PROCEEDINGS

OF THE

LONDON MATHEMATICAL SOCIETY.

VOL. XVI.

TWENTY-FIRST SESSION, 1884–5.

November 13*th*, 1884.

ANNUAL GENERAL MEETING, held at 22 Albemarle Street, W.

Professor HENRICI, F.R.S., President, in the Chair.

Prof. Karl Pearson, M.A., University College, London, Fellow of King's College, Cambridge, was elected a Member.

The President then announced, in the following words, the losses the Society had sustained by death during the Recess :—

"Before our ordinary business begins, I have to address you once again,* to say a few words in memory of two of our Members, Prof. Rowe, of Cambridge, and Prof. Townsend, of Dublin, who have died since our last meeting.

"Prof. Rowe never enjoyed good health, and died when still a young man. I first saw him in the examination-room of the London University, when Prof. Townsend and I were examiners. We were both struck by the brilliancy of his answers, but shocked by the extreme delicacy of his appearance. Later on, when he became my colleague at University College, this impression was somewhat effaced by his vivacity. He was a man of great amiability and geniality, and these qualities, together with his great musical gifts, made him friends

* [Alluding to the previous announcements of the deaths of Mr. Merrifield and Dr. Todhunter during the President's tenure of office.]

wherever he went. The principal thing he has left us as a mathematician is his paper in the *Phil. Trans.* of the Royal Society, on Abelian Functions, which showed that we might have expected good work from him, had he lived.

"Prof. Townsend had come to ripe years, though he was by no means an old man. He, like Prof. Rowe, was characterised by great geniality. He was beloved by his numerous pupils, and held in the greatest esteem by all who knew him intimately for his kindness and warm-heartedness. As a teacher he is said to have possessed singular qualifications.

"Personally, I have had the privilege of working with him for four years as examiner at the London University, and learned to value him very highly. As a mathematician, he was devoted to Pure Geometry, though he has published many papers belonging to Applied Mathematics. He treated his subject with considerable elegance. His chief work is his 'Chapters on the Modern Geometry of the Point, Line, and Circle,' which, though they are confined to a somewhat narrow field, are very rich in methods, and many of the results can at once be extended to Conics in general. He had just obtained a Senior Fellowship at Trinity College, Dublin, when the illness began which caused his death."

After a slight pause, he presented the De Morgan Memorial Gold Medal to Prof. Cayley, accompanying the presentation with the following address:—

"You will remember that, two years ago, it was announced from this chair that the Council had settled the conditions under which the De Morgan Medal should be given, and that the first award would be made at the anniversary meeting of 1884.

"I have now to make the announcement that the Council has decided that the first medal should be given to Professor Cayley in acknowledgment of his work in the 'Theory of Invariants.'

"As this is the first award of the medal, I may remind you of its origin. Soon after the death of De Morgan, some of his admirers started a subscription, for the double purpose of having a bust executed and founding a medal to be given in his memory. The *Bust* now adorns the Library of the London University, where also his valuable collection of books is preserved. The *Medal* was offered to the Mathematical Society, and its Council accepted the honourable duty of determining its award. There is a peculiar fitness in the medal being thus connected with our Society; for this Society was founded with the active co-operation of De Morgan by a number of his advanced students, among whom his talented son George, who

died soon afterwards, took the lead. De Morgan himself was the first President, and our *Proceedings* began with a very characteristic opening speech by him.

"The Medal is to be given for eminent original work in Mathematics, and no more fitting memorial than this could in my opinion be devised for a man who spent his whole life in carefully preparing the foundation for such work by his teaching and his writings.

"De Morgan was pre-eminently a teacher. His most original work does not so much increase our stock of mathematical knowledge, but is concerned with mathematical reasoning, and with exact reasoning in general.

"In the opening speech referred to, De Morgan himself divides exact science into two branches, *the analysis of the necessary laws of thought*, and *the analysis of the necessary matter of thought*. His own work belongs to the former. He was a logician much more than a mathematician in the ordinary sense of the word, and, when reading his mathematical works, I have always had the feeling that he studied Mathematics not so much for its own sake as on account of the logic contained and exemplified in it. I once made this remark in the Professors' Common Room of University College, when an old colleague of his turned round and said, 'You are quite right; he told me so himself.'

"In this work De Morgan did not stand alone; we may almost take him as a type of his period. It has often struck me as a noteworthy fact that in England, after a long pause in mathematical activity, the work taken first in hand was investigation into the very bases of Mathematics, and more particularly into mathematical reasoning. These investigations became partly a mathematical analysis of Logic itself, and partly a logical analysis of the laws followed by the symbols and operations used in Mathematics. De Morgan worked in both directions; we have his 'Formal Logic' and his 'Double Algebra.' Operations were studied quite independently of the meaning given to the symbols. Originally, the symbols stood for concrete things, and each operation had its concrete meaning. At present, symbols are sometimes used without giving them any meaning whatsoever, and without defining them at all; and then the operations for combining these symbols are arbitrarily defined, with the sole restriction that they do not contradict each other.

"Each new set of operations thus establishes a calculus. If afterwards any entities can be found which can be combined by operations answering the characteristics of the operations used in the new calculus, then the latter may be employed for a theory of those entities, and its results will allow of an interpretation. These entities

themselves may be anything—concrete things, or logical concepts, or ordinary algebraical quantities.

"Thus the ground was already prepared for greatly extending the realm of Algebra, and the scope and power of algebraical operations, when the genius of Prof. Cayley conceived the idea of Invariants, which has given rise to that marvellous growth of our science which has suddenly brought England again far to the front.

"It was known from Gauss's investigations that for Quadratic expressions a certain combination of its constants, its determinant, exists which has the following property:—

"If the Quadratic expression be transformed into another by a linear substitution, then the determinant of the transformed expression is obtained from that of the original expression by multiplying it by a factor which depends solely on the substitution used.

"Afterwards Eisenstein discovered that a similar theorem holds for a cubic expression of one variable. These isolated facts suggested to Cayley that combinations of constants having this property must exist for all algebraical expressions. The problem was, how to find these.

"The manner in which this has been solved I need not restate here, but I wish to call your attention to the fact that the symbolic methods worked out by the school of mathematicians referred to have been of the greatest use in the development of the Theory of Invariants, which could scarcely have been brought to its present perfection without it.

"It would be an impertinence for me to say much either in praise of Prof. Cayley's work or in justification of the Council's choice. Prof. Cayley has invented and worked out the Theory of Invariants, and, in steady life-long work, connected it with nearly every branch of Mathematics, enriching everything he touches, and everywhere throwing open new vistas of future work.

"The Council of the Mathematical Society, in selecting Prof. Cayley as the first recipient of the De Morgan Medal, and thus doing homage to his genius, did so not so much with the idea that it could add honour to his name, as that they might add honour to the Medal, by connecting his great name with it, and thus increase its value for all future recipients. And it is befitting that a body like the London Mathematical Society should give formal expression to the reverence and admiration in which it holds the greatest among its members.

"I shall now have the honour, the greatest which has ever fallen to me, of handing this medal to Prof. Cayley, and I call upon him to receive it.

"Professor Cayley, I hand this medal over to you, in the name of

the London Mathematical Society, as a token of their respect and admiration."

Prof. Cayley briefly returned thanks, and waived all claim to priority of discovery of Invariants.*

The Treasurer (Mr. A. B. Kempe) read his report. Its reception was moved by Mr. S. Roberts, seconded by Prof. Greenhill, and carried unanimously.

At the request of the Chairman, Captain P. A. Macmahon, R.A., consented to act as Auditor.

From the Report of the Secretaries, it appeared that the number of members since the last General Meeting, held November 8th, 1883, had increased from 159 to 173, of these 61 being Life Members.

The Obituary of the Society comprised the following names :—

Dr. Isaac Todhunter, F.R.S., elected June 18th, 1865, died March 1st, 1884.

C. W. Merrifield, F.R.S., elected March 19th, 1866, died January 1st, 1884.

Rev. R. Townsend, F.R.S., elected April 16th, 1866, died October 17th, 1884.

R. C. Rowe, M.A., elected November 13th, 1879, died September 21st, 1884.

The following communications had been made :—

Symmetric Functions, and in particular on certain Inverse Operators in connection therewith: Captain P. A. Macmahon, R.A.

On a Certain Envelope: Prof. Wolstenholme, D.Sc.

On certain results obtained by means of the Arguments of Points on a Plane Curve: R. A. Roberts, M.A.

Multiple Frullanian Integrals, Part iii.: E. B. Elliott, M.A.

Note on Jacobi's Transformation of Elliptic Functions: J. Griffiths, M.A.

Symmedians and the Triplicate-Ratio Circle: R. Tucker, M.A.

The Form of Standing Waves on the surface of Running Water: Lord Rayleigh, F.R.S.

A Method of finding the Plane Sections of a Surface, and some Considerations as to its extension to Space of more than Three Dimensions: W. J. C. Sharp, M.A.

On a Deduction from the Elliptic-Integral Formula

$$y = \sin(A+B+C+\ldots):$$

J. Griffiths, M.A.

* [*Cf.* Dr. Salmon's "Lessons on Higher Algebra," pp. 103, 295.]

An Extension of Pascal's Theorem to Space of Three Dimensions; and on the Theory of Screws in Elliptic Space: A. Buchheim, B.A.

On Contacts and Isolations, a Problem in Permutations: H. Fortey, M.A.

On the Induction of Electric Currents in Cylindrical and Spherical Conductors: Prof. H. Lamb, M.A.

On a Group of Circles which are connected with the Triplicate-Ratio Circle: R. Tucker, M.A.

On the Intersections of a Triangle with a Circle: H. M. Taylor, M.A.

On the Function which denotes the difference between the number of $(4m+1)$-divisors and the number of $(4m+3)$-divisors of a Number: J. W. L. Glaisher, F.R.S.

On a General Theory including the Theories of Systems of Complexes and Spheres: A. Buchheim, B.A.

On the Square of Euler's Series: J. W. L. Glaisher, F.R.S.

Further Results from a Theory of Transformation of Elliptic Functions: J. Griffiths, M.A.

Note concerning the Pellian Equation: S. Roberts, F.R.S.

On the closed Link Polygons belonging to a system of Coplanar Forces having a Single Resultant: Prof. M. J. M. Hill, M.A.

On the direct Application of the Principle of Least Action to Dynamical Analogues, Parts i., ii.: Prof. J. Larmor, M.A.

On Double Algebra: Prof. Cayley, F.R.S.

On Direct Investigation of the Complete Primitive of the Equation $F(x, y, z, p, q) = 0$, with a way of remembering the Auxiliary System: J. W. Russell, M.A.

On Electrical Oscillations and the effects produced by the Motion of an Electrified Sphere: J. J. Thomson, F.R.S.

Motion of a Network of Particles, with some analogies to Conjugate Functions: E. J. Routh, D.Sc., F.R.S.

On a Subsidiary Elliptic Function $pn\,(u, k)$: J. Griffiths, M.A.

On the Homogeneous Equation of a Plane Section of a Geometrical Surface: J. J. Walker, F.R.S.

On a Birational Transformation of Space of Three Dimensions, of the Sixth Degree, the Inverse of which is of the Fifth: Prof. Cremona, F.R.S.

Some Properties of Two Lines in the Plane of a Triangle: R. Tucker, M.A.

Note on the Induction of Electric Currents in a Cylinder placed across the Lines of Magnetic Force: Prof. H. Lamb, M.A.

Minor communications were made by Prof. Sylvester, F.R.S., and J. Hammond, M.A.

Additional exchanges of *Proceedings* were made with Dr. Bierens de Haan (*Nieuw Archief vur Wiskunde*), and Prof. Liguine (Mathematical Society of Odessa)*.

The same journals had been subscribed for as in the preceding Session.

The meeting then proceeded to the election of the new Council. The Scrutators (Mr. G. Heppel and Prof. M. J. M. Hill) having examined the Balloting Lists, declared the following gentlemen duly elected :—

President, J. W. L. Glaisher, F.R.S.; Vice-Presidents, Dr. Henrici, F.R.S., Prof. Sylvester, F.R.S., J. J. Walker, F.R.S.; Treasurer, A. B. Kempe, F.R.S.; Hon. Secs., M. Jenkins, M.A., and R. Tucker, M.A.; other members, Prof. Cayley, F.R.S., Sir J. Cockle, F.R.S., E. B. Elliott, M.A., Prof. Greenhill, M.A., J. Hammond, M.A., Prof. H. Hart, M.A., Dr. Hirst, F.R.S., S. Roberts, F.R.S., R. F. Scott, M.A.

Dr. Henrici having thanked the members for "the kind indulgence they had shown him" during his Presidency, Mr. J. W. L. Glaisher then took the chair, and thanked the Society for the high honour they had conferred upon him.

Mr. Tucker read abstracts of the following papers :—

On the Theory of Screws in Elliptic Space (Supplementary Note), and on the Theory of Matrices: Mr. A. Buchheim.

On Sphero-Cyclides: Mr. H. M. Jeffery.

Results from a Theory of Transformation of Elliptic Functions: Mr. J. Griffiths.

On the Limits of Multiple Integrals: Mr. H. MacColl.

On the Motion of a Viscous Fluid contained in a Spherical Vessel: Prof. H. Lamb.

On Certain Conics connected with a Plane Unicursal Quartic: Mr. R. A. Roberts.

Note on Elliptic Functions, on an Integral Transformation, and a Theorem in Plane Conics: Mr. A. Mukhopâdhyây.

The President (Dr. Henrici taking the Chair) then read a paper on Certain Systems of q-series in Elliptic Functions in which the Exponents in the Numerators and in the Denominators are connected by Recurring Relations.

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* For list of exchanges, see Vol. xiv., p. 315.

The following presents were received:—

"Educational Times," for November.

"Scientific Transactions of the Royal Dublin Society" (Series II.), Vol. I., Parts XX.—XXV.; Vol. III., Parts I.—III.

"Scientific Proceedings of the Royal Dublin Society" (New Series), Vol. III., Vol. VI., VII.; Vol. IV., Parts I.—IV.

"Smithsonian Report for 1882," 8vo; Washington, 1884.

"Bulletin des Sciences Mathématiques et Astronomiques," October and November.

"Beiblätter zu den Annalen der Physik und Chemie," Band VIII., No. 10, 1884.

"Archives Néerlandaises des Sciences Exactes et Naturelles," T. XIX., L. 3; Harlem, 1884.

"Jahrbuch über die Fortschritte der Mathematik," XIV. 1, Jahrgang 1882; Berlin, 1884.

"Cours de Mécanique," par M. Despeyrous, avec des Notes par M. G. Darboux, Tome I., 8vo; Paris, 1884.

"Crelle," Bd. XCVII., Heft 3.

M. E. Lemoine presented the following pamphlets:—

Association Française pour l'Avancement des Sciences:

Congrès de Lyon, 1873—"Sur quelques propriétés d'un point remarquable d'un triangle."

Congrès de Lille, 1874—"Note sur les propriétés du centre des médianes antiparallèles dans un triangle." (*Suite.*)

Congrès de Nantes, 1875—"Note sur le tetraèdre dont les arêtes opposées sont égales deux à deux."

Congrès de Reims, 1880—"Questions de Probabilités et valeurs relatives des pièces du Jeu des Echecs."

Congrès d'Alger, 1881—"Quelques questions de géométrie de position sur les figures qui peuvent se tracer d'un seul trait."

Congrès de Rouen, 1883—"Sur les quatre groupes de deux points d'un triangle ABC qui sont en même temps les foyers d'une conique inscrite et d'une conique circonscrite à ce triangle," et "Sur les nombres formés des mêmes chiffres écrits en sens inverse."

"Bulletin de la Société Mathématique de France," 1883—"Quelques questions de Probabilités résolues géométriquement."

"Journal de Mathématiques spéciales," 1883—"Nouveaux points remarquables du plan d'un triangle."

"Mémoires de la Société des Ingénieurs civils"—"Note sur le losange articulé du Colonel Peaucellier."

"Nouvelles Annales de Mathématiques," 1884—"Sur une question de Probabilité."

"Comptes Rendus," Oct. 1882—"Décomposition d'un nombre entier N en ses puissances $n^{\text{èmes}}$ maxima."