spheres, the change of direction of the relative velocity due to collision, and therefore $\lambda'\nu'-\lambda\nu$ of Art. 20, depends only on θ and ϕ, and not on the magnitude of V.

In the case of the molecules being centres of repulsive force, let c be the distance at which when two molecules approach each other that force first becomes sensible, and call c the diameter of a molecule, and define θ, ϕ as in Art. 17. It is then easy to see that the change of direction of V between the beginning and end of an encounter depends on V as well as on θ and ϕ; whence we may infer that, in this and in all cases where finite forces act between molecules, b and D are functions of h, *i.e.*, of the temperature, as well as of the density. It seems reasonable to expect that $D=0$, the point at which the mathematical formula ceases to be applicable, would, if we could solve the problem, express that relation between density and temperature at which the gas changes its form and becomes liquid.

Concerning the Abstract Groups of Order $k!$ and $\frac{1}{2}k!$ Holohedrically Isomorphic with the Symmetric and the Alternating Substitution-Groups on k Letters. By ELIAKIM HASTINGS MOORE, of Chicago. Received November 24th, 1896. Read December 10th, 1896.

Introduction.

Every abstract group may be defined by a system of generators which are compounded in accordance with a table of generational relations. This fundamental theorem is due to Cayley.

So far as I know, no such abstract generational definitions of the groups abstractly equivalent respectively to the symmetric and the alternating substitution-groups on k letters have been given. Of course from any one such definition many other such definitions may easily be deduced.

In this paper I prove the following theorems:—

Theorem A.—The abstract group $G(k)$ $(k \geq 2)$ defined by the $k-1$ generators

(1) $$B_d \qquad (d=1, 2, \ldots, k-1),$$

with the generational relations

(2) $$B_d^2 = 1 \qquad (d = 1, 2, \ldots, k-1),$$

(3) $$(B_d B_{d+1})^3 = 1 \qquad (d = 1, 2 \ldots, k-2),$$

(4) $$(B_d B_e)^2 = 1 \qquad \begin{pmatrix} d = 1, 2, \ldots\ldots\ldots\ldots, k-3 \\ e = d+2, d+3, \ldots, k-1 \end{pmatrix}$$

[whence follow at once

(5) $$(B_{d+1} B_d)^3 = 1, \quad B_d B_{d+1} B_d = B_{d+1} B_d B_{d+1},$$

(6) $$(B_e B_d)^2 = 1, \qquad B_d B_e = B_e B_d],$$

has the order $O(k) = k!$, and is holohedrically isomorphic with the symmetric substitution-group on k letters.

Theorem A'.—The abstract group $G'(k)$ $(k \geqq 3)$ defined by the two generators

(7) $$B, C,$$

with the generational relations

(8) $$B^2 = 1, \quad C^k = 1, \quad (BC)^{k-1} = 1,$$

(9) $$(BC^{\mp 1} B C^{\pm 1})^3 = 1,$$

(10) $$(BC^{\mp l} B C^{\pm l})^2 = 1 \qquad (l = 2, 3, \ldots, k-2)$$

[where, however, every relation with lower sign is a consequence of the corresponding relation with upper sign], is the group $G(k)$ of Theorem A.

Theorem B.—The abstract group $G\{k\}$ $(k \geqq 3)$ defined by the $k-2$ generators

(11) $$E_d \qquad (d = 1, 2, \ldots, k-2),$$

with the generational relations

(12) $$E_1^3 = 1, \; E_d^2 = 1 \quad (d = 2, 3, \ldots, k-2),$$

(13) $$(E_d E_{d+1})^3 = 1 \quad (d = 1, 2, \ldots, k-3),$$

(14) $$(E_d E_e)^2 = 1 \quad \begin{pmatrix} d = 1, 2, \ldots\ldots\ldots\ldots, k-4 \\ e = d+2, d+3, \ldots, k-2 \end{pmatrix},$$

has the order $O\{k\} = \frac{1}{2}k!$, and is holohedrically isomorphic with the alternating substitution-group on k letters.

[*Addition, June 28th,* 1897.—If two groups G, G' are generated respectively by the systems of generators $\{A_1, \ldots, A_h\}$, $\{A'_1, \ldots, A'_h\}$, and if the generators A' satisfy by the correspondence

$$A_i \sim A'_i \qquad (i = 1, 2, \ldots, h)$$

all the generational relations of the group G, then this correspondence determines an isomorphism between the two groups G, G'. In this isomorphism every element of G corresponds to one element of G'. The identity-element of G' corresponds to the elements of a self-conjugate sub-group H of G. The group G' is holohedrically isomorphic to the quotient-group G/H.

Whence, on the basis of known theorems of the theory of substitution-groups:

Theorem C.—If the generators of a group G of order greater than $\begin{matrix} 2;\ 6 \\ 1;\ 3 \end{matrix}$ satisfy at least the generational relations of any generational determination of the abstract $\begin{matrix}\text{symmetric}\\\text{alternating}\end{matrix}$ group of degree k for $\begin{matrix} k \neq 4;\ k = 4 \\ k \neq 4;\ k = 4 \end{matrix}$, then G is a form of the abstract $\begin{matrix}\text{symmetric}\\\text{alternating}\end{matrix}$ group, and its generators satisfy no further generational relations.]

1. *Four Lemmas—Theorems in the General Theory of Abstract Groups.*

Lemma A.—A group G contains two sub-groups K_m and L_n. K_m of order m with the m elements $k_1, \ldots, k_m$ is generated by the g generators $\kappa_1, \ldots, \kappa_g$. L_n of order n with the n elements $l_1, \ldots, l_n$ is generated by the h generators $\lambda_1, \ldots, \lambda_h$. If now every product $l_s \kappa_p \begin{pmatrix} s = 1, \ldots, n \\ p = 1, \ldots, g \end{pmatrix}$ can be written as a product $k_i l_j$, or, if every product $\lambda_q k_r \begin{pmatrix} q = 1, \ldots, h \\ r = 1, \ldots, m \end{pmatrix}$ can be so written, then the sub-groups K_m and L_n of G determine as sub-group H of lowest order containing them, the group H *of order at most* mn, whose elements are the distinct ones of the mn products $k_i l_j \begin{pmatrix} i = 1, \ldots, m \\ j = 1, \ldots, n \end{pmatrix}$.

Lemma B.—If the group G_n of order n with the n elements $g_1, \ldots, g_n$ has an involutoric holohedric isomorphism with itself, viz.,

$$(g_1, \ldots, g_i, \ldots, g_n) \sim (g_{s_1}, \ldots, g_{s_i}, \ldots, g_{s_n})$$
$$(s_{s_i} = i,\ i = 1, \ldots, n),$$

where $(s_1, \ldots, s_i, \ldots, s_n)$ is a certain permutation of the indices $(1, \ldots, i, \ldots, n)$, then, by the adjunction of a new generator ϵ with the generational relations

$$\epsilon^2 = 1, \quad g_i\epsilon = \epsilon g_{s_i} \qquad (i = 1, \ldots, n),$$

the group G_n of order n is extended to a group G'_{2n} of order $2n$ with the elements

$$g_i, \; g_i\epsilon \qquad (i = 1, \ldots, n).$$

Lemma C.—The group G_n of order n is generated by the f generators $\gamma_1, \ldots, \gamma_f$, which are subject to the generational relations Σ. Every relation Σ is taken in the convenient form

$$\gamma_{i_1}^{e_1}\gamma_{i_2}^{e_2}\ldots\gamma_{i_t}^{e_t} = 1.$$

If the relations Σ' obtained from the relations Σ by changing in every relation the sign of every exponent $e_1, \ldots, e_t$ are group-theoretic consequences of the relations Σ, then the correspondence of generators $(\gamma_1, \ldots, \gamma_f) \sim (\gamma_1^{-1}, \ldots, \gamma_f^{-1})$ defines an involutoric holohedric isomorphism of the group G_n with itself.

Lemma D.—If the generational relations Σ of the group G_n of Lemma C are all of the form

$$\gamma_i^{c_i} = 1, \quad (\gamma_i^a\gamma_j^b)^{c_{i,j;a,b}} = 1,$$

then the relations Σ' follow from the relations Σ.

These lemmas may be easily proved. A is generally known. *B* is the simplest case of a general theorem given by Mr. Hölder for use in the construction of composite groups ("Die Gruppen der Ordnungen p^3, pq^2, pqr, p^4," *Mathematische Annalen*, Vol. XLIII., pp. 301–412, 1893. See p. 329).

2. *Proof of Theorem* A.

We consider the group $G(k)$ generated by the $k-1$ generators $B_1, \ldots, B_{k-1}$ (1) under the fundamental relations (2, 3, 4), and the derivative relations (5, 6).

The symmetric substitution-group $G^k_{k!}$ on k letters $l_1, \ldots, l_k$ is generated by the $k-1$ transpositional substitutions

(15) $$S_d = (l_d l_{d+1}) \qquad (d = 1, 2, \ldots, k-1).$$

These generators S_d of the substitution-group $G^k_{k!}$ satisfy the relations (2, 3, 4) prescribed for the generators B_d of our abstract group

$G(k)$ (and conceivably, though not actually, other relations not therefrom derivable). Hence, the order $O(k)$ of $G(k)$ is at least $k!$, and, further, if the order $O(k)$ is exactly $k!$, then $G(k)$ is holohedrically isomorphic with $G^k_{k!}$.

That $G(k)$ has the order $O(k) = k!$ is proved by a one-based induction; it is obviously true for $k = 2$; we prove that, if it is true for $k = m$, then it is true for $k = m+1$.

The group $G(m+1)$ with the generators $B_1, \ldots, B_{m-1}, B_m$ has the order $O(m+1) \geqq (m+1)!$. $G(m+1)$ contains the sub-group

$$K_{m!} = G(m),$$

with the generators $B_1, \ldots, B_{m-1}$, and the $m!$ elements $k_1, \ldots, k_{m!}$ ($m!$, since its order is by hypothesis $m!$).

Now the substitution-group $G^m_{m!}$ on m letters $l_1, \ldots, l_m$ is most conveniently extended to the substitution-group $G^{m+1}_{(m+1)!}$ on $m+1$ letters $l_1, \ldots, l_m, l_{m+1}$ by the cyclic substitution

$$T = (l_1 l_2 \ldots l_m l_{m+1}),$$

which has the period $m+1$, and which is expressed in terms of the generators $S_1, \ldots, S_m$ (15) by the formula

$$T = S_m S_{m-1} \ldots S_2 S_1. \tag{16}$$

We extend our abstract group $K_{m!} = G(m)$ to $G(m+1)$ by the extending generator B_m, or, what amounts to the same thing, by the extender

$$C = B_m B_{m-1} \ldots B_2 B_1. \tag{17}$$

The relations of C to the m generators of $G(m+1)$ and the period of C (which is surely $\geqq m+1$) must be investigated.

Noticing at once that

$$C^{-1} = B_1 B_2 \ldots B_{m-1} B_m, \tag{18}$$

and defining B_{m+1} by the formula

$$B_{m+1} = C^{-1} B_m C, \tag{19}$$

we have the transformation-relations

$$B_{d+1} = C^{-1} B_d C \qquad (d = 1, 2, \ldots, m, m+1), \tag{20}$$

where, as in the sequel, *the indices of the B_d are to be reduced modulo*

$m+1$ *to indices of the series* $1, 2, \ldots, m, m+1$. $(20)_{d=m}$ is true by (19). $(20)_{d<m}$ is equivalent to

$$B_d C = C B_{d+1},$$

and, by (17, 6), this is equivalent to

$$B_m B_{m-1} \ldots B_{d+2} \,.\, B_d B_{d+1} B_d \,.\, B_{d-1} B_{d-2} \ldots B_2 B_1$$
$$= B_m B_{m-1} \ldots B_{d+2} \,.\, B_{d+1} B_d B_{d+1} \,.\, B_{d-1} B_{d-2} \ldots B_2 B_1,$$

and, by (5), this is true. (20) is then true for $d \leq m$. Now

$$C^{-1} \,.\, C \,.\, C = C;$$

hence, by (17), and $(20)_{d \leq m}$,

$$C^{-1} \,.\, B_m B_{m-1} \ldots B_2 B_1 \,.\, C = B_{m+1} B_m \ldots B_3 B_2 = C;$$

that is, $$B_{m+1} C = C B_1.$$

Thus (20) is true also for $d = m+1$.

Hence, more generally, from (20, 2, 5, 6, 17)

(21) $$C^{-j} B_d C^j = B_{d+j}, \quad B_d C^j = C^j B_{d+j} \qquad \begin{pmatrix} d = 1, 2, \ldots, m+1 \\ j = 0, \pm 1, \pm 2, \ldots \end{pmatrix},$$

(22) $$B_d^2 = 1, \quad B_d B_{d+1} B_d = B_{d+1} B_d B_{d+1}, \quad B_d B_e = B_e B_d \qquad \begin{pmatrix} d, e = 1, 2, \ldots, m+1 \\ e \neq d-1, d, d+1 \end{pmatrix},$$

(23) $$C = B_m B_{m-1} \ldots B_2 B_1 = B_{m+1} B_m \ldots B_3 B_2$$
$$= B_1 B_{m+1} B_m \ldots B_4 B_3 = \ldots = B_{m-1} B_{m-2} \ldots B_2 B_1 B_{m+1},$$

or (24) $$C = B_l B_{l-1} \ldots B_{l-m+2} B_{l-m+1} \quad (l = 1, 2, \ldots, m+1).$$

Further, the period of C is $m+1$,

(25) $$C^{m+1} = 1.$$

This is proved by a one-based induction. Denoting the C of (17) by C_{m+1}, we have

$$C_2^2 = B_1^2 = 1,$$

and we suppose that

$$C_m^m = (B_{m-1} B_{m-2} \ldots B_2 B_1)^m = 1.$$

Now $$C_m = B_m C_{m+1}.$$

Hence, by $(24)_{i=m-1}$, we have

(26) $$(B_m B_{m-1} \dots B_2 B_1 B_{m+1})^m = 1,$$

or (27) $$(B_m \dots B_1)(B_{m+1} \dots B_2)(B_1 \dots B_3)(B_2 \dots B_4) \dots$$
$$\dots (B_{m-2} \dots B_m)(B_{m-1} \dots B_{m+1}) = 1,$$

that is, by (24), $$C_{m+1}^{m+1} = 1.$$

[The specialization of C to T shows that period of C is $\geqq m+1$, and this result $C^{m+1} = 1$ shows that it is exactly $m+1$.] We notice (for use in § 3) the relations

(28) $$C_{m+1} B_1 = C_{m+1}^{-1} C_m C_{m+1}, \quad (C_{m+1} B_1)^m = 1, \quad (B_1 C_{m+1})^m = 1.$$

Now we consider in the group

$$G(m+1) = G(m+1 : B_1, \dots, B_m),$$

the two sub-groups

$$K_{m!} = G(m) = G(m : B_1, \dots, B_{m-1}),$$

with the $m!$ elements $k_1, \dots, k_{m!}$, and the cyclic L_{m+1} generated by C with the $m+1$ elements C^j $(j = 1, 2, \dots, m+1)$. The sub-group H of lowest order containing $K_{m!}$ and L_{m+1} is $G(m+1)$ itself with the order $O(m+1) \geqq (m+1)!$. But Lemma A (§ 1) applies, and so $O(m+1) \leqq (m+1)!$, and hence, indeed, $O(m+1) = (m+1)!$, and our Theorem A is proved. In fact, we have, (21),

(29) $$C^s B_p = B_{p-s} C^s \qquad (p, s = 1, 2, \dots, m+1).$$

For the cases $p-s \equiv 0, -1 \pmod{m+1}$, writing these relations in the more desirable forms, by the use of (17),

(30) $$C^{p+1} B_p = B_{m-1} B_{m-2} \dots B_2 B_1 . C^p,$$
$$C^p B_p = B_1 B_2 \dots B_{m-2} B_{m-1} . C^{p+1},$$

we have in the relations (29, 30), $(p, s = 1, 2, \dots, m)$ the relations needed to make Lemma A apply.

The $G(k)$ is then the abstract symmetric group $G_{k!}^k$.

It is obvious from the substitution-group form $G_{k!}^k$ of the abstract $G(k)$ that all the $k-1$ generators B_d are necessary; if one is omitted, a sub-group is generated.

3. *Proof of Theorem* A′.

The group $G'(k)$ $(k \geqq 3)$ has the generators B, C with the generational relations

$$B^2 = 1, \quad C^k = 1, \quad (BC)^{k-1} = 1, \quad (BC^{-1}BC)^3 = 1, \quad (BC^{-l}BC^l)^2 = 1$$
$$(l = 2, 3, \ldots, k-2).$$

Defining

$$(31) \qquad B_{i+1} = C^{-i}BC^i, \quad B_1 = B, \qquad (i = 0, 1, \ldots, k-1),$$

we have, taking indices of B, modulo k, from the indices $1, 2, \ldots, k$,

$$(32) \qquad B_{d+j} = C^{-j}B_dC^j \qquad (d, j = 1, 2, \ldots, k).$$

The relations

$$(33) \qquad B_1^2 = 1, \ (B_1B_2)^3 = 1, \ (B_1B_e)^2 = 1 \ (e = 3, 4, \ldots, k-1)$$

yield

$$(34) \qquad B_d^2 = 1, \quad (B_dB_{d+1})^3 = 1, \quad (B_dB_e)^2 = 1$$
$$\begin{pmatrix} d, e = 1, 2, \ldots, k \\ e \neq d-1, d, d+1 \end{pmatrix};$$

Further, since $\qquad (BC)^{k-1} = 1,$

and so $\qquad (CB)^{k-1} = 1,$

$$(35) \quad C_k = B_{k-1}B_{k-2}\ldots B_2B_1 = C^{-(k-2)}(BC)^{k-2}B = C(CB)^{k-1} = C.$$

The relations 31, 32, 33, 34, 35 show that the group $G'(k : B, C)$ is the group $G(k : B_1, \ldots, B_{k-1})$, perhaps reduced by additional generational relations. The relations (2, 3, 4, 21, 25, 28) of §§ 1, 2 show that $G(k : B_1, \ldots, B_{k-1})$ is the $G'(k : B, C)$, perhaps reduced by additional generational relations. Hence the $G'(k : B, C)$ and the $G(k : B_1, \ldots, B_{k-1})$ are identical.

It should be added that, for $k = 3$, the three relations

$$B^2 = 1, \quad C^3 = 1, \quad (BC)^2 = 1$$

involve the fourth $\qquad (BC^{-1}BC)^3 = 1,$

and thus define the (dihedron) group G_6.

4. *Proof of Theorem* B.

The generational relations of the abstract symmetric group $G(k)$ involve the generators B_d evenly. Hence, the different expressions for an element of the group in terms of the generators involve all either an even or an odd number of generators. We may then speak of even and of odd elements.

There are $\frac{1}{2}k!$ even elements. They constitute the abstract alternating group $G_{\frac{1}{2}k!}$. This group $G_{\frac{1}{2}k!}$ is *within the* $G(k)$ generated by the $k-2$ generators

(36) $$E_d = B_{d+1}B_1 \quad (d = 1, 2, ..., k-2),$$

for we have

(37) $$B_cB_d = B_cB_1 . B_1B_d = E_{c-1}E_{d-1}^{-1} \quad (c, d = 1, ..., k-1),$$

where we have written $E_0 = B_1B_1 = 1.$

These generators E_d satisfy the relations

(38) $$E_1^3 = 1, \quad E_d^2 = 1 \quad (d = 2, 3, ..., k-2),$$

(39) $$(E_dE_{d+1})^3 = 1 \quad (d = 1, 2, ..., k-3),$$

(40) $$(E_dE_e)^2 = 1 \quad \begin{pmatrix} d = 1, 2, ..., k-4 \\ e = d+2, d+3, ..., k-2 \end{pmatrix}$$

(and conceivably, but not in fact, others not derivable from them), and are connected with the generator B_1, which extends the $G_{\frac{1}{2}k!}$ to the $G_{k!} = G(k)$ by the relations

(41) $$B_1^2 = 1, \quad B_1E_d = E_d^{-1}B_1 \quad (d = 1, 2, ..., k-2).$$

Now the abstract group $G\{k\}$ of Theorem B has the $k-2$ generators E_d subject *only* to the generational relations (38, 39, 40). In accordance with Lemmas D and C (§ 1), the correspondence

(42) $$(E_1, ..., E_{k-2}) \sim (E_1^{-1}, ..., E_{k-2}^{-1})$$

defines an involutoric holohedric isomorphism of $G\{k\}$ with itself. Then (Lemma B, § 1) the group $G\{k\}$ of order $O\{k\}$ is extended by the adjunction of a new generator B_1 with the generational relations

(43) $$B_1^2 = 1, \quad B_1E_d = E_d^{-1}B_1 \quad (d = 1, 2, ..., k-2),$$

to a group of order $2O\{k\}$. This new group is generated by the $k-1$ generators $B_1, \ldots, B_{k-1}$, where we set

$$B_{d+1} = E_d B_1 \qquad (d = 1, 2, \ldots, k-2).$$

These generators satisfy the relations prescribed for the generators B_a of the $G(k)$, and conceivably others not derivable from them.

Thus we have

$$\tfrac{1}{2}k! \leqq O\{k\} \quad \text{and} \quad 2O\{k\} \leqq k!.$$

Hence $$\tfrac{1}{2}k! = O\{k\},$$

and the abstract group $G\{k\}$ is indeed the abstract alternating group $G^k_{\frac{1}{2}k!}$.

If, for $k \geqq 5$, we replace the generator E_1 by the generator

$$E_1' = E_1 E_3,$$

we have a system of generators $E_1', E_2, E_3, \ldots, E_{k-2}$, all of which are of period 2. But the generational relations for this system are more complex.

Another system of generators suggests itself, viz., those corresponding to the cyclic substitutions of period 3,

$$(l_1 l_2 l_3), \quad (l_2 l_3 l_4), \quad (l_3 l_4 l_5), \quad \ldots, \quad (l_{k-2} l_{k-1} l_k).$$

The generational relations for this system are still more complex.

Thursday, January 14th, 1897.

Prof. E. B. ELLIOTT, F.R.S., President, in the Chair.

Present—fourteen members and a visitor.

Mr. William Henry Blythe, M.A., late Scholar of Jesus College, Cambridge, and Prof. Eliakim Hastings Moore, B.A. and Ph.D. Yale University, Professor of Mathematics, University of Chicago, were elected members.

Prof. Sylvester spoke at some length "On the Partition of an Even Number into Two Primes," and answered questions put to him by the members present.

Mr. J. J. Walker gave a solution of a certain quadratic vector equation.

The following papers were communicated :—

Supplementary Note on Matrices: Mr. J. Brill.

Some Properties of Bessel's Functions: Dr. Hobson.

Mr. T. I. Dewar exhibited a large number of stereoscopic pictures of the Algebraic Catenary.

The following presents to the Library were received :—

"Nautical Almanac for 1900," 8vo; London, from the Lords Commissioners of the Admiralty, 1897.

"Proceedings of the Royal Society," Vol. LX., Nos. 363, 364.

"Beiblätter zu den Annalen der Physik und Chemie," Bd. xx., St. 11; Leipzig, 1896.

"Nyt Tidsskrift for Matematik," A. Aargang 7, Nos. 5, 6, 7; B. Aargang 7, No. 3; Copenhagen, 1896.

"Proceedings of the Physical Society," Vol. XIV., Pt. 12; December, 1896.

Lorenz, L.—"Œuvres Scientifiques," revues et annotées, par H. Valentiner, Tome I., Fasc. 1, 8vo; Copenhague, 1896.

"Jornal de Sciencias Mathematicas e Astronomicas," Vol. XII., No. 6; Coimbra, 1896.

"Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen," Math.-Phys. Kl., 1896, Heft 3; Geschäftliche Mittheilungen, Heft 2, 1896.

"Bulletin des Sciences Mathématiques," Tome xx., Nov., Dec., 1896; Paris.

"Bulletin of the American Mathematical Society," 2nd Series, Vol. III., No. 3, December, 1896; New York.

"Rendiconto dell' Accademia delle Scienze Fisiche e Matematiche," Serie 3, Vol. II., Fasc. 11; Napoli, 1896.

"Rendiconti del Circolo Matematico di Palermo," Tomo x., Fasc. 6; Nov., Dec., 1896.

Hartmann, J.—"Die Beobachtung der Mondfinsternisse," No. 5, roy. 8vo; Leipzig, 1896.

Fischer, O.—"Beiträge zur Muskelstatik," No. 4, roy. 8vo; Leipzig, 1896.

"Zur fünfzigjährigen Jubelfeier der K. Sächs. Gesells. der Wissenschaften zu Leipzig," roy. 8vo; Leipzig, 1896.

"Educational Times," January, 1897.

"Annals of Mathematics," Vol. XI., No. 1; Sept., 1896.

"Atti della Reale Accademia dei Lincei," Sem. 2, Vol. v., Fasc. 1, Dec. 1896; Roma.

"Indian Engineering," Vol. xx., Nos. 20-25 (wanting 22), Nov. 28-Dec. 19, 1896.

"Astronomical, Magnetic, and Meteorological Observations made during 1890 at the U. S. Observatory," 4to; Washington, 1895.

Cayley, A.—"Collected Mathematical Papers," Vol. XI., 4to; Cambridge, 1896.