Estimation of the induced neutron noise

- Comparisons diffusion/transport (discrete ordinates) for perturbations in capture cross-sections in both energy groups at 1 Hz (OECD/NEA C3G2 benchmark configuration)



Fast group – relative difference in amplitude



Thermal group – relative difference in amplitude

Estimation of the induced neutron noise

- The Green's function can also be estimated:
  - probabilistically using an equivalence to subcritical problems (demonstrated hereafter on diffusion theory in 2 energy groups):

$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \delta\phi_1^{real}(\mathbf{r},\omega) \\ \delta\phi_2^{real}(\mathbf{r},\omega) \end{bmatrix} + i \begin{bmatrix} \delta\phi_1^{im}(\mathbf{r},\omega) \\ \delta\phi_2^{im}(\mathbf{r},\omega) \end{bmatrix}$$

Estimation of the induced neutron noise

- Although the whole problem is solution of:

$$\left\{\nabla\cdot\left[\mathbf{D}(\mathbf{r})\nabla\right]+\Sigma_{dyn}(\mathbf{r},\omega)\right\}\times\begin{bmatrix}\delta\phi_1(\mathbf{r},\omega)\\ \delta\phi_2(\mathbf{r},\omega)\end{bmatrix}=\begin{bmatrix}S_1(\mathbf{r},\omega)\\ S_2(\mathbf{r},\omega)\end{bmatrix}$$

The coupling to the imaginary (real, respectively) part of the neutron noise when solving for the real (imaginary, respectively) balance equations is treated as additional noise sources

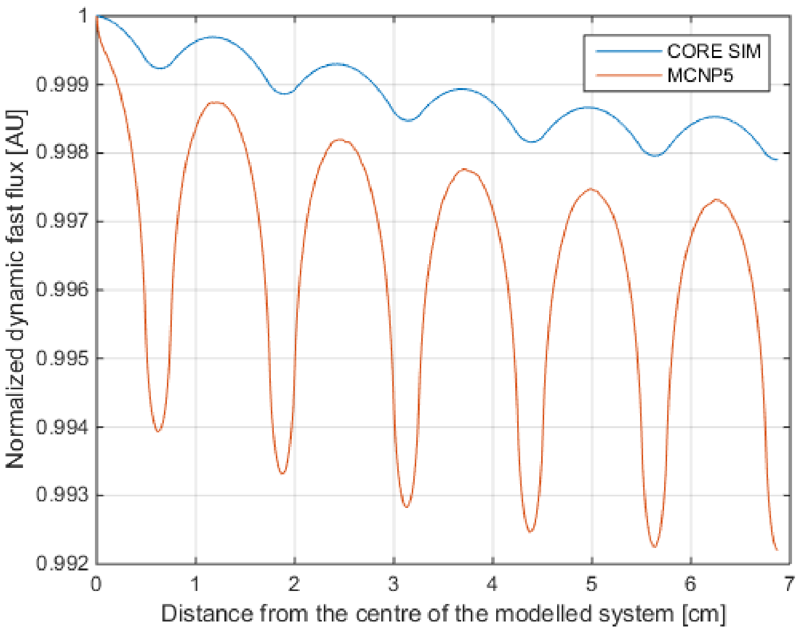
$$\begin{bmatrix}S_1^{real\text{ or }im}(\mathbf{r},\omega)\\ S_2^{real\text{ or }im}(\mathbf{r},\omega)\end{bmatrix}=\begin{bmatrix}\mathrm{Re}\ \text{ or }\ \mathrm{Im}\left\{S_1(\mathbf{r},\omega)\right\}\\ \mathrm{Re}\ \text{ or }\ \mathrm{Im}\left\{S_1(\mathbf{r},\omega)\right\}\end{bmatrix}+\mathbf{M}\times\begin{bmatrix}\mathrm{Im}\ \text{ or }\ \mathrm{Re}\left\{\delta\phi_1(\mathbf{r},\omega)\right\}\\ \mathrm{Im}\ \text{ or }\ \mathrm{Re}\left\{\delta\phi_2(\mathbf{r},\omega)\right\}\end{bmatrix}$$

- Induced neutron noise:

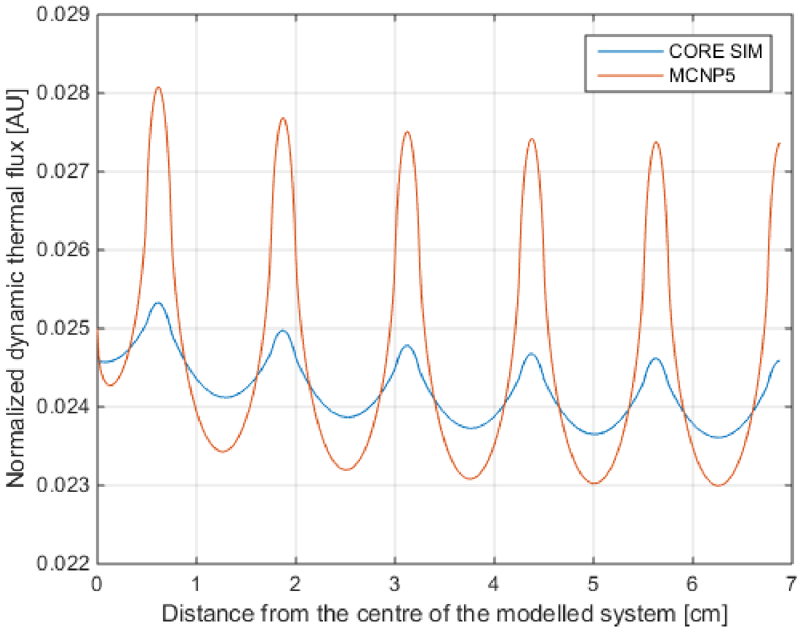
$$\begin{bmatrix}\delta\phi_1^{real\text{ or }im}(\mathbf{r},\omega)\\ \delta\phi_2^{real\text{ or }im}(\mathbf{r},\omega)\end{bmatrix}=\begin{bmatrix}\int\left[\tilde{G}_{1\to1}(\mathbf{r},\mathbf{r}',\omega)S_1^{real\text{ or }im}(\mathbf{r}',\omega)+\tilde{G}_{2\to1}(\mathbf{r},\mathbf{r}',\omega)S_2^{real\text{ or }im}(\mathbf{r}',\omega)\right]d^3\mathbf{r}'\\ \int\left[\tilde{G}_{1\to2}(\mathbf{r},\mathbf{r}',\omega)S_1^{real\text{ or }im}(\mathbf{r}',\omega)+\tilde{G}_{2\to2}(\mathbf{r},\mathbf{r}',\omega)S_2^{real\text{ or }im}(\mathbf{r}',\omega)\right]d^3\mathbf{r}'\end{bmatrix}$$

Estimation of the induced neutron noise

- Comparisons diffusion/transport (Monte Carlo) for perturbations in all cross-sections in both energy groups at 1 Hz (infinite system of 11 pins with central perturbation)



Fast group – amplitude



Thermal group – amplitude

Conclusions and outlook

- Many successful past applications of noise analysis for core diagnostics
- Most of the past applications use simple models of the reactor transfer function or no model at all
- Taking full advantage of noise analysis requires:
  - A correct modelling of the noise source
  - The estimation of the reactor transfer function
  - Its inversion

Conclusions and outlook

- **CORTEX**: CORe monitoring Techniques and Experimental validation and demonstration – EU funding.

CORTEX
Core monitoring techniques and experimental validation and demonstration

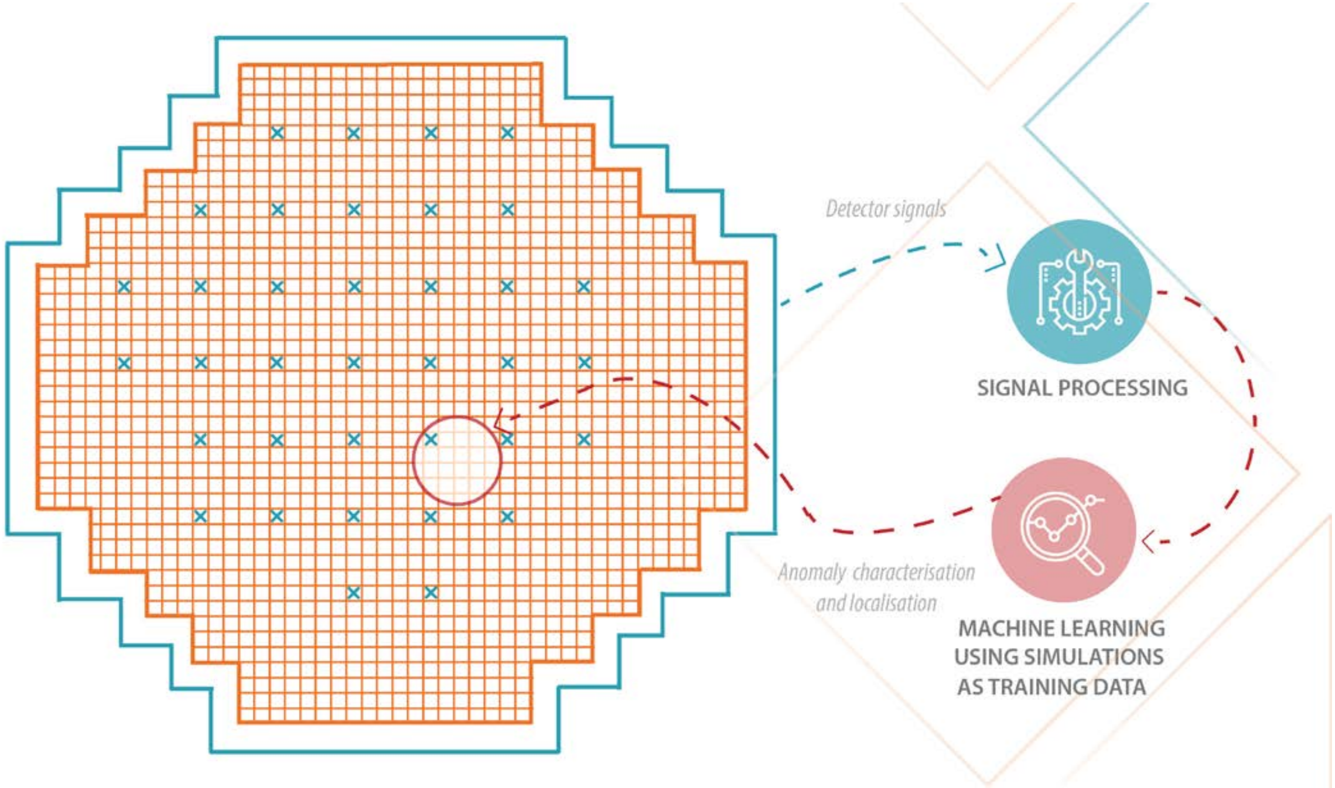
  - Chalmers coordinating the project
  - 20 partners (18 from EU + 1 from Japan + 1 from USA)

http://cortex-h2020.eu

This project has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 754316.

Conclusions and outlook

- Method to be developed in CORTEX:



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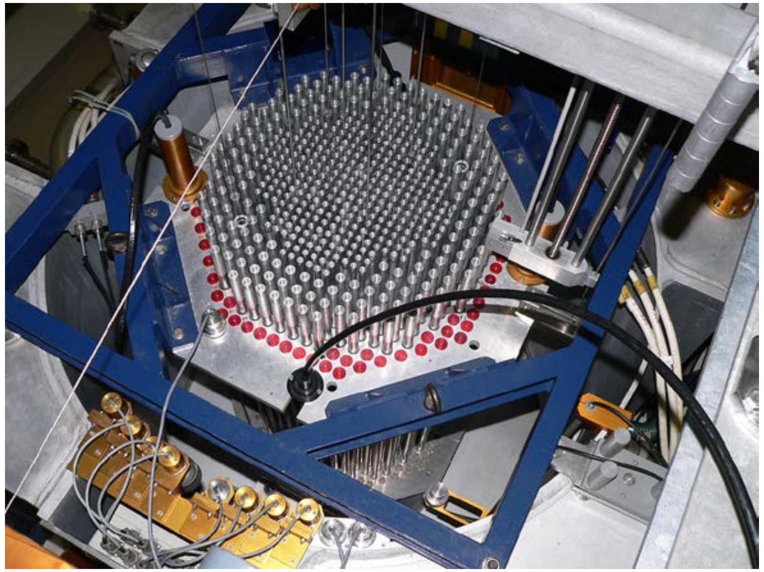
CHALMERS DREAM

Conclusions and outlook

- CORTEX aims:
  - Developing high fidelity tools for simulating stationary fluctuations
  - Validating those tools against experiments to be performed at research reactors



CROCUS reactor @EPFL, Switzerland

AKR-2 reactor @TU Dresden, Germany

This project has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 754316.

Conclusions and outlook

- CORTEX aims:
  - Developing high fidelity tools for simulating stationary fluctuations
  - Validating those tools against experiments to be performed at research reactors
  - Developing advanced signal processing and machine learning techniques (to be combined with the simulation tools)
  - Demonstrating the proposed methods for both on-line and off-line core diagnostics and monitoring

This project has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 754316.

Conclusions and outlook

- Core diagnostics leading to improved reactor safety and becoming increasingly important

- CORTEX project potentially having a large impact if successful

POWER REACTOR NOISE: from the modelling of noise sources to their effect onto the neutron flux

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DREAM

Task Force on Deterministic REActor Modelling
at Chalmers University of Technology

Additional slides

"Vibrating absorber" type of noise source

- Lateral movement of the absorber represented as (weak absorber):

$$\delta\Sigma_{a,2}\left(\mathbf{r},t\right)=\gamma\theta\left(z-z_0\right)\left[\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}-\boldsymbol{\varepsilon}\left(t\right)\right)-\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}\right)\right]$$

- A first-order Taylor expansion in the time-domain gives:

$$\delta\Sigma_{a,2}\left(\mathbf{r},t\right)=-\gamma\theta\left(z-z_0\right)\boldsymbol{\varepsilon}\left(t\right)\cdot\delta'\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}\right)$$

and in the frequency-domain:

$$\delta\Sigma_{a,2}\left(\mathbf{r},\omega\right)=-\gamma\theta\left(z-z_0\right)\boldsymbol{\varepsilon}\left(\omega\right)\cdot\delta'\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}\right)$$

"Vibrating absorber" type of noise source

- Induced neutron noise:

$$\delta\phi_g(\mathbf{r},\omega) = \int G_{2\to g}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) d^3\mathbf{r}' \equiv \int\int G_{2\to g}(\mathbf{r},\mathbf{r}_{xy}',z',\omega) S_2(\mathbf{r}_{xy}',z',\omega) d^2\mathbf{r}_{xy}'dz'$$
$$= -\gamma\boldsymbol{\varepsilon}(\omega)\cdot\int \hat{G}_{2\to g}(\mathbf{r},\mathbf{r}_{xy}',\omega)\delta'(\mathbf{r}_{xy}' - \mathbf{r}_{p,xy}) d^2\mathbf{r}_{xy}'$$

with

$$\hat{G}_{2\to g}(\mathbf{r},\mathbf{r}_{xy}',\omega) = \int G_{2\to g}(\mathbf{r},\mathbf{r}_{xy}',z,\omega)\theta(z'-z_0)\phi_{2,0}(\mathbf{r}_{xy}',z')dz'$$

- Integrating by parts gives:

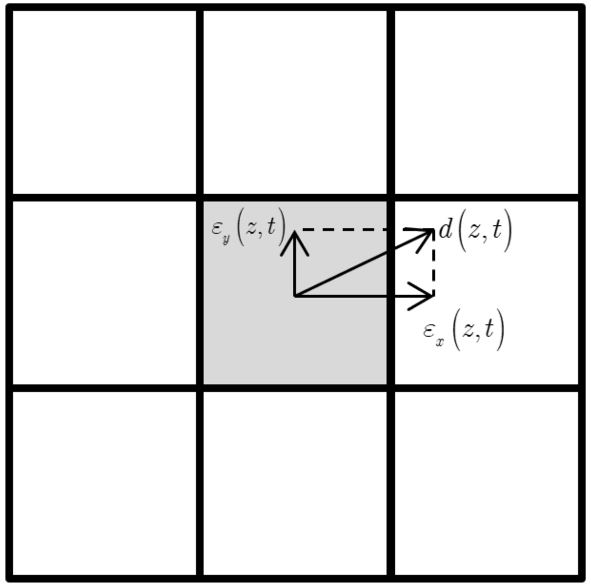
$$\delta\phi_g(\mathbf{r},\omega) = -\gamma\boldsymbol{\varepsilon}(\omega)\cdot\boldsymbol{\delta\varphi}_g(\mathbf{r},\omega)$$

with

$$\boldsymbol{\delta\varphi}_g(\mathbf{r},\omega) = \nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}(\mathbf{r},\mathbf{r}_{p,xy},\omega)$$

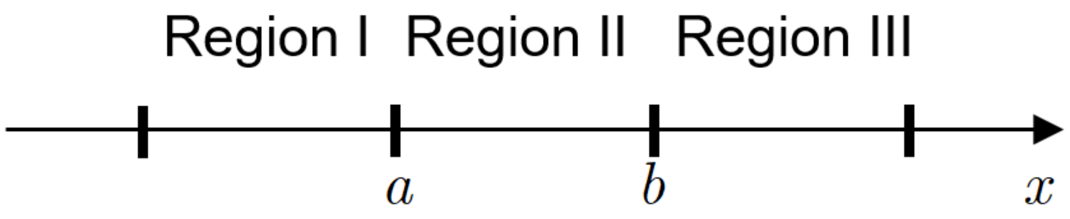
Fuel assembly vibrations

- Lateral vibrations represented as:



Fuel assembly vibrations

- In e.g. the x-direction, one has:



with static cross-section given as:

$$\Sigma_{\alpha,g}^{x}\left(x\right)=\left[1-\Theta\left(x-b\right)\right]\Sigma_{\alpha,g,II}+\Theta\left(x-b\right)\Sigma_{\alpha,g,III}$$

Fuel assembly vibrations

- For a time-dependent boundary:

$$b(z,t) = b_0 + \varepsilon_x(z,t)$$

one obtains after a first-order Taylor expansion in the time-domain:

$$\Sigma_{\alpha,g}^{x}(x,z,t) = \left[1 - \Theta(x - b_0)\right]\Sigma_{\alpha,g,II} + \Theta(x - b_0)\Sigma_{\alpha,g,III} + \varepsilon_x(z,t)\delta(x - b_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

- Noise source in the frequency-domain:

$$\delta\Sigma_{\alpha,g}^{x}(x,z,\omega) = \varepsilon_x(z,\omega)\delta(x - b_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

- Point-like source!

Fuel assembly vibrations

- Factorizing the noise source as:

$$\varepsilon_x(z,\omega) \equiv \varepsilon_x(\omega) h(z) \quad \text{and} \quad \varepsilon_y(z,\omega) \equiv \varepsilon_y(\omega) h(z)$$

leads to

$$\delta\Sigma^x_{\alpha,g}(x,z,\omega) \equiv \varepsilon_x(\omega)\delta\Sigma^x_{\alpha,g}(x,z) \quad \text{and} \quad \delta\Sigma^y_{\alpha,g}(y,z,\omega) \equiv \varepsilon_y(\omega)\delta\Sigma^y_{\alpha,g}(y,z)$$

with

$$\delta\Sigma^x_{\alpha,g}(x,z) = h(z)\delta(x-a_0)\left[\Sigma_{\alpha,g,I} - \Sigma_{\alpha,g,II}\right] + h(z)\delta(x-b_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

$$\delta\Sigma^y_{\alpha,g}(y,z) = h(z)\delta(y-c_0)\left[\Sigma_{\alpha,g,IV} - \Sigma_{\alpha,g,II}\right] + h(z)\delta(y-d_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,V}\right]$$

Estimation of the induced neutron noise

– Although the whole problem is solution of:

$$\left\{\nabla\cdot\left[\mathbf{D}\left(\mathbf{r}\right)\nabla\right]+\Sigma_{dyn}\left(\mathbf{r},\omega\right)\right\}\times\begin{bmatrix}\delta\phi_1\left(\mathbf{r},\omega\right)\\\delta\phi_2\left(\mathbf{r},\omega\right)\end{bmatrix}=\begin{bmatrix}S_1\left(\mathbf{r},\omega\right)\\S_2\left(\mathbf{r},\omega\right)\end{bmatrix}$$

The coupling to the imaginary (real, respectively) part of the neutron noise when solving for the real (imaginary, respectively) balance equations is treated as additional noise sources

$$\begin{bmatrix}S_1^{real\text{ or }im}\left(\mathbf{r},\omega\right)\\S_2^{real\text{ or }im}\left(\mathbf{r},\omega\right)\end{bmatrix}=\begin{bmatrix}\mathrm{Re}\text{ or }\mathrm{Im}\left\{S_1\left(\mathbf{r},\omega\right)\right\}\\\mathrm{Re}\text{ or }\mathrm{Im}\left\{S_1\left(\mathbf{r},\omega\right)\right\}\end{bmatrix}+\mathbf{M}\times\begin{bmatrix}\mathrm{Im}\text{ or }\mathrm{Re}\left\{\delta\phi_1\left(\mathbf{r},\omega\right)\right\}\\\mathrm{Im}\text{ or }\mathrm{Re}\left\{\delta\phi_2\left(\mathbf{r},\omega\right)\right\}\end{bmatrix}$$

➢ Induced neutron noise:

$$\begin{bmatrix}\delta\phi_1^{real\text{ or }im}\left(\mathbf{r},\omega\right)\\\delta\phi_2^{real\text{ or }im}\left(\mathbf{r},\omega\right)\end{bmatrix}=\begin{bmatrix}\int\left[\tilde{G}_{1\to1}\left(\mathbf{r},\mathbf{r}',\omega\right)S_1^{real\text{ or }im}\left(\mathbf{r}',\omega\right)+\tilde{G}_{2\to1}\left(\mathbf{r},\mathbf{r}',\omega\right)S_2^{real\text{ or }im}\left(\mathbf{r}',\omega\right)\right]d^3\mathbf{r}'\\\int\left[\tilde{G}_{1\to2}\left(\mathbf{r},\mathbf{r}',\omega\right)S_1^{real\text{ or }im}\left(\mathbf{r}',\omega\right)+\tilde{G}_{2\to2}\left(\mathbf{r},\mathbf{r}',\omega\right)S_2^{real\text{ or }im}\left(\mathbf{r}',\omega\right)\right]d^3\mathbf{r}'\end{bmatrix}$$

Estimation of the induced neutron noise

with the modified Green's function solution of:

$$\left\{\nabla_{\mathbf{r}}\cdot\begin{bmatrix} D_{1,0}(\mathbf{r}) & 0 \\ 0 & D_{2,0}(\mathbf{r})\end{bmatrix}\nabla_{\mathbf{r}} + \begin{bmatrix} -\Sigma_{a,1,0}(\mathbf{r}) - \Sigma_{r,0}(\mathbf{r}) + \dfrac{\nu\Sigma_{f,1,0}(\mathbf{r})}{k_{eff}}\dfrac{\lambda^2+\omega^2(1-\beta)}{\lambda^2+\omega^2} & \dfrac{\nu\Sigma_{f,2,0}(\mathbf{r})}{k_{eff}}\dfrac{\lambda^2+\omega^2(1-\beta)}{\lambda^2+\omega^2} \\ \Sigma_{r,0}(\mathbf{r}) & -\Sigma_{a,2,0}(\mathbf{r})\end{bmatrix}\right\}\times\begin{bmatrix}\tilde{G}_{g\to 1}(\mathbf{r},\mathbf{r}',\omega) \\ \tilde{G}_{g\to 2}(\mathbf{r},\mathbf{r}',\omega)\end{bmatrix}$$

$$= -\begin{bmatrix}\delta(\mathbf{r}-\mathbf{r}') \\ 0\end{bmatrix}_{g=1} \quad \text{or} \quad -\begin{bmatrix} 0 \\ \delta(\mathbf{r}-\mathbf{r}')\end{bmatrix}_{g=2}$$

Estimation of the induced neutron noise

and with the modified noise sources defined as:

$$\begin{bmatrix} S_1^{real}(\mathbf{r},\omega) \\ S_2^{real}(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\left\{S_1(\mathbf{r},\omega)\right\} \\ \mathrm{Re}\left\{S_1(\mathbf{r},\omega)\right\} \end{bmatrix} + \begin{bmatrix} -\dfrac{\omega}{v_1} - \dfrac{\nu\Sigma_{f,1,0}(\mathbf{r})}{k_{eff}} \dfrac{\omega\beta\lambda}{\omega^2+\lambda^2} & -\dfrac{\nu\Sigma_{f,2,0}(\mathbf{r})}{k_{eff}} \dfrac{\omega\beta\lambda}{\omega^2+\lambda^2} \\ 0 & -\dfrac{\omega}{v_2} \end{bmatrix} \times \begin{bmatrix} \mathrm{Im}\left\{\delta\phi_1(\mathbf{r},\omega)\right\} \\ \mathrm{Im}\left\{\delta\phi_2(\mathbf{r},\omega)\right\} \end{bmatrix}$$

$$\begin{bmatrix} S_1^{im}(\mathbf{r},\omega) \\ S_2^{im}(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \mathrm{Im}\left\{S_1(\mathbf{r},\omega)\right\} \\ \mathrm{Im}\left\{S_2(\mathbf{r},\omega)\right\} \end{bmatrix} - \begin{bmatrix} -\dfrac{\omega}{v_1} - \dfrac{\nu\Sigma_{f,1,0}(\mathbf{r})}{k_{eff}} \dfrac{\omega\beta\lambda}{\omega^2+\lambda^2} & -\dfrac{\nu\Sigma_{f,2,0}(\mathbf{r})}{k_{eff}} \dfrac{\omega\beta\lambda}{\omega^2+\lambda^2} \\ 0 & -\dfrac{\omega}{v_2} \end{bmatrix} \times \begin{bmatrix} \mathrm{Re}\left\{\delta\phi_1(\mathbf{r},\omega)\right\} \\ \mathrm{Re}\left\{\delta\phi_2(\mathbf{r},\omega)\right\} \end{bmatrix}$$