Address to the Economic Science and Statistics Section of the British Association for the Advancement of Science, York, 1906
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Source: *Journal of the Royal Statistical Society*, Vol. 69, No. 3 (Sep., 1906), pp. 540-558
Published by: Wiley for the Royal Statistical Society
Stable URL: http://www.jstor.org/stable/2339344
Accessed: 15/01/2015 09:19

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I.—*Address to the Economic Science and Statistics Section of the British Association for the Advancement of Science, York,* 1906. By A. L. BOWLEY, M.A., President of the Section.

FROM 1835 to 1855 Section F of the British Association was devoted to "Statistics," and it is only from 1856 onwards that it has received its curious name, "Economic Science and Statistics." It is interesting to recall that Babbage was its first President, and that in its earlier career such well-known pioneers in the application of statistical method to industrial phenomena as Porter and Tooke occupied the chair. In its later course economics and statistics have shared the honour with public administration, whether related to trade or finance; and in recent years the professorial economist has alternated with the official administrator. It may be hoped that a new category will soon be added to the interesting and varied list—that of those engaged in practical or applied economics, the organisers of the army of industry; and in this connection it is much to be regretted that Sir George Gibb was unable to take the place which I now occupy. With a list which includes the names of Baines, Newmarch, Chadwick, Jevons, Booth, Giffen, and Edgeworth, no complaint can be made that statistical science and statistical art have not been worthily represented, and it would seem that there was no species of the exponents of our group of sciences not already scheduled in this roll; but I find that I have the unique position of being the first professed—or, shall I say, armchair—statistician, with few economic credentials, to hold this position, and this fact leads me to direct my address mainly to the claims of statistics to be an exact science, worthy to rank as such with those which form the subject-matter of Sections A to L. Since, however, the title of our section and the names of its officers, past and present, suggest the essential connection of economics with statistics, and the establishment of both on a quadruple basis of theory, history, experiment, and practice, my intention is to show that our work resembles the natural sciences in the respect that the most delicate researches in theory lead directly to visible and important practical results. The graduation of the income-tax, the supply of fish in the North Sea, and the expenditure of a labourer's wage, are among the subjects to which I have recently had to apply mathematical statistical analysis.

It is a long step from Arthur Young's tours to Professor Edgeworth's "generalised law of great numbers," but there is no distinction in the nature of things between arithmetical and mathematical statistics; the distinction to be made is not between the various methods of accumulating and tabulating data, but between the truth and falsity of the reasoning based on the tabulation. Mathematical treatment in the end only furnishes us with a microscope to observe differences which are blurred to the naked eye of arithmetic, and with a method of measurement to aid the judgment too immature to seize the significant fundamental fact concealed by its diverse manifestations. Purely arithmetical work is, however, limited to the tabulation of exact records, where the whole field to be surveyed can be covered, where no approximation or interpolation is necessary, and where statistics becomes only another name for accountancy; whereas the application of mathematical principles makes it possible to measure the inaccessible, to describe the animal from the single bone, to make firm observations from a shifting base, to dispense with the fixed meridian which the base practice of industrial and official needs obscures.

Great progress has been made in recent years on what I am calling the arithmetical side of statistics. With the encouragement of the very careful work done by the International Statistical Institute, whose labours have been mainly in this direction, official statisticians are aiming at logical systems of classification, on such natural lines as may be applicable for the majority of civilised nations and for long intervals of time. There are many difficulties in this direction: economic categories—such as skilled and unskilled labour, manufactured goods and raw materials, animal or vegetable products, occupied or unoccupied—do not admit of such simple definition as divides an acid from an alkali; statistical definitions are rather the delimitation of boundaries, and it is a matter of convention on which side particular persons or things are to be placed. Different conventions have become established in different countries, and the rapid change of conditions frequently induces an alteration in the conventions of a single country. Progress is being made in the direction of uniformity as between country and country, and it is here that the classifications of the International Institute are so important. It is still the case, however, that we are so far from agreement that it is impossible to understand the published statistics of this or any other country without intimate knowledge of the methods of compilation and classification in each group. The comparison of wages or prices, for example, in England and Germany is so difficult as to be hopelessly misleading, except in the hands of those few who have made a special study of these statistics in both countries.

It has till recently been the custom of departments publishing statistical returns to issue them without explanation of the particular conventions adopted, and then to complain that the ignorant public misquote them, till there was a danger that statistics should be issued only by officials for officials, and even by an official for himself alone, while the use of them (necessarily erroneous) by the general public was regarded as objectionable trespass in a private preserve. The growth of popular interest, and of a certain blind and misguided confidence in statistical statements, resulted in the printing of cautions that the statistics did not mean what they appeared to mean; and thus boards were erected to the effect that this table was dangerous to statisticians, and newspaper writers should drive with caution; but it did not for long occur to those responsible that it was their business to put the public roads in good order for the convenience of travellers.

The Board of Trade—and especially the Labour Department—have gone a long way now in the direction of explicitness of statement as to the exact meaning of their tables, and of carefully-thought-out improvements in classification and nomenclature; and herculean efforts have been made to improve the Occupation Census, the difficulties of which task have hardly been realised by its many critics. The returns on a greatly altered basis for 1901 illustrate the permanent dilemma that compilers of periodic returns have to try to avoid. If the old classification is retained, modern conditions throw it out of date; if a new one is adopted, comparisons may be made impossible. In 1881 veterinary surgeons and farriers were classed together in the census list of occupations; in 1891 the latter were put under "Workers in Metals," and in West Ham (for instance) were included under the heading "Blacksmiths and Whitesmiths." In 1901 whitesmiths are merged either in "Others in Engineering" or in "Miscellaneous Metal Trades," blacksmiths are given together with "Strikers," and the whole group of metal industries re-arranged. The result is that in comparing 1881 and 1901 the smallest comparable group must comprise all workers in metals and veterinary surgeons together. The old classification, so far as it is retained, still leads to curious anomalies. The word "postman" does not occur even in the detailed list of occupations, and it is a doubtful question whether a telegraph-boy is not considered as occupied in "The Government of the Country," while the post-office clerk is engaged in the "Conveyance of Men, Goods, and Messages."

It is a sad reflection that, while so much care and labour are spent in accumulating and printing statistical tables, so few of them are of any real importance, and so few are intelligible, even to one who studies them carefully. This topic was handled so ably by Professor Mandello in London last year* that it is only its great and immediate practical importance that leads me to refer to it.

* See *Journal of the Royal Statistical Society*, 1905, pp. 752, *seq.*

We need a central thinking department in statistics. There is already collected by the various Government departments, partly in their routine work, partly for the dissemination of information, an immense amount of valuable facts; but whenever a scientific inquirer endeavours to describe accurately some social or industrial development, or wishes to bring to the test of statistics the effect of some proposed reform, whether in taxation, regularity of employment, care of young children, or whatever it may be, some essential information is found lacking, for the reason that it has been no one's business to collect it. The details of returns of income remain uncodified in the offices of the local surveyors. Baxter's first estimate of—or, rather, guess at—the amount of income below the exemption limit still holds the field, for no inquiry has ever been made, and we continue in our ignorance of the aggregate national income and of its distribution. We have no adequate knowledge of the age, physical condition, or former occupation of the persons who receive public relief. Illustrations such as these could be multiplied by everyone who has tried to use official statistics. Owing to the enterprise of the Board of Trade, we are to have a second Wage Census and an Industrial Census, and thus many important gaps will be filled in; but there are as yet no signs of the consideration of the general question of what statistical measurements of the wealth, industry, occupations, and physical condition of the nation should and can successfully be undertaken.

Official publications have in general been restricted to arithmetical statistics, except in the case of the Table of Survivors in the Census reports, and quite recently to the measurement of the significance between the death-rates of different occupations in the Registrar-General's report on Scotland. The official view appears to be, quite correctly, that nothing should be published under the sanction of Government which is not an ascertained fact; but the briefest study is sufficient to show that the very nature of the measurements which have to be made, if only because of the necessary arbitrariness of definition, precludes exactness. The result is that the official counting is numerically correct, but the things counted are not coextensive with the quantity that the scientific inquirer needs to measure. He is left in the position of a man who inquires a distance in France, and is told that it is 8·543 kilomètres along the high-road, and then some way along a path; the precision of the first measurement is useless to him. It must be recognised that most statistics are necessarily approximate; and just as in other scientific measurements the quantity is given as correct to so many significant figures, so in statistics the possible and probable limits of error should be estimated, and the false show of so-called mathematical accuracy given up.

It is in this direction that the application of mathematical methods is necessary; but before dealing with them I wish to consider the provision made for the supply of persons capable of dealing with statistics by scientific methods.

I have made inquiries at the principal universities of the United Kingdom, with a view to ascertaining what facilities were afforded for the study of statistics, whether arithmetical or mathematical.

A knowledge of the statistics of trade and some acquaintance with the main sources and ordinary non-mathematical methods of statistics is required in the Faculties of Commerce in Manchester and Birmingham, and the nature of averages and index numbers is dealt with in lectures and examinations. In Manchester statistics may be taken as one of a long group of special subjects. In Liverpool elementary knowledge of statistics is expected of students in economics, and an effort is now being made to introduce a course of statistics. In Glasgow and in Edinburgh statistics has no formal place, but an attempt is being made in the latter city to recognise it. In Dublin part of the ordinary lecture course is devoted to elementary statistical methods, and the subject "Elements of Statistics" counts as an essential part of the examination for the annual prizes in political economy; the questions are non-mathematical. At Cambridge statistics is not distinguished as such in the Syllabus for the Tripos in Economics, though statistical methods are implied in the Part II papers on "Advanced Economics, mainly Analytic." So far as I can learn, there is no provision in lectures or examinations at Cambridge or Oxford for the application of mathematics to statistics. More complete recognition is given in the Faculty of Economics in London. There statistics up to the point reached in Birmingham and Manchester is demanded of all Pass students, and a considerable amount beyond is expected of those who have preferred mathematics to logic in their first year. Regular courses of lectures are specially devoted to the subject in two of the constituent colleges or schools of the university, and a large number of students have passed through them. Though the mathematics required stops just short of the infinitesimal calculus, there is no such limitation in statistics as one of eleven possible special subjects for honours.

It may reasonably be held that the application of mathematical reasoning to tabular information is so special a subject that it may safely be left to post-graduate study and individual initiative for men who are working at so wide a subject as political economy or taking a practical course in commerce; but no one can, I think, reflect seriously on the statements just made without coming to the conclusion that, in view of the immense importance attached to statistical reasoning in modern times—whether in trade disputes, or in proposals for social reforms, or in political pamphlets and speeches, or in the public Press—men who have not mastered the main criteria of the adequacy of such statements, who are not acquainted with the possibilities of such measurement, and who do not know the main statistical facts already common property, are not completely prepared in their professed subjects, and that there is not much likelihood that they will obtain this knowledge while the universities give so low a formal place to, and so little organised teaching in, these subjects.

If, however, it is admitted that the Pass student in economics cannot be expected to have more mathematics than is required for matriculation, and that there is not a sufficient demand for a course which shall apply more advanced analysis to economics and statistics, we might expect that other means would be taken to supply the country with those expert statisticians that so many public departments, whether central or local, so obviously need. Those responsible for the syllabus for First Division clerks in the Civil Service were not of this opinion. Out of twenty subjects, political economy is one, and in its syllabus the application of statistical methods to economic inquiries is named; but only two questions have been set in the last three years, and these do not involve mathematics. Mathematics itself occupies a conspicuous place, but there is no sign that its application to statistics or the theory of probability are included. It may be said that the examination is intended to be a test of non-specialised education, and that technical methods are best studied in the departments themselves. But though I find that the Record Office demands a knowledge of history and of a language from its recruits, there seems no evidence that an adequate knowledge of mathematics is required among those who are drafted to offices where the public statistics are handled. It may happen that those who are responsible for statistical analysis have gained high honours in mathematics; but in the existing routine it seems just as likely that they gained their distinction in Latin verse or mediæval history. Should a department discover that the handling of statistics did not come naturally to an ordinary educated person, it can perhaps fall back on the seventh clause of the Order in Council of 1870:—"In case the chief of a department to which a situation belongs . . . shall consider that the qualifications in respect of knowledge and ability deemed requisite . . . are . . . professional, or otherwise peculiar, and not ordinarily to be acquired in the Civil Service," a person "who has acquired such qualifications in other pursuits" may be appointed.

I have dealt with this subject at length in order to ask the question: Have we any guarantee that the public service, whether official or unofficial, will be supplied with a sufficient number of persons who are qualified to handle statistics expertly, to follow the rapid mathematical developments which alone can get the full significance of records, and to inform the public with reasoned knowledge of the measurable phenomena of national life? There is no dearth of capable mathematicians streaming from our universities, but there are relatively few who apply their special knowledge to public affairs; they rather dissipate it in elementary teaching or put it aside as a useless weapon. There is a very plentiful supply of expert arithmeticians entering the lower grade of the public service, but there is no provision for their developing into educated statisticians.

The use of mathematical reasoning in statistics is very imperfectly understood, partly because the passage from numbers to symbols and back to numbers suggests an air of mystery, or even of prestidigitation, to the unmathematical mind; partly because, even with mathematicians, the application of the theory of probability to the determination of the precision of an estimate is unfamiliar; partly because the method, though fully sixty years old,* has only recently been developed, and the methods and limitations of its use are still a matter of analysis and discussion among its advocates. In many respects its position resembles that of mathematics in economic theory, a subject handled at length by Professor Edgeworth, my predecessor in this chair in 1889. There are those that hold, in both cases, that verbal or numerical reasoning, unassisted by symbols, is sufficient for the elucidation of all truth. Whatever may be said in favour of this view as regards economic theory—a discussion so familiar to my audience that I need not dwell on it—I do not think that in the case of statistics the argument can be seriously maintained, and it is my intention to give such reasons for this statement as the limitations of a presidential address make possible.

Scientific measurement is in general approximate, and in the physical sciences much attention is given to the determination of the accuracy of experiments, and their result is given as not absolute, but as correct to so many significant figures. Statisticians frequently find that their second significant figure is doubtful, as in the case of the national income, which is estimated as between 1,700,000,000*l.* and 2,000,000,000*l.* Sometimes even the first figure is doubtful, as in the erroneous quotation that 13,000,000 persons are on the verge of hunger. In such cases as this the physicist would stop, and set to work to elaborate his measurements. Not so the popular statistician, who delights in guessing in tens of millions and mixing up these bold round numbers with others correct to ten significant figures. These guesses must be rigorously excluded from serious work, and, lest they should come in unawares, the exact limitation of the quantity actually measured and its relation to the total in question must always be carefully studied. We must candidly accept the fact that our raw material is imperfect, and our business is to remove the imperfections as far as we can, and, above all, to measure those we cannot remove. It is in these two directions that mathematical methods are generally necessary, and sometimes sufficient. The material is improved by methods of interpolation and graduation; the general law of grouping or direction of movement is discovered, and the accidental variations eliminated; or conversely, the general direction is neutralised and the variations measured. The adequacy of the material is discovered from internal evidence of consistency and conformity to the laws of continuity, and improved by carefully selected samples. The last-named method will be dealt with presently.

When the material is improved and tested there arise questions of causation—as to whether two series or groups are connected, or whether the observed variations are independent, or as to whether a difference between two measurements is significant or the accident of observation, or as to whether a change observed in average or grouping can be accounted for without assuming a change in the plexus of causes governing the phenomena.

* Quetelet's "Lettres sur la Théorie des Probabilités" was published in 1846.

It may be well to give commonplace instances of each of these methods. The ages of persons, as tabulated from the Census forms, are systematically smoothed for the Table of Survivals. Records of prices are averaged to give index numbers independent of individual variations. Records of unemployment are averaged to give the seasonal variation apart from the general trend. Records of wages may be rejected or doubted if they show too close a grouping at round numbers, if the grouping found in two similar establishments is markedly different, or if the relation of the various grades is not that obtained in properly chosen samples. The relation between infantile mortality and the employment of married women is a problem in correlation very difficult from the dearth of data. The observed difference in death-rates in two occupations requires a delicate mathematical test before it can be established as a real phenomenon. The proof that a known change in tariff has or has not affected prices or trade requires an adequate measurement of the variations when there has been no such change. The question whether the national income has in recent years become more or less uniformly distributed supplies a mathematical problem of considerable difficulty. Most of these illustrative problems can be treated arithmetically; the essential thing to observe is that the choice of the right method of treatment requires mathematical analysis.

The time is not ripe, nor have I the knowledge, for writing the history of mathematical statistics; but a slight sketch may be offered of some of the main developments, from Gauss and Laplace to Quetelet, and to Professors Edgeworth and Karl Pearson. I leave on one side the mathematics of graduation and interpolation, Newton's interpolation formula, Farr's life table, Hain's application to the smoothing of statistical series, and Mr. Sheppard's central differences; and I omit references to the method of least squares, used by Gauss in 1795 and Legendre in 1806, since this has been developed on non-statistical lines and its statistical use is merged in the development of the law of error.

The fundamental formula of the normal curve was known at least as early as 1809. Hagen (of Berlin) used it in 1837, deducing it from the binomial form. In 1837 Poisson defined "the law of great numbers," a phrase whose meaning has been enlarged by Edgeworth. In 1846 Quetelet showed its very extensive application to anthropometry, and enriched his letters with illustrations culled from a very wide field. In the same year Bravais discussed the surface of error for two variables. In 1852 Hain (of Vienna) applied Quetelet's method to the observation of the constancy of many statistical totals and to the measurement of their variation. He measured the deviation by the method of mean square $\left(\sqrt{\frac{\Sigma d^2}{n}}\right)$, whereas Quetelet had dealt mainly with the binomial measure $(\sqrt{2pqn})$. In 1877 Lexis showed the importance of the difference between the two methods of measuring deviation just stated, and applied the law of great numbers to the ratio of male

2 N 2

to female births and to the normal span of man's life (a subject continued in Professor Karl Pearson's *Chances of Death*). In 1885 Professor Edgeworth brought into prominence the means of testing the significance of observed differences between the averages of kindred groups, and showed the very large practical field of possible applications of this and other methods based on the law of error. More recently, and especially in the last ten years, the theoretical foundation and the extensions and modifications of the normal law have been examined and the formulæ developed. The skewness of the binomial form is shown in Laplace's formula (1814), and was commented on by Quetelet (in 1846); Poisson gave the second approximation to the normal curve; this has been developed, varieties of treatment suggested, and further approximations given, by Fechner, Lipps, Bruns, Werner, Ludwig, Charlier, Kapteyn, and especially by Edgeworth, who has arrived at a temporary conclusion of his labours this year in his analysis of the complete form of the law of great numbers and of the conditions under which it may be expected to hold. On the other hand, mathematical formulæ on a double basis of *a priori* hypothesis and of empirical justification have been elaborated (primarily for biological purposes) by Professor Karl Pearson, whose method of fitting by moments has proved fundamental for work of this kind.

The measurement of correlation, implied by Bravais in 1846,* received a great impetus from Galton in 1888, and after its analysis in successive papers by Edgeworth, Pearson, and Yule, is now in general use. The more elementary processes of measurement by averages have been examined and extended by Galton, Venn, and Fechner, till the ideas of median and quartiles (used implicitly in Quetelet's method of fitting), mode, arithmetic average, dispersion, and mean and standard deviation, are common property with even the least advanced statisticians, and are coming into use in official statistics here and in the United States of America. The most important inroad into official statistics has been made by index numbers. This method has grown very gradually, and has received impetus from many economists and statisticians: the most complete analysis of its mathematical basis is in the report of this section's Committee in 1887-89.

In recent years progress in the development of theory has, indeed, been rapid, and a great number of important and thoroughly criticised methods are ready for use, and are, in fact, in constant use by biologists and botanists; but there has been remarkably little application to practical statistical problems. In the thirty years following the publication of Quetelet's *Lettres*, attention was mainly given to establishing the constancy of great numbers and averages based thereon, an important but limited work, while the relation of the frequency of deviations to the

* Bravais' formula relates to the position of a point given by two co-ordinates, the sources of error of which are not independent. The term arising from this interdependence proved to be essentially the same as that reached by the later writers working from quite different standpoints.

law of error was regarded rather as a statistical curiosity. Professor Edgeworth's illustrations in 1885 of the importance of mathematical methods in testing the truth of practical deductions has as yet borne singularly little fruit. The attention of mathematical statisticians has been mainly directed to theory, and to actual measurement of anthropometrical and biological correlations; it is time that it was brought to bear on the criticism and analysis of existing industrial statistics. Something has been done by Yule and Hooker in England, by Norton in the United States of America, and others, to test correlation and periodicity, and in other practical problems; but most of our statistics remain untested and their significance not analysed. The simple method of samples, illustrated below, for which all the materials have existed for at least twenty years, has (so far as I know) been completely ignored.

The region to which I am devoting particular attention is that where the theory of probability is invoked, not because there are not many other directions in which mathematical methods are useful, but because this is of the greatest importance and the least generally understood. All depends on a complete grasp of the nature of the measurement when we say, for example, that from certain data the most probable estimate of average wages is 24*s.*; it is as likely as not, however, to be as much as 4*d.* from this value: the standard deviation is 6*d.*; the chances are 10 to 1 against the average being over 24*s.* 8*d.*, 100,000 to 1 against it being over 26*s.* This is the kind of statement to which calculations lead. The result may be briefly indicated as 24*s.* ± 6*d.*, when the "standard deviation" is adopted as the measure of accuracy. In a normal curve of frequency about two-thirds of the area is within the standard deviation: the chance that a given observation should be within this distance of the true average is 2 : 1. The unit of measurement thus devised is most subtle and most complex. When it is applicable it gives the only complete measure of precision. When the initial difficulty of appreciating the nature of mathematical probability is overcome, a difficulty which rather grows than diminishes as one works at it, there still remains the greater task of deciding in what cases it can properly be applied and on the method of calculation. It has, in my opinion, often been used where it is not appropriate, where the chances of deviation are not those indicated by a normal curve, where it is a mere numerical value without involving the superstructure that makes the measurement of precision real. Thus it has sometimes been argued that if pn cases of a particular kind are found in n instances, then (without further analysis of the relation of the cases to the whole group) the "statistical coefficient" for the class is $p \pm \sqrt{\frac{p(1-p)}{n}}$, a deduction not based on sound theory; if in fact (here I follow Lexis) the deviation found from this formula is compared with that actually found from several observed values of p, the two do not in general coincide. In general, two lines of analysis are possible: we may find an empirical formula (with Professor Karl Pearson) which fits this class of observations, and by evaluating the constants determine an appropriate curve of frequency, and hence allot the chances of possible differences between our observation and the unknown true value; or we may accept Professor Edgeworth's analysis of the causes which would produce his generalised law of great numbers, and determine *a priori* or by experiment whether this universal law may be expected or is to be found in the case in question. It is to the latter method that my next remarks apply, and on which the example I give depends. It can be shown* that if quantities are distributed according to almost any curve of frequency satisfying simple and common conditions, the average of successive groups of say, 10, 20, 100 . . . n of these conform to a normal curve (the more and more closely as n is increased) whose standard deviation diminishes in inverse ratio to the number in each sample. My own practice is to take, first, a number of small samples (say 4 or of 10 in each) and observe the curve of frequency for these; if there is a reasonable indication of the shape of the normal curve appearing, I calculate the "standard deviation" for this grouping, say σ, and proceed with confidence to deduce that the average of a much larger sample, say of n, will have a normal curve of frequency, with deviation nearly $\sigma\sqrt{\frac{10}{n}}$, where 10 was the number in the first group of samples. If we can apply this method—and for clearness I give an example immediately—we are able to give not only a numerical average, but a reasoned estimate for the real physical quantity of which the average is a local or temporary instance. It is the main weakness of statistical estimates, whether of those on a great scale supplied by English or foreign public departments or of more intensive inquiries by private investigation, that no measure of precision is given, and consequently that no determination can be made as to whether observed differences (in wages, in death-rates, in diet, in prices) are the accidents of observation or are really significant.

The example I have taken for illustrating the use of samples is worked rather roughly, but when we are calculating the chances of unknown deviations it seems unnecessary to go beyond the first decimal place. I took a copy of the *Investor's Record* in which is given the yield per cent. to an investor at current prices on the basis of last year's dividends for 3,878 companies, and set to work to find the average of these percentages and the numbers giving various rates per cent. by sampling. Having numbered the list consecutively, I took the *Nautical Almanac* and read down the last digits of one of the tables, in groups of four: if the number so read was over 3,878 I ignored it, if under (including such a number as 0063) I wrote down the corresponding interest from the table I was sampling. I thus secured equal chances for each of the

* See Professor Edgeworth's paper in the Jubilee number of the Statistical Society, and subsequent papers there and elsewhere till that of June, 1906.

3,878 entries, and took one at a venture 400 times. It was necessary to make certain, in some such way as this, that the *chances are the same for all the items of the group to be sampled, and that the way they are taken is absolutely independent of their magnitude.*

The forty averages of 10, so obtained, should by Professor Edgeworth's theory be grouped in a normal curve of error, and, in fact, they are, with modulus 1·096*l.* The average of the 400 is found to be 4·7435*l.*, with standard deviation ·122*l.* The original items vary* from nil to 103*l.* The average, deduced from the samples, is thus known with practical certainty to be between 4*l.* 7*s.* and 5*l.*, and the chances are even that it is or is not between 4*l.* 13*s.* 3*d.* and 4*l.* 16*s.* 6*d.*† Actually, when the whole 3,878 were added together, the average proved to be 4*l.* 15*s.* 7*d.*

It is to be noticed that the *precision of this and the following measurements does not depend in any way on the size of the group sampled, but only on its nature and on the number of samples taken, if the area of choice is co-extensive with the group.* Here I have taken 2

* They are, in the list used, grouped according to the nature of the securities, Government, Municipal, Railways, Mines, &c., and the averages and standard deviations on successive pages differ materially. An artificial method of sampling is therefore necessary. This aggregation is very similar to that found in wages in different occupations and localities, and in many other practical examples.

† 400 samples taken at random from a list containing 3,878.

Average of 400, 4·7435*l.*

Modulus for 40 averages of 10, deduced from these averages, 1·096 – c, where $c = \sqrt{\frac{2\Sigma d^2}{n}}$.

Hence *modulus for the average of 400* is $\frac{c}{\sqrt{40}}$ = *·173l.* "Probable error" = ·082*l.* Standard deviation, ·122*l.*

Hence *average of all is as likely as not to be between 4·826l. and 4·661l.*

The modulus for the average of 100 is ·346, and, in fact, the deviations of the four sample averages of 100 taken from the average for 400 are + ·06 + ·20 – ·27 + ·003.

The 40 averages of 10 each conform fairly with a normal curve of error thus:—

Between	And	Actual.	In Normal Curve.
Average + c	average + $\frac{7}{5}c$	3	2·2
+ $\frac{3}{5}c$	+ c	5	4·8
+ $\frac{1}{5}c$	+ $\frac{3}{5}c$	7	7·6
– $\frac{1}{5}c$	+ $\frac{1}{5}c$	6	8·9
+ $\frac{3}{5}c$	– $\frac{1}{5}c$	8	7·6
– c	– $\frac{3}{5}c$	7	4·8
– $\frac{7}{5}c$	– c	3	2·2

And 1 instance at + $2c$

When the curve of frequency is normal with standard deviation σ, the chance that any particular case shall differ from the central value by as much as σ is ·317; 2σ, ·046; 3σ, ·0027.

in 19 as samples, but the results would apply equally well if my original list were extended a hundredfold or to any size; but then the task of verification would be prohibitive. If information were required as to the incomes, for example, of 1,000,000 persons, the labour of sampling to obtain results of given precision would be no greater than for 10,000 persons, except that precautions would need to be taken that each of the 1,000,000 had an equal chance of inclusion.

Having forecasted the average, I proceed to forecast the grouping. 7 per cent. are found with no dividend, 9 per cent. between 3*l.* and 3*l.* 10*s.*, and so on, as in the table below. The precision of these measurements is found from themselves, and varies jointly as the square roots of the number in the whole sample and (nearly) of the fraction the class selected is of the whole. The *precision can be made as great as we please*, the probable and possible errors as little, *by increasing the size of the sample.* It is to be noticed that the deviations in the separate classes are not independent, since their sum is zero, and the problem is thus complicated. If an unlucky sample is taken for one group, there must be one or more bad samples for others. Where, in the table below, no standard deviation is given, it is considered that the class is too small for any good forecast.*

* *Distribution of Samples.*

	First 100.	Second 100.	Third 100.	Fourth 100.	Forecast.		Actual Distribution.
					Average per Cent.	Standard Deviation.	
Dividend—							
Nil	6	5	8	9	7	± 1·27	6·0
£ s. d. — £ s. d.							
1 – – to 2 19 9	3	0	3	0	1½	—	1·5
3 – – „ 3 9 9	10	10	7	10	9¼	± 1·46	8·4
3 10 – „ 3 19 9	24	13	22	12	17¾	± 1·90	18·8
4 – – „ 4 9 9	14	14	16	20	16	± 1·83	17·3
4 10 – „ 4 19 9	11	16	12	14	13¼	± 1·68	13·8
5 – – „ 5 19 9	13	18	16	13	15	± 1·78	17·7
6 – – „ 7 19 9	9	16	9	14	12	± 1·63	10·8
8 – – „ 10 19 9	8	8	7	6	7¼	± 1·29	3·8
Above 11 – –	2	0	0	2	1	—	1·9
	100	100	100	100	—	—	100
Aliter— £ s. d.							
Below 3 – –	9	5	11	9	8½	± 1·4	7·5
£ s. d.							
3 – – to 3 19 9	34	23	29	22	27	± 2·2	27·2
4 – – „ 4 19 9	25	30	28	34	29¼	± 2·3	31·1
5 – – „ 5 19 9	13	18	16	13	15	± 1·78	17·7
6 – – „ 7 19 9	9	16	9	14	12	± 1·63	10·8
Above 8 – –	10	8	7	8	8¼	± 1·4	5·7
	100	100	100	100	—	—	100

The standard deviation is thus calculated: If m examples are found in

We are thus able to forecast that about 7 per cent. of the investments yield nil, 27 per cent. between 3*l.* and 4*l.*, 29 per cent. between 4*l.* and 5*l.*, 15 per cent. between 5*l.* and 6*l.*, and the remaining 20 per cent. yield over 6*l.* per 100*l.*

While the above was in the press I tabulated the whole group, and entered the numbers in the column "Actual Distribution" in the proof. It is seen that the agreement between prediction and fact is most satisfactory except in the case of the group above 8*l.**

The average yield calculated must not be confused with the average return to capital invested; it is simply the average of the rates tabulated, taking all the companies as of equal importance.

The method of sampling is, of course, only one of many instances of the application of the theory of probability to statistics. I have taken it at length because the method is so persistently neglected, and even when it is used the test of precision is ignored. We are thus throwing aside a very powerful weapon of research. It is frequently impossible to cover a whole area, as the census does, or as Mr. Rowntree here and Mr. Booth in London successfully accomplished, but it is not necessary. We can obtain as good results as we please by sampling, and very often quite small samples are enough; the only difficulty is to ensure that every person or thing has the same chance of inclusion in the investigation.

The use of such methods must remain for some time in the hands of specialists for several reasons. The theory is still in the making; no general rules can be laid down, and considerable judgment is needed as to how far a particular theory is applicable to a given problem. There is still some difference of opinion as to the best method of grounding the theory, and there is not complete agreement as to the meaning to be attached to the technical terms or as to the best terms to be used. Again, the observations we are able to make are so few and so rough that special skill is required to handle them; the worse the tools the better must be the workman. So far, too, there is dearth of raw material for educative or laboratory work, though suitable groups of statistics are now

400 samples, the deduced frequency for the class is $\frac{m}{400} = p$, with standard deviation for the percentage found in class approximately $100\sqrt{\frac{p(1-p)}{400}}$.

The numbers in the second to the fifth columns belong, row by row, to normal curves, with the centre given in the sixth and twice the standard deviation given in the seventh column. Thus, in the third row, the average is $9\frac{1}{4}$, the standard deviation for 100 is $2 \times 1{\cdot}46$ (1·46 being the deviation for the average of the four columns), and the four cases (10, 10, 7, 9) are all within the standard deviation.

* Among the 3,878 cases there are 27 yielding exactly 10*l.* per cent.; of these 8 appeared in my sample of 400. There are in all 149 yielding from 8*l.* to 10*l.* 19*s.* 9*d.*; of these 29 appeared in my sample. The chance that so many as 29 should come in perfectly random sampling is only ·0008, say the chance of drawing two named cards from a pack of 52. I cannot discover anything in my process to lead to such a result.

accumulating rapidly. These conditions strengthen, rather than weaken, my demand for an adequate supply of specialists, and for the proper preparation of those whose duty it will be to handle statistics in a few years' time, when the methods will be more perfect. On the other hand, they support the general practice of the non-inclusion of the more advanced mathematical methods in undergraduate courses; but I could wish that the line were not drawn quite so low.

The use of the methods I have indicated can be illustrated from almost any branch of statistical inquiry. In dealing with wages we have to determine whether the samples we have are sufficient. In studying trade records we need to know the precision of our index numbers, the theory of weighted averages, and the measurement of trend and of fluctuations. In questions of income and income-tax we have to consider what help Pareto's law of graduation can give. In sociological and anthropometrical investigations, whether we are considering the malnutrition of children, or the alleged deterioration of national physique, or the birth-rate, or the employment of women, we have again and again to take mathematical means for describing the groups or estimating the precision of the averages and the significance of observed differences. In most cases of cause and effect, and in general in testing the interdependence of phenomena, we have to use the mathematical measure of correlation, a subject whose importance demands much more than the brief mention here given.

For illustration of the immense practical importance of such methods I offer a brief analysis of the most important and pressing of modern economic problems, showing in what respects mathematical statistics are needed. What do we actually know of, and how should the nation deal with, the classes who do not fit in with the ordinary and normal economic life of society—who do not, as a fact, attain any reasonable standard of life in our *régime* of free competition: the sweated, the casual labourer, the frequently unemployed, the permanently unemployed, and their forerunner, the underfed and uneducated child?

As economists and statisticians we are not concerned with palliatives or methods of expediency, but with a correct knowledge and true diagnosis of the extent of the evils, on which can be built reasoned and permanent remedies.

As is generally the case, our information as to the facts is hopelessly incomplete. There is no agreement as to scientific classification, no complete estimate of numbers; nothing but most limited records, supplemented by ill-informed guesswork. This is the case at the present moment, when public attention has been focussed on these questions for some time. Still less do we know anything about conditions thirty, twenty, or even ten years ago. In these circumstances we cannot say whether the very serious conditions, which are obvious at present, are better or worse than those of previous decades; whether, for example, the number of able-bodied men who are earning less than 40*l.* per annum has increased or decreased, absolutely or in proportion to the population. Remedies depend not only in extent, but in kind, on the numbers to be dealt with. Private resources may be sufficient to re-establish on a sound economic basis a small number of men who have been unable to weather an economic storm, but quite other means are necessary if a large class has lost the means or habit of earning a livelihood. Similar remarks apply to all the classes to whom I am referring. Till we know the facts we cannot prescribe the remedies, and it is during this period of trade activity that we have the leisure to gather the facts.

To learn the actual economic condition of all the 40,000,000 persons of the United Kingdom, or even of those who are not obviously above any possible poverty line, seems at first sight an impossible task; and so indeed it is, but only because of general apathy as to the subject. We must, therefore, proceed by some method of samples. Before we can get sound information from samples we must have a method of numbering or classification by persons or by districts. If we had a definite system of registration and identification, as in Germany, it would be easy to choose, say, 1 in 100 or 1 in 1,000 at random from among all the persons whose record satisfied certain conditions, and then to investigate more carefully the history and circumstances of those chosen. A similar method could be applied to any particular district. There is no need to make a house-to-house visitation to learn the conditions of a district; it is sufficient to enumerate the houses, to choose a certain proportion at random, and investigate carefully the status of their inhabitants. But the area of choice must be coincident with the area to be investigated.

When we have the sample, and have tested its precision by internal evidence, there are still difficulties of classification, but these can be overcome without mathematical analysis.

The economic analysis of these problems is constantly in need of help from statistics. What is the cause of, and what the remedy for, the existence of a large body of able-bodied persons frequently out of work or working for a wage below any reasonable standard? The least acquaintance with economic theory will lead us to deny any permanent absence of demand for a large body of existing labour in normal conditions; the difficulty must lie in the unfitness of the supply. The root cause economically is the fact that these persons are not fit for any of the work which society as an organisation needs. The unfitness may arise from the permanent loss of the trade to which the persons belonged; or to mental or physical deterioration following a bad spell of periodic unemployment (a phenomenon to which I return); or may be, and is, I think, more likely to be due to an absence of preparation for any of the employments which need more labour. In fact, it appears that at present in England the demand for labour is not sufficiently definite, and the supply too badly organised, to obtain equilibrium.

In a progressive or changing society new trades are continually growing, old trades altering their character or dying out. The latter process does not necessarily, nor, I think, generally, mean the throwing out of work of existing employees; it rather means the checking the demand for recruits, who should enter the newer trades, which in normal circumstances attract them with higher real wages. There is, however, no information available by which an intelligent artisan can decide into what occupation to put his son. A good deal could be done by mathematical and actuarial work, based on the successive occupational censuses (if these could be improved), to forecast what trades were relatively overcrowded. More could be done by a very careful organisation of technical schools, directed to educating the young for the trades of the immediate future. At present the choice of a trade is too much a matter of chance, decided by the immediate vacancy in the neighbourhood, or by an ignorant observation of the temporary prosperity of a particular industry. For example, superficial observation suggests that too many lads have entered the building trades in the last twenty years; but, as usual, our sources of information break down when this is examined.

It is true that, even at present, new trades and growing trades are very rapidly supplied. Skilled labour as a whole is very fluid: witness the manufacture of cycles in the eighties and nineties, the more recent motor-car industry, the great increase in the number of coal miners. On the other hand, the unemployment statistics in years of good trade show that the process of transmutation is not sufficiently rapid. The possibility of improvement lies in regulating the supply. An even more serious difficulty is that of moving from one grade to another. We are very ignorant on the subject, but it is commonly alleged that the son of an unskilled workman in general must also be unskilled. The father's wages being low, the lad must get to work at once, at the first thing that opens. There is a permanent demand for errand and messenger boys, and generally for quite unskilled labour at the bottom of an industry, which if not checked throws a great many young men adrift to begin the world at 18 in total ignorance of any useful occupation. There is, therefore, a tendency for a permanent oversupply of the unskilled relative to the skilled. It is not known whether in modern industry the proportions of skilled, partly skilled, and unskilled have changed or not. I have not found any significant alteration in such inquiries as I have been able to make. But this proportion is not fixed by any natural law. A deficit of unskilled would soon be supplied by machinery; processes are rapidly adapted to the labour supply. The labour market could readily absorb a greater supply of skilled men, if their skill was that in demand in the growing trades. If we want to check the growth of ignorant and unadaptable labour, we must save the boys of 13 and 14 from entering occupations that offer no future, and provide them with that knowledge and technique which industry will need five years later. The reason why a not unwilling worker cannot find an employer is not the want of sufficient capital, but the uselessness of the workman to society. So far we can get by *a priori* reasoning; whether the facts are correctly stated can only be decided by careful inquiry, applying the mathematical methods of sampling, averaging, and grading. A purely arithmetical inquiry, as that conducted at the London Docks by Mr. C. Booth, and at Liverpool by Miss E. F. Rathbone and Mr. G. H. Wood, will, however, throw a flood of light on such a question as to how many men are wanted, and how many in fact are present, in a trade. We may also hope to learn a great deal from a study of the information collected by the various relief agencies in the recent period of unemployment.

The question of periodic unemployment (as opposed to chronic want of work) is easier to handle and is better understood. It is, however, in need of very careful investigation; and I may remark that the most recent inquiry put to me as to mathematical processes related to the question of forecasting the turning-point towards better or worse trade. The cycle of commercial credit, which is very intimately connected with employment, is best studied by index numbers of prices and of quantity, and the most advanced mathematical work done by Section F* related to these numbers. The more the nature of a crisis is understood, the better it can be discounted and its worst effects mitigated, and there is some evidence that this is now done. When the recurring wave of unemployment is sufficiently well known, proper rates of insurance for want of work can be established, and the very extensive insurance in this direction by trade unions and other bodies can be put on a safer basis. It is a curious point, and one little noticed, that in the high tide of trade, work is plentiful and wages high; but prices are also high, and therefore the purchasing power of a sovereign low. This is the time to save, whether privately or in a society; for when the tide falls there is both more leisure to spend and the purchasing power of money is greater. Those whose occupation is affected by the commercial cycle have their salvation in their own hands.

There remain those who are physically or mentally unfit for work, who must always be a burden on their more fortunate fellows, and in considering them we pass out of the region of economics. But in this, as in other sociological questions, we still need statistics —perhaps most those methods of measurement we associate with Galton's name—to enable us to understand the magnitude and nature of the burden to be supported.

Again I would urge that in regard to all these questions we are in a condition of great ignorance. If the numbers of unemployed or unfit are increasing relatively to the population at large, the position is very serious and heroic remedies are wanted; but if they are diminishing, while we lament the present evil we may be hopeful as to the future. What light do statistics, mathematical or otherwise, throw on this question? We know that the wages of regular workers have increased steadily for many decades, whether measured in cash or in purchasing power, and that hours of labour have diminished progressively. The consumption of necessary commodities and of common luxuries has increased more rapidly

* *Report of Committee on Variations in the Monetary Standard*, 1888-90.

than the population. Working-class savings and investments have grown enormously. What evidence we have indicates that aggregate wages and aggregate national income have increased at nearly the same rate. Unemployment, so far as it is registered, has, period for period, been at nearly the same level for forty years, except that the years of good employment were specially numerous in the nineties. But this is only one side—the visible side—of the picture. For the permanently unemployed and unfit our only records are the singularly inadequate and imperfect statistics of pauperism. We have nothing to go on but guesses as to the real extent of poverty. We cannot recover records for previous years, and statistical science must remain powerless where there are no data. We are not taking any steps as yet to learn our existing condition in any complete way, though the work done in intensive inquiries would have been sufficient, if directed over the whole field, to have given us an adequate sample.

It is because of the immediate and pressing need of information before we commit ourselves to dangerous remedies on an erroneous diagnosis that I have spent my allotted time in pressing the importance of scientific method in statistical research.

II.—*The Criminal.* By Professor BÉLA FÖLDES, of Budapest.

FROM a moral and political standpoint, the investigation of the circumstances of the criminal is of much more importance than that of the crime. In those circumstances, whether personal, domestic, or social and economic, we find the explanation of the crime. The most valuable branch of the statistics of criminality, therefore, is that which relates to the criminal in the above relations. Regarding these, statistical *data* are at present, it must be admitted, very incomplete. Those relating to judicial proceedings have always been held to be of the greatest interest, yet the importance of these is trifling compared with that of facts which give the key to the secret motive, the soul, of the criminal. Amongst those facts we place in the first rank the sex, age, social position of the family, and the intellectual, economic, and religious qualifications of the individual.

Sex.—The influence of sex as an element in criminality may best be illustrated by the case of the female. On this, statistics are somewhat at a discount, since we must take into consideration the fact that a great part of the criminality of woman is confined to the family or domestic circle,[1] and is for that reason more difficult to be brought home to the offender than is the case with men, whose life is spent more in public and outdoor society. Then, again, owing to the weakness, timidity, and shame of their sex,

[1] *Atti di Commissione Giudiziaria,* 1885, p. 70.