16. On the Centroid of a Trapezoid

Author(s): D. Quint
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For the application of Carlyle's solution to a problem set for Woolwich, Dec. 1894, see "Solutions," No. 20.

The solution by Pappus of the original problem is worthy of notice—

Draw AK, BL as before, let the circle on AB as diameter cut KL in y, and then the perpendicular to KL through y cuts AB in the required points (Figs. 10, 11).

All these constructions have important bearings on the geometry of conics, P being a focus of a central conic, having AB for its transverse axis and KL for a tangent. Carlyle's solution reminds us that the intercept KL made on any tangent to a central conic by the tangents at the vertex subtends a right angle at each of the foci, while that by Pappus gives the well-known property of the auxiliary circle. The figures occur in this connection in Apollonius and Gregory St. Vincent. E. M. LANGLEY.

14. On the C. G. of a circular arc.

(i) Let G_1, G_2 be the centroids of the circular arcs $CA(2\alpha)$ and $CB(2\beta)$, L and M their mid points. Then the centroid of the whole arc $BA(\overline{2\alpha+\beta})$ lies on ON, the bisector of $\angle BOA$, and it is easily seen that $\angle NOL = \beta$, $\angle NOM = \alpha$ (Fig. 12).

Hence by taking moments about ON—

$$\text{arc } CA \,.\, OG_1 \sin\beta = \text{arc } CB \,.\, OG_2 \sin\alpha,$$

$$\therefore\ \alpha \,.\, OG_1 \sin\beta = \beta \,.\, OG_2 \sin\alpha,$$

$$\therefore\ OG_1 : OG_2 = \frac{\sin\alpha}{\alpha} : \frac{\sin\beta}{\beta} \qquad (1).$$

Let β diminish indefinitely, then OG = limit of $OG_1 = r\dfrac{\sin\alpha}{\alpha}$.

(ii) For the sectors COA, COB we can prove equation (1) as before; and letting β diminish indefinitely G_2 becomes the C. G. of the evanescent triangle COB. Hence $OG_2 = \frac{2}{3}r$,

and $OG = \frac{2}{3}r\dfrac{\sin\alpha}{\alpha}$.

(iii) For the surfaces or volumes of lunes of a sphere bounded by secondaries of the great circle ACB, we can prove equation (1) as before, and putting $2\beta = \pi$ the lune (2β) becomes a hemisphere.

Hence *for surface* $OG_2 = \frac{1}{2}r$,

$$\therefore\ OG_1 = \tfrac{1}{2}r\frac{\sin\alpha}{\alpha} \cdot \frac{\frac{1}{2}\pi}{\sin\frac{1}{2}\pi} = \frac{r}{4\pi} \cdot \frac{\sin\alpha}{\alpha};$$

while *for volume* $OG_2 = \frac{3}{8}r$,

$$\therefore\ OG_1 = \frac{3r}{16\pi} \cdot \frac{\sin\alpha}{\alpha}.$$

G. H. BRYAN.

15. On another proof of XI. 4.

Let CO be perpendicular to OA, OB and OP any line through O in the plane AOB. Draw AB perpr. to OA.

Square on CB = squares on CO, OB = squares on CO, OA, AB = squares on CA, AB; therefore $\angle CAB$ is right.

Therefore square on CP = squares on CA, AP = squares on CO, OA, AP = squares on CO, OP, therefore $\angle COP$ is right. W. GALLATLY.

16. On the centroid of a trapezoid.

As in the investigation in No. 3 the parallel sides AB, DC of the trapezoid ABCD being supposed to contain p and q units of length respectively, the triangles ABD, BDC may be regarded as containing $3p$ and $3q$ units of mass respectively. Hence the centroid of the trapezoid is that of the system of particles containing p units each at A,B,D and q units each at B,D,C—*i.e.* of the system of 4 at A,B,C,D containing p, $p+q$, q, $p+q$ units respectively. Similarly it must be that of system of 4 at A,B,C,D containing $p+q$, p, $p+q$, q units respectively. Hence, superposing the two systems it must be that of $2p+q$ at A, $2p+q$ at B, $p+2q$ at C, $p+2q$ at D, and therefore of $2(2p+q)$ at E and $2(p+2q)$ at F. D. QUINT.

17. On the theorem *ad quatuor lineas*.

It may be worth noticing that the value of k obtained in Note 4 (No. 3) comes at once for *the ellipse* by projection thus: If α, β, γ, δ be the perpendiculars from any point P on a circle upon the sides of an inscribed quadrilateral ABCD, we have by elementary geometry $\alpha\gamma = \beta\delta$ or $\alpha : \beta :: \delta : \gamma$.

If then the sides of the quadrilateral be produced so that $AA' = BB' = CC' = DD'$ = diameter of circle, we have

$$\Delta OAA' : \Delta OBB' :: \Delta ODD' : \Delta OCC' \qquad \text{(i)}$$

(their bases being equal).

Now let the circle be projected into an ellipse; (i) will still hold, and we deduce at once—

$$\alpha \,.\, Oa : \beta \,.\, Ob :: \delta \,.\, Od : \gamma \,.\, Oc$$

(since the bases of the triangles will still be equal to the *parallel* diameters $2Oa$, $2Ob$, $2Oc$, $2Od$), *i.e.*

$$\frac{\alpha\gamma}{\beta\delta} = \frac{Ob \,.\, Od}{Oa \,.\, Oc}.$$

PERCY J. HEAWOOD.

SOLUTIONS OF EXAMINATION QUESTIONS.

The Editor will be glad to avail himself of the help of all classes of readers towards making this section of the Gazette *as useful as possible.* MATHEMATICAL TUTORS *are invited to send neat solutions;* STUDENTS *to call attention to classes of problems presenting exceptional difficulties, and* EXAMINERS *who sympathise with us to forward copies of their papers. The help of foreign readers is especially requested in obtaining copies of papers set in the public examinations of other countries.*

The Editor acknowledges with thanks the receipt of the following sets of papers:—

University of Texas (Dec. 1894, Modern Geometry).

18. *If θ be the acute angle between a tangent to the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ and the focal distance of the point, prove that the distance of the point from the centre is $\sqrt{a^2-b^2\cot^2\theta}$.* [W. 1881.]

With the usual notation we have

$$b^2 = SY\cdot S'Y' = SP\cdot S'P \sin^2\theta = CD^2\sin^2\theta,$$

since $SP\cdot S'P = CD^2$, therefore $CD^2 = b^2\operatorname{cosec}^2\theta$.