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L. *On Bi- and Unilateral Galvanometer Deflection.* By G. CHRYSTAL, B.A., Fellow of Corpus Christi College, Cambridge*.

I WAS led to study the subject of this paper during a series of experiments undertaken for the purpose of directly testing Ohm's law.

The results arrived at are, I think, interesting, not only in connexion with galvanometry, but also in relation to the theory of induced magnetism. In the first instance I shall describe the phenomena as simply as possible from the first point of view, and then consider a little more closely some points which arise when the matter is looked at from the second point of view. This order is to a great extent that in which the facts came under my notice; and it has the additional advantage that it leads us incidentally to see that the phenomena in question really have their seat solely and entirely within the galvanometer, and have nothing to do with any phenomenon of the nature of unilateral conductivity or with any other exception to Ohm's law.

Dr. Schuster has described† an experiment in which a small current of constant direction is superposed on the alternating currents of a sine inductor, and the whole sent through a galvanometer. Such an experiment affords (under certain suppositions) a test of Ohm's law; for the average intensity of the current in the direction of the small constant current is greater than that in the opposite direction; hence, if the re-

* Communicated by the Author.

† Phil. Mag. [IV.] vol. xlviii. p. 340.

sistance of the circuit depends on the intensity of the current, one of the currents will prevail, and the presence of the alternating currents will affect the permanent deflection due to the small constant current, whereas, if Ohm's law were true, there would be no such effect. Without discussing Dr. Schuster's results here, we may remark that a similar test of Ohm's law could be obtained by merely passing through a galvanometer the currents from the secondary coil of an inductorium. It is well known that if the primary be made and broken periodically, there will be an alternating current in the secondary, which will have the same period. The whole quantity of electricity which passes during a complete oscillation is zero; but the maximum intensities of the positive and negative parts of the current are very different. The positive part, starting immediately after the break, has a considerable initial intensity, which is independent of the resistance of the secondary; the negative part, starting at the make, begins, on the contrary, with zero intensity, and never reaches so large a maximum as the positive*.

It follows that if the resistance of the circuit depends on the intensity of the current, then the two parts of the current will not experience equal resistances, and we shall get a galvanometer indication in the direction of that which has the advantage. Any such effect would be much increased by the introduction into the secondary circuit of a resistance composed of very fine wire. It is easy enough to calculate what this resistance should be, in order to produce the greatest effect on the galvanometer.

Such a resistance I used in the shape of a fine German-silver wire ($\cdot 002$ inch diameter) wound on a cylindrical piece of vulcanite about 9 inches long, the turns being insulated from each other by the thread of a screw of one hundred turns to the inch cut in the vulcanite. The whole is enclosed in a glass tube with brass caps and copper terminals. For this instrument I am indebted to Mr. Garnett, of St. John's, Demonstrator at the Cavendish Laboratory. The induction-coil used was of the ordinary lecture-room form by Apps; the primary was made and broken by electric tuning-forks of various pitch.

The results I obtained indicated an *apparent* departure from Ohm's law, sometimes in one direction, sometimes in the other. The presence or absence of the fine wire in the circuit did not seem to be an essential condition of the phenomenon.

* This supposes the period of alternation long compared with the time-constants of the coil; the same description applies, in a modified degree, to other cases.

I was therefore led to suspect that the cause lay in the galvanometer itself—a suspicion which became certainty when I found that reversing the galvanometer-connexions with the secondary, or reversing the primary, had no effect whatever on the character of the phenomena. It appeared that the effects observed could be analyzed as follows:—Suppose we are using a Thomson's galvanometer with mirror, lamp, and scale as usual, and let the scale be placed parallel to the coil-windings, a common perpendicular passing through the centre of the mirror, and the slit through which the light comes from the lamp; then two distinct states appear, according to the relation between the strength of the alternating currents in the secondary and the strength of the magnetic field in the axis of the coil due to the earth and deflecting magnet. 1st. If the currents are powerful enough and the magnetic field weak enough, the spot of light goes off the scale completely, either to one side or the other, and remains there. It can be made to go to either side and remain there by starting it off properly, which is easily enough managed by throwing on the alternating currents after it has passed the zero towards the side to which it is desired to send it. The spot will not remain at zero, even when placed there very carefully. This phenomenon I call *bilateral deflection* *. 2nd. If the strength of the currents be *decreased* sufficiently, whether by interpolating resistance in the secondary or primary, or by reducing the electromotive force in the primary, or by shunting the galvanometer—or if, on the other hand, the strength of the magnetic field be sufficiently increased, say, by lowering the deflecting magnet,—

I. If the spot of light be brought, when there are no electrical oscillations, to zero on the scale, then on setting the coil in action it comes to rest at zero and remains steady there.

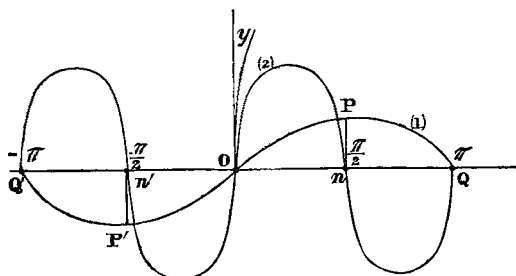
II. If the spot be brought to any position right or left of zero, then when the coil is in action it comes to rest in some position a little further to the right or left respectively, and remains steady there. The difference between these positions is greater the greater the original deflection of the spot from zero. This phenomenon I call *unilateral deflection*.

III. No difference of any kind was produced in any of these phenomena by reversing the connexions of the secondary with the galvanometer. Nor did the character of the phenomenon depend on the number of alternations per second, which in my experiments varied from 10 to 200. It was the second of the last-mentioned set of phenomena

* After Poggendorff, who originally observed the phenomenon and called it “Doppelsinnige Ablenkung.”—Pogg. *Ann.* vol. xlv. p. 353 (1838).

for different relative positions. These points of intersection correspond to the positions of equilibrium of the needle ; and the stability or instability of the equilibrium is seen at a glance. For example, take the case $\alpha=0$. The points corresponding to $\theta=0$ in (1) and (2) must be superposed ; thus, for a *large* value of B we have fig. 1. Here O , P , P' , Q , and Q' give

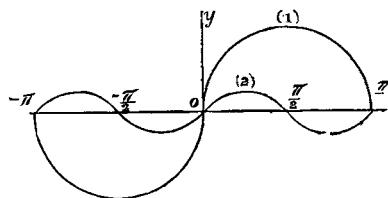
Fig. 1.



positions of equilibrium ; but O is obviously unstable, because on going to the right (increasing θ) the curve (2) lies above the curve (1), *i. e.* the force tending to increase θ preponderates ; and similarly, if we go to left (decrease θ), the force tending to decrease θ preponderates. Similarly, Q and Q' are unstable positions ; but P and P' are stable positions symmetrically situated with respect to O . The positions of P and P' lie nearer a point distant $\frac{\pi}{2}$ from O , the greater B . This case is best illustrated experimentally with a tangent-galvanometer of the usual construction, where the motions of the needle can be traced all the way round. We have thus explained Poggendorff's case, which is the limit to the state of the phenomenon*.

Next, suppose B to be small, we have fig. 2. The positions

Fig. 2.

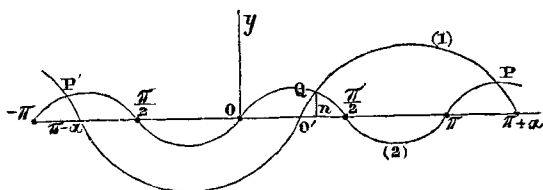


* There are some points in Poggendorff's observations which did not appear in mine ; but the difference might arise from the inductive influence of the earth being sensible in his experiments, which was not the case in mine.

P and P' have now disappeared, and the equilibrium at O has become stable. This corresponds to case (1) of the second state of the phenomenon.

Lastly, suppose B small and α not zero, then we must *displace* the origin in curve (1) to the *right*, say. We thus get fig. 3. Here $O O' = \alpha$, so that O' represents the position of

Fig. 3.



equilibrium when there are no currents. P and P' are *unstable*; and Q represents a single stable position when the currents are going. The deflection corresponding to this position is $O n$, where n is the foot of the ordinate of Q, and $O n$ is $> O O'$, in accordance with the experimental facts above stated.

Similar reasoning would show that if the magnet had been deflected in the opposite direction, the alternating currents would have increased the deflection in the same direction. In fact the above simple graphical representation embraces the experimental facts, as I have observed them, completely.

It may render the above clearer to give the results of one of the earlier experiments with a Thomson's galvanometer. No care was taken to adjust the scale parallel to the windings of the coil; so that the results are not symmetrical; but they illustrate perfectly the nature of the phenomenon in its second state.

Position of spot with no current.	Position of spot with currents going.	Difference.
+173	+350	177
+ 50	+108	58
- 2	+ 10	12
- 14	- 15	1
- 20	- 30	10
- 64	-134	70
-193	-355	162

In this case the mirror was very nearly parallel to the coils when the spot of light was at -14 on the scale. It will be seen from the above that the effects are of a very decided and unmistakable character. I may also add that I have got similar results with three different galvanometers.

This may be sufficient so far as the subject is connected with galvanometry merely. I need hardly call attention to the importance of these phenomena in relation to experiments such as those of Dr. Schuster, where alternating currents are used.

In attempting to get a clearer insight into the nature of the effect on the magnetism of the needle, I was led to make some numerical verifications, and to give some variations to the experiment which may perhaps be of interest.

The phenomenon of magnetic induction is more or less complicated according to circumstances. We have to deal, in fact, with several distinct phenomena, which may for the present purpose be classified under two heads:—I. Temporary Magnetism; II. Residual Magnetism and its gradual decay. These phenomena are analogous to (1) the temporary strain, (2) the permanent set and “elastic recovery”* in a solid subjected to stress in any way.

In what follows I shall assume that we have to do with temporary magnetism merely. Even when thus simplified the present case is to some extent peculiar. It is probable that the maximum magnetization producible by a given force is attained only after the force has been in action for some time†. Now, if the effects we are considering be due to induced magnetism at all, it is obvious that a very considerable fraction of the induced magnetism due to a given magnetic force must be developed in an interval of time incomparably smaller than the $\frac{1}{400}$ of a second, whereas in the ordinary experiments on induced magnetism the time allowed for the development of the magnetization is practically unlimited. This peculiarity gives the present case additional interest.

I shall, in what follows, assume that when the magnetic force to which an element of iron or steel is subject varies, the corresponding variation of the magnetic moment of the element follows at an interval of time which is incomparably shorter than any other we have at present to deal with (*e. g.* the time during which either of the induced currents remains in the neighbourhood of its maximum).

With this assumption, we may apply the ordinary theory of magnetic induction. Three general conclusions may at once be drawn from the simple consideration that an elongated body tends to place its axis parallel to the lines of magnetic force:—

* By elastic recovery is meant what in Germany is familiarly known under the name of “Elastische Nachwirkung.” I do not know of any English name for it which has the sanction of good authority.

† Wiedemann, *Galvanismus*, Bd. II. 2, p. 160. Also Faraday, *Phil. Mag.* [IV.] vol. ix. p. 92.

I. An elongated magnet magnetized axially would give phenomena analogous to those observed.

II. A spherical magnet would give *no such* phenomena.

III. An elongated magnet magnetized transversely would give similar phenomena, except that in unilateral deflection the sign of the effect would be reversed; *i. e.*, the spot of light being brought, by means of the deflecting magnet, to the right or left of zero, the effect of the alternating currents would be to *diminish* this deflection.

Conclusion III. was directly verified; and the result was in complete agreement with theory. Conclusion II. was also verified experimentally. A small spherical steel magnet was fitted with a mirror hung up in the galvanometer and observed, as will be afterwards described in the case of an elongated magnet magnetized axially. The magnetic moment of the sphere was roughly determined for me by Mr. Shaw, B.A., of Emanuel College; the maximum horizontal earth-couple on the sphere was about $\cdot 22 \text{ grm.} \left(\frac{\text{cm.}}{\text{sec.}} \right)^2$.

Two cells of Grove were used in the primary of the induction-coil. The result was a feeble *unilateral* deflection. An observation was made with an elongated magnet consisting of a piece of thin watch-spring magnetized longitudinally. The maximum earth's couple in this case was about $\cdot 27 \text{ grm.} \left(\frac{\text{cm.}}{\text{sec.}} \right)^2$. All the other arrangements were exactly a before. The result was strong *bilateral* deflection. Here the two magnets were very nearly in the same circumstances, the advantage being somewhat in favour of the sphere, owing to its smaller moment. It appears, then, that the form of the magnet has a very powerful effect on the phenomenon.

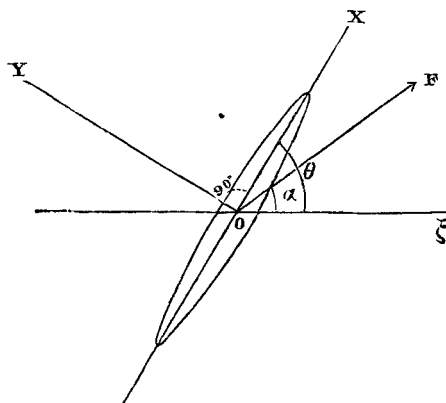
That the effect should be absolutely *nil* with the sphere was not to be expected; for we know* that a piece of steel once permanently magnetized by a force F is never in the same state as it was originally. We can demagnetize the steel apparently completely by a force less than F ; but it requires a force greater than F to magnetize to an equal degree in the opposite direction. The particles of a magnetized steel sphere have therefore a *quasi*-crystalline structure related to the magnetic axis; so that the perfect symmetry which causes a sphere to behave neutrally in a field of uniform force, as far as induced magnetism is concerned, is probably in some degree lost, and can only be restored by heating the steel over red heat and

* *E. g.* Rowland, Phil. Mag. [IV.] vol. I. p. 358. Also Fromme, Pogg. Ann. Ergbd. vii. p. 421; Phil. Mag. [IV.] vol. I. p. 299.

allowing it to cool apart from magnetic influence, or by some other equivalent process of molecular revolution.

A closer examination into the behaviour of an elongated magnet further confirms the above theory. Let us take as type of such a magnet a very elongated ellipsoid of revolution magnetized parallel to its axis, and suspended from a point in its equator. Let $o\xi$ be the plane of the galvanometer-wind-

Fig. 4.



ings, oF the direction of the resultant magnetic force (F) when there is no current, oX the direction of the axis of the magnet at any time,

$$\alpha = Fo\xi,$$

$$\theta = Xo\xi,$$

m = permanent magnetic moment,

κ = coefficient of induced magnetization,

V = volume of magnet,

i = current at any time t ,

g = constant of galvanometer,

n = number of breaks per second in primary.

The component forces parallel to oX and oY at time t tending to magnetize the needle inductively are

$$X = gi \sin \theta + F \cos (\theta - \alpha),$$

$$Y = gi \cos \theta - F \sin (\theta - \alpha).$$

Hence (see Maxwell's 'Electricity,' vol. ii. pp. 65 & 67) the couple tending to increase θ is

$$V \left(\kappa - \frac{\kappa}{1 + 2\pi\kappa} \right) XY = \frac{\pi\kappa^2 V}{1 + 2\pi\kappa} \{ g^2 i^2 \sin 2\theta + 2Fgi \cos(2\theta - \alpha) - F^2 \sin 2(\theta - \alpha) \}.$$

Considering now what happens during a complete oscillation, let P be the uniform force whose action during that time is equivalent to the action of the varying force due to induced magnetism, then

$$P \frac{1}{n} = \frac{\pi \kappa^2 V}{1 + 2\pi \kappa} \left\{ g^2 \sin 2\theta \int_0^{\frac{1}{n}} i^2 dt - F^2 \cdot \frac{1}{n} \cdot \sin 2(\theta - \alpha) \right\}.$$

The middle term disappears because $\int_0^{\frac{1}{n}} i^2 dt = 0$. Hence, if I^2 denote $\int_0^{\frac{1}{n}} i^2 dt \div \frac{1}{n}$, i. e. the mean square of the induced currents, then

$$P = A'I^2 \sin 2\theta - B' \sin 2(\theta - \alpha), \quad (1)$$

where

$$A' = \frac{\pi \kappa^2 V g^2}{1 + 2\pi \kappa}, \quad B' = \frac{\pi \kappa^2 V F^2}{1 + 2\pi \kappa}.$$

Adding now the couple due to the permanent moment of the needle, we get for the whole force tending to decrease θ ,

$$mF \sin(\theta - \alpha) - A'I^2 \sin 2\theta + B' \sin 2(\theta - \alpha). \quad (2)$$

In all the experiments of verification to be afterwards described (and in all the experiments discussed in this paper) the permanent field on the galvanometer was very much weakened by properly adjusting permanent magnets. Under these circumstances B' is very small compared with mF and $A'I^2$. It is easy to verify this by comparing the observed values of the times of oscillation of the needle when the currents are going and when they are not, with the values calculated by means of (2). The verification, however, is suppressed, as it is long and uninteresting.

The expression for the couple therefore becomes

$$mF \sin(\theta - \alpha) - A'I^2 \sin 2\theta, \quad (3)$$

the same as that obtained above by looser reasoning. It appears, then, in the first place, that if we make a series of observations of unilateral deflection, and if α define the position of the needle for no current, and θ the position when the currents are going, then, other things being equal,

$$\frac{\sin(\theta - \alpha)}{\sin 2\theta} = \frac{A'I^2}{mF} = \text{const} = C \text{ say.}$$

To verify this law I used the following arrangement:—The galvanometer (designed by Professor Maxwell and originally made by Warden, Clark, and Muirhead, but rewound by myself last summer) has two coils wound in channels $\frac{7}{8}$ inch broad, $1\frac{1}{8}$ inch deep, and 4 inches in external diameter. The number

of windings, partly of thin and partly of thick wire, is about 2668, making up a resistance of about 68 ohms. The channels are cut in the same piece of boxwood, along the axis of which is drilled a cylindrical hole in which hangs the magnet. The ends of this cylindrical cavity are closed by two caps, one of which is fitted with a plano-convex lens, the other with a piece of plane-parallel glass. The torsion-head is fitted with a Weber's suspension-screw in the usual way. The whole of the upper part is supported on a foot, with placing screws in such a way that the coils can be turned about a vertical axis.

The magnet consisted of a piece of silvered glass fitted to a brass frame, with weights for increasing the moment of inertia. To the back of this was fitted a thin piece of magnetized watch-spring about 10 millims. long and 2 millims. broad. Under the earth's force the period of the needle's oscillation was 22.2 sec. This was raised to 49.2 sec. by properly weakening the field.

The deflection of the magnet was measured by means of a scale and telescope in the ordinary way, the position of rest being deduced from three elongations. The deflection of the coils was measured by means of the faint image of the scale from the plate-glass cap which closes the cylindrical core.

The battery used was six cells of Smee; and the primary circuit was made and broken by means of an electric tuning-fork lent me by Mr. Dew Smith, driven by the primary current itself. The induction-coil was of the ordinary construction, the resistance of the secondary being about 2714 ohms.

By making an observation with the planes of the plate-glass cap and mirror very nearly parallel, the value of the correction for the deviation of magnetic axis of needle in this position from the true plane of the coils was found. Using this correction, a series of observations with different deflections of the coils gave the following result:—

α .	θ .	C.
+1° 28'	1° 55'	.1175
+4 0	5 13	.1172
+5 0	6 29	.1156
+5 16	6 54	.1195
-4 34	6 3	.1225

Considering the means employed in obtaining them, these values of C, differing from the mean by less than 4 per cent., agree as well with each other as was to be expected. The two disturbing elements were the inconstancy of the battery and

the varying residual magnetism of the iron core in the induction-coil.

I next made some experiments to determine whether, other things being equal, C varies as I^2 , which by the above theory it ought to do. With this object in view different resistances were interpolated in the secondary, every thing else being kept the same. The resistance of the secondary, including the galvanometer, was about 2768 ohms. Resistances of 1000 and upwards were put in, α and θ observed, and the resulting values of C calculated.

If we suppose the time which elapses between two successive interruptions of the primary to be so long that the current in the primary arrives at the steady state in the interval, and the induced currents in the secondary due to make and break do not interfere, then it is very easy to calculate the value of I^2 for the induction-currents. The result is

$$I^2 = \frac{nM^2j^2}{2NQ} \left\{ 1 + \frac{NP}{LQ + NP} \right\}, \quad \dots \quad (A)$$

where L , N are the coefficients of self-induction for the primary and secondary, M the coefficient of mutual induction, and P and Q the respective resistances, j the steady current in primary, and n the number of interruptions per second as before. Of the two terms within the bracket the first is contributed by the current due to the break, the second by that due to make.

Now with an induction-coil such as I used—where $L = \cdot 013$, $M = \cdot 79$, $N = 52$, and $P = 2$ (say) and $Q = 2768$ (these numbers are very rough estimates deduced from experiments performed for practice)—the time-constants of the coil are such that with tuning-forks such as I used for producing the break (which gave $n = 50$, 100, or 200) the above formula is very far from being applicable.

In fact the result is much nearer what we should get by assuming that the primary current followed the sine law, in which case we should get, A being the maximum electromotive force in primary, and $\nu = 2\pi n$,

$$I^2 = \frac{\frac{1}{2}\nu^2 M^2 A^2}{(LN - M^2)^2 \nu^4 + (N^2 P^2 + 2M^2 P Q + L^2 Q^2) \nu^2 + P^2 Q^2}, \quad \dots \quad (B)$$

in which case it is easy to see that if ν be big enough, the effect on I^2 of doubling and trebling the resistance Q will be comparatively small. This is confirmed by experiment, as the following Table will show :—

Q.	Values of C for different values of n .		
	50.	100.	200.
2780	·2448	·2211	·2128
3780	·2359	·2126	·2062
8780	·1798	·1829	·1822
12780	·1424	·1563	·1657

When the value of n is less, the value of C ought to fall quicker as Q increases. This is confirmed by the following result of a series of experiments in which n had the value 10 very nearly :—

Q.	C.	D.
2780	·0774	1237
5480	·0475	1637
10480	·0242	1774
12780	·0194	1796

The column D gives the first four figures of the reciprocal of

$$\frac{1}{CQ} \left\{ 1 + \frac{NP}{NP + LQ} \right\},$$

which ought to be constant if I^2 followed formula (A). It will be seen that D is not very far from being constant, the differences getting less as Q increases. This is what we should expect.

If instead of using the induction-coil in the usual way we throw the battery into the coil usually used as the secondary, and put the galvanometer into the small-resistance coil commonly used as the primary, then the induced currents, even with the 200 fork, will not interfere. We must in this case put in formula (A), which is now perfectly applicable,

$L=52$, $M=·79$, $N=·013$, $P=2720$ (say), $Q=68$, the last arising practically from the galvanometer resistance. Hence

$$\frac{NP}{LQ + NP} = ·010,$$

and when Q is doubled its value is $·005$; so that the fraction contributed by the current at make to the value of I^2 is now comparatively small. Hence C will approximately vary inversely as the resistance Q , and C ought to drop to nearly half its value when Q is doubled.

The experiment was tried with twenty-five small Leclanché cells in the secondary of the induction-coil; the breaking-fork

was now driven by an auxiliary battery. The value of n was 50. The galvanometer (resistance about 68) was put in the primary as above described.

The above calculation represents the case thus realized pretty closely; for although the self-induction of the galvanometer has been neglected and the resistance of the battery only roughly estimated, yet neither of these affects the important term. The result, therefore, of experiment ought to be nearly as above predicted. It was so nearly as could be seen, taking account of unavoidable experimental errors.

It appears, therefore, that the above theory stands so far the test of experiment. When I can get the use of a sine-inductor or a sufficiently delicate electro-dynamometer (both of which will probably soon be added to the collection of instruments at the Cavendish Laboratory), it will be easy to test the theory still further.

If it be accepted, it seems to me that an interesting conclusion follows, viz. that, of the total induced magnetism which a given field of force is capable of generating in any body placed in it, a very considerable fraction must be developed in a time very much less than $\frac{1}{400}$ of a second.

Perhaps a method for measuring the inductive capacities for temporary magnetism of *strongly* magnetic substances might be built on the experiments I have described; but this can hardly be done until it is better known what degree of accuracy can be ascribed to the law

$$C \propto I^2.$$

Possibly by sufficiently increasing the speed of revolution we might with a sine-inductor be able to introduce the element of *time* into magnetic measurements, and thereby get new light on the difficult subject of magnetic induction.

Cavendish Laboratory, Cambridge,
October 2, 1876.

LI. *Attempt at a Theory of the (Anomalous) Dispersion of Light in Singly and Doubly Refracting Media.* By Professor E. KETTELER.

[Continued from p. 345.]

7. IF we now make the attempt to extend our theory to anisotropic media also, only one procedure will lead to the end in view, and that totally different from the usual one. According to Fresnel's method, namely, the mathematical treatment has hitherto been restricted exclusively to the differential equations of the vibratory motion of the æther perpendicular to the normal of the waves; and by means of them the