LETTERS TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* Nature. *No notice is taken of anonymous communications.*]

Euclid, Newton, and Einstein.

Since the results of the Eclipse Expedition of May last have been made public a very great deal of general interest has been displayed in a theory which, until a few weeks ago, was known only to mathematicians and physicists. Even among these, not many could offer any adequate explanation of the new view of space and time and their mutual relations, while some regarded the whole question as a mathematical joke which led to interesting results of no practical value; and probably not a few thought that a non-Euclidean system of geometry was inadmissible in any physical theory of the universe. On the other hand, there are some who have gone so far as to advocate that non-Euclidean geometry should be taught to boys and girls in secondary schools. The published books on this subject do not come into touch with any ordinary experience, and the whole subject, consequently, has been regarded as a mathematical fiction. So far from this being so, most people have actually seen the ordinary operations of life proceeding in non-Euclidean space, though they have not realised the meaning of all they have seen. In the space behind a plane mirror objects are reversed right and left (perverted), though in all other respects they correspond precisely to the real objects in front of the mirror of which they are the images, but in the space behind a convex mirror this is not the case. The geometry of this space and the behaviour of moving bodies therein, as viewed by the external observer and as studied by an intelligent being within the image space, say, the image of the external observer, who applies to the images and their movements the same standards of measurement as the external observer applies to the real objects in his own space, introduce us to a non-Euclidean space which is the subject of common observation, and prepare the mind for the reception of many of the conclusions of the now famous theory of relativity. In the discussion of that theory two observers are supposed to be moving relatively to one another, each with his own set of measuring instruments and each living in his own world or system, and the differences between the phenomena which occur in each system as measured by the dweller in that system and by the external observer form the basis of the theory. Corresponding to these two observers we propose to consider the actual observer outside the convex mirror and his supposed intelligent image behind the mirror, and to consider how the images behind the mirror, treated as real objects, appear to behave to both observers.

In the first place, it is necessary to consider the size and shape of the objects, or, in other words, the geometry of the space. To save repetition it will be convenient to call the external observer A and his intelligent image B. The line joining the middle point of the mirror with the centre of the sphere of which the surface of the mirror is a part is the axis of the mirror, and may be supposed to be extended indefinitely outside the mirror. The image of an infinitely distant star on the axis of the mirror will be formed at a point half-way between the surface of the mirror and the centre of the sphere. This point is called the principal focus, and its distance from the mirror is the focal length, which is half the radius. It will be convenient to call this point F. A series of lines drawn from the circumference of the mirror outwards and all parallel to the axis encloses a cylindrical space to which the external objects considered are to be confined. All these lines produced indefinitely will at length meet the star on the axis of the mirror. Their images will, therefore, all converge to the principal focus F, and the whole of the infinite cylinder in the external world will correspond to a cone behind the mirror having F for its vertex and the mirror for its curved base. If an object outside moves away to infinity its image will never get beyond F, and the images of straight lines meeting the mirror and extending parallel to the axis as far as the distant star will all meet at F. We shall suppose the radius of curvature of the mirror to be very large as compared with the dimensions of the mirror itself or of the observer.

There is a very simple geometrical law connecting the distance of an object from the mirror and the distance of its image from F. This law need not concern us except to point out that as the object recedes from the mirror its image approaches F, and, as seen by the external observer, the dimensions of the image in all directions at right angles to the axis are proportional to its distance from F, but the dimensions parallel to the axis are proportional to the square of the distance from F of the image. This is the peculiar property of convex mirror space. If a cricket-ball is placed in front of the mirror at a distance equal to the focal length, its image will be half-way between the mirror and F, but the image will not be spherical. In all directions at right angles to the axis the dimensions will be reduced to one-half, but along the axis they will be reduced to one-quarter, so that the sphere will be represented by an oblate spheroid (an orange) with a polar axis one-half of the equatorial diameter. If the ball moves farther from the mirror the oblateness of the spheroid will be increased, and when the image is three-quarters of the way between the mirror and F the polar axis will be only one-quarter of the equatorial diameter of the spheroid, which will itself be only one-quarter of the diameter of the cricket-ball. If a circular hoop is placed with its plane at right angles to the axis its image will be circular, but if it is turned round so that its plane is parallel to the axis the image will be an ellipse, which will become more and more eccentric as the hoop recedes from the mirror and the image diminishes on approaching F. A top set spinning with its axis perpendicular to the axis of the mirror will appear in its image to the external observer to be elliptic, with its axes fixed in space, so that as any line of particles in the top approach parallelism to the axis of the mirror they will be squeezed together and expand again as they recede from parallelism. Midway between the mirror and F the density of the top will appear to A to be twice as great in the direction of the axis as in any direction at right angles to the axis, for the same number of particles will be squeezed into half the length.

All this has been written from the point of view of A, the external observer. But how will all these things appear to B, who is living and moving in the mirror space? Like A, the observer B may use a foot-rule for measuring length, breadth, and thickness, and a protractor for measuring angles. As A proceeds to measure the real object, B proceeds to measure the image, but as he approaches the focus his foot-rule, like himself and the image he is going to measure, gets smaller and in precisely the same proportion, so that if the image measured 6 in. in height when close to the mirror, it would always appear to measure 6 in. in height, for, as seen by A, the foot-rule would contract just as the image contracted, though B would be unconscious of the contraction. Moreover, half-way between the mirror and the focus B's foot-rule will appear to A to be only 6 in. long when held perpendicular to the axis, but when turned parallel to the axis it will appear to A to be only 3 in. long, and if it is turned round it will contract in exactly the same way as the image which it is used to measure. B, therefore, will be quite unable by means of his foot-rule to ascertain that the cricket-ball is no longer spherical, or the top or hoop no longer circular. The judgment of A and that of B will therefore be entirely discordant.

If a circle divided by radii, say 5° apart, into equal angles is held with its plane perpendicular to the axis, the image will appear to both A and B to be circular and the angles equal, but if it is turned with its plane parallel to the axis the image to A will appear an ellipse and the angles in each quadrant unequal, but B will have no means of detecting these inequalities, and he will place implicit faith in the accuracy of his protractor.

The question will naturally be asked: Cannot B see that his circle has become an ellipse? When the plane of the circle is at right angles to the axis and B looks straight at it, the image on B's retina, as it appears to A as well as to B, is circular, but when the circle is turned round and B turns round to look at it, B's retina undergoes precisely the same changes as the circle itself, and still the image occupies the same portion of B's retina as before, and therefore produces the same mental impression of a circle on B, though A recognises the ellipticity of B's retinal image (which A is supposed to see in the mirror).

If A walks straight away from the mirror to an indefinite distance, B will walk towards the focus, but as A can never reach the star, so B, walking, as he thinks, uniformly, can never reach F. In fact, his speed of walking as seen by A appears to diminish in proportion to the square of his distance from F, as all small distances measured along the axis diminish in this ratio, but B can never discover this, for he always appears to walk the same number of feet in a minute, as measured by his own diminishing foot-rule. It is true that when B's height and the length of his legs appear to A to be reduced to one-half, the length of his step appears to be reduced to one-quarter, and the angle between his legs as he walks to be reduced correspondingly; but if B tries to measure this angle, his protractor suffers the same distortion, as recognised by A, and B thinks he is walking always in precisely the same way.

It appears, then, that to B the principal focus F is infinity. He can never reach it, however long or however quickly he walks; and there is nothing in his world beyond it. All straight lines drawn from F to the mirror appear to B to be parallel, for they meet only at infinity, and he can never reach their point of meeting. They correspond to parallel lines in the Euclidean space outside the mirror. The image of a square held with its plane perpendicular to the axis will appear to both A and B to be square, but, held with two of its sides parallel to the axis, the angles of the square will appear to A to be unequal, for the two sides parallel to the axis will converge to F, and the dimensions of the square along the axis will be less than its dimensions at right angles, but neither the foot-rule nor the protractor in the hands of B will detect these irregularities. In convex mirror space straight lines which meet at F are parallel.

If two of the straight lines which appear to B to be parallel are cut by a third line, and the figure is examined by A, the two interior angles on the same side of the cutting line do not appear to be equal to two right angles, and the exterior angle does not appear to be equal to the interior and opposite angle. This is the essential feature of convex looking-glass space, but B will not agree with A on either question. To B, Euclid's propositions respecting parallel straight lines will appear to hold. He will think that he is living in Euclidean space, though A knows better, or thinks he knows.

To the external observer, then, convex looking-glass space has different properties as the focus is approached, or, in technical phrase, it is not homoloidal, and it has different properties in different directions, like a uniaxial crystal—that is, it is not isotropic, but differs from the crystal since its lack of isotropism increases as the focus is approached. The image of a metre rod nine-tenths of the distance from the mirror to the focus will appear to the external observer to measure a decimetre when at right angles to the axis, but only a centimetre when parallel to the axis.

This "distortion" of space is precisely what happens according to the theory of relativity in the neighbourhood of a gravitating body, though the distortion is very small even at the surface of the sun. In the direction of the gravitation pull space is contracted, and a foot-rule is actually shorter than when it lies at right angles to the force to the extent of about 43 parts in 10,000,000 at the sun's surface. The effect is greater the greater the intensity of gravitation, and, consequently, it increases on approaching a gravitating body.

If space is supposed to be occupied by points, and the length of a line to be measured by the number of points in it, then in space free from gravitation the points are equally distributed in all directions, but when gravity acts the points are closer together in the direction of gravity than in other directions, as soldiers in column are closer together from right to left than from front to rear, or as the images of evenly distributed points in space are more closely packed along the axis of a convex mirror than in other directions. This representation of the effect of gravity is due to Prof. Eddington. Light always goes from one point to another in the shortest possible time. This principle leads to the ordinary laws of reflection and refraction. In passing through space in the presence of gravitation it will take the path which necessitates passing through the smallest number of spatial points, and this means refraction similar to that produced when it passes into a denser medium in which its velocity is reduced. The effect on light in passing near to the sun will be the same as if the sun were surrounded by an atmosphere extending to a distance of many millions of miles, and diminishing in density as the distance from the sun is increased. This will act like a convex lens refracting the light, which will travel more slowly as it approaches the sun. A comet approaching the sun with the velocity of light would, according to the laws of Newton, travel more quickly as it approached, but its orbit would be bent towards the sun as the light is bent, but only to one-half the extent. If light from a star were passing the sun close to its limb, and behaved like a comet under the sun's attraction, it would be deflected about seven-eighths of a second of arc. On the theory of relativity it would be deflected through $1\frac{3}{4}$ seconds. It was this deflection which the Eclipse Expedition set out to measure. The behaviour of comets shows that there is no solar atmosphere to account for the refraction at distances from the sun at which the refraction was observed.

In all that has been said respecting the space behind a convex mirror the size of the mirror is supposed to be very small as compared with its radius of curvature, and the objects and images much smaller still. If a complete spherical mirror is suspended in free space the geometrical images of the stars will be distributed over a sphere of half the radius of the mirror, and this spherical surface is infinity to all the dwellers in the mirror space. The image of an object which subtends a large angle at the centre of the mirror will be bent. In Fig. 1, *ab*, *cd*, and *ef* are the images of straight lines all passing through the same point distant half the radius from the face of the mirror. These lines are all curved and concave to the centre of the mirror, but they are straight lines in convex mirror space, and pass through the smallest number of spatial points of any line joining the extreme points. They are the paths which would be taken by rays of light in space in which the spatial points were packed as in convex mirror space. In every case the light is refracted towards the portion of space in which the point density is greatest. In the figure PQ represents the mirror, RS the focal sphere of half the radius, while the images correspond to straight lines cutting FA produced in the same point at 90°, 45°, and $22\frac{1}{2}^\circ$ respectively. It will be seen that the curvature of *ab* enables it to pass through a region in which the points are less closely packed than along the line joining *a* and *b*, which appears to the external observer to be straight. On the Einstein theory, light passing a gravitating body like the sun is refracted in the same way. In convex mirror space strings stretched between the points *a* and *b*, *c* and *d*, and *e* and *f* would take the forms shown. A person in a hurry and endeavouring to pass through a crowd will make a detour to avoid the more densely packed portions of the crowd.

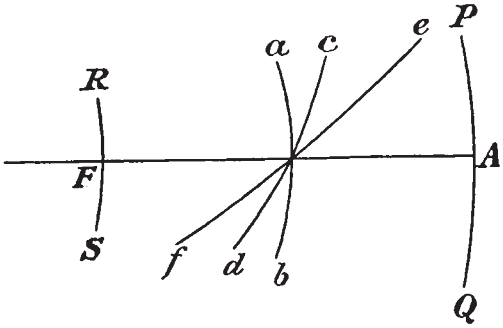

FIG. 1.

According to the theory of relativity, motion and force, involving time, change the properties of space. In convex looking-glass space position and direction only are involved, so that the problem is much simpler, while many of the results are very similar.

If the two great mechanical principles of the conservation of momentum and the conservation of energy are applied to the movement of bodies in B's space a consistent system of dynamics can be constructed, and B with his measuring instruments will be quite unable to detect any divergence from Newton's laws of motion. To A, however, the laws will appear very different. For example, a body under the action of no external force moving along the axis of the mirror will move with a velocity varying as the square of its distance from F. This means that the apparent mass will vary inversely as the square of the distance of the body from F, and as the body approaches F the mass appears to increase indefinitely. This corresponds to the increase of mass according to the theory of relativity when the velocity of a body increases, becoming infinite as the velocity of light is approached. According to the theory of relativity, the mass of a body is greater in the direction of its motion than in directions at right angles to its direction of motion. In convex looking-glass space the mass is greater, when measured by the accelerative effect of a force, in the direction of the axis than in directions at right angles to the axis, and greater the nearer the focus. The reason why B cannot detect any of these changes is that all his standard units change in the same way; and, as all physical measurements ultimately reduce themselves to a comparison with standard units, if the units change a corresponding change in the quantity measured cannot be detected. We cannot, for instance, detect the variation in the weight of a body between the equator and the poles by means of standard weights and a pair of scales, though we may detect it by a spring-balance or a pendulum. It is always the looker-on, A, who sees most of the game.

Some thirty or more years ago a little *jeu d'esprit* was written by Dr. Edwin Abbott entitled "Flatland." At the time of its publication it did not attract as much attention as it deserved. Dr. Abbott pictures intelligent beings whose whole experience is confined to a plane, or other space of two dimensions, who have no faculties by which they can become conscious of anything outside that space and no means of moving off the surface on which they live. He then asks the reader, who has consciousness of the third dimension, to imagine a sphere descending upon the plane of Flatland and passing through it. How will the inhabitants regard this phenomenon? They will not see the approaching sphere and will have no conception of its solidity. They will only be conscious of the circle in which it cuts their plane. This circle, at first a point, will gradually increase in diameter, driving the inhabitants of Flatland outwards from its circumference, and this will go on until half the sphere has passed through the plane, when the circle will gradually contract to a point and then vanish, leaving the Flatlanders in undisturbed possession of their country (supposing the wound in the plane to have healed). Their experience will be that of a circular obstacle gradually expanding or growing, and then contracting, and they will attribute to *growth in time* what the external observer in three dimensions assigns to motion in the third dimension. Transfer this analogy to a movement of the fourth dimension through three-dimensional space. Assume the past and future of the universe to be all depicted in four-dimensional space and visible to any being who has consciousness of the fourth dimension. If there is motion of our three-dimensional space relative to the fourth dimension, all the changes we experience and assign to the flow of time will be due simply to this movement, the whole of the future as well as the past always existing in the fourth dimension.

The theory of relativity requires a fourth dimensional term to be introduced into its dynamical equations. This term involves time and the velocity of light. Generally, the easiest method of expressing algebraically position and motion in three-dimensional space is by reference to three directions mutually at right angles, like the edges of a cube which meet at one corner. These lines may, for example, be drawn through the observer north and south and east and west, like the reference lines on a map, while the third line is up and down. The observer's point of reference is where these three lines meet. In four-dimensional geometry there is a fourth direction at right angles to each of the three. Most of us are unable to form any clear picture of such a direction as a purely geometrical conception. To us the only figure which is at right angles to every straight line drawn through a point O is a sphere, or any number of spheres, having O as centre. As stated above, the fourth co-ordinate involves time and the velocity of light together. Imagine these spheres to be always moving inwards towards O with the velocity of light, and then to expand again from O with the same velocity, and this to take place quite uniformly, however O may move in relation to other points of observation, so that the centre of the system of contracting and expanding spheres travels with the observer, and each observer has his own system of spheres. The approaching and contracting spheres contain within them the whole future; the receding and expanding spheres contain the past. The present is the passage of a sphere through O, the observer, when that sphere is concentrated on a point. This conception of a fourth dimension is thus not that of a simple spatial dimension like the other three, but, as required in the theory of relativity, it is intimately associated with time and motion, and the observer's experience of it is simply the happening of events with the flux of time. It is very like the Flatlander's conception of the third dimension derived from the invading sphere. It will be noticed that to different observers the impressions of the present are not quite the same. We observe an event in a star. It is present to us. To an observer in the star it happened years ago.

The theory of relativity involves a change in the unit of time, according to the motion of the observer relative to the object observed. This complication did not enter into the consideration of the space behind the convex mirror, so that the dynamical problems in that space were relatively simple. According to the theory of relativity, if the observer is moving with the velocity of light, time remains unchanged. This must have been the case with the Mad Hatter. With him it was always six o'clock, and always tea-time.

W. G.

Thermionic Valves on Aircraft.

In a paper just published in the Proceedings of the Royal Society (A, vol. xcvi.) Drs. W. H. Eccles and J. H. Vincent give an account of some experiments on the small variations of wave-length introduced when changes are made either in filament temperature or plate voltage of a thermionic valve supplying oscillating energy to a wireless circuit. It may be of interest to readers of Nature to know how this effect influenced the design of wireless aircraft generators used in the war.

In 1916, when experimenting with continuous-wave telegraphy and telephony from aircraft, I noticed a small outstanding variation of wave-length radiated from an aeroplane, which variation seemed to depend mainly on the speed of flight, and therefore, possibly, on the voltages supplied by the windmill-driven generator.

Following up this clue, I found in the Air Force Laboratory that the changes of wave-length introduced by variations of filament temperature and plate voltage were more considerable than I had thought, especially on short wave-lengths.

It was the knowledge of this fact which led to the inclusion of special regulating devices in the aircraft dynamo circuits, so that the wave-length variation, at the best of times noticeable owing to aerial sway, banking, etc., should be reduced, at any rate, to a minimum.

R. Whiddington.

The University, Leeds, February 5.

Popular Science.

I should like to be allowed to underline a few remarks that occur in a review entitled "Scientific Biography" in Nature for January 29. The writer urges that science has neglected the populace and offered its wares for popular edification in a highly unedifying way. I believe this is very true. I am old enough to remember different times, and can recall with truth and gratitude the feeling of enthusiasm, and even of exaltation, which I had in early days on hearing or reading popular science lectures. I think of Huxley, Tyndall, Clifford, W. B. Carpenter, Lockyer, Roscoe, and some others. Science lectures then were aimed at showing how science did its work, and they brought into view something of the personality of the real scientific worker.

Remembering how much I had gained, I endeavoured in my turn to carry on the good work within the much-restricted range of my own powers, but in the same spirit. In time I realised two things: one, the debilitating tendency of publicity and easily won applause; the other, the invasion of the science platform by the mere entertainer and his *entrepreneur*. The work became suspect to all self-respecting people. The degenerated Press has completed the havoc.

Is it not possible to improve matters? I believe it is. No doubt some knowledge of science is more prevalent than it was, but there is yet ample room for the simple, popular lecture of the genuine kind by men who are the real workers. It is a serious tax, but I am inclined to think a justifiable one, on the time of these men to give, say once a year in some large city, a really popular account of their latest discoveries and have it printed to sell at a popular price. That, and a vocal public opinion in the world of science against comic, pyrotechnic, mystic, or other profane tickling of the groundlings, might do much in a good cause.

Victorian.

Mirage Effects.

The mirage effect noticed by Mr. Quilter and Miss Botley is very common on Woolacombe Sands, especially on hot, sunny days when the observer is looking south. The apparently wet patch keeps at a half to three-quarters of a mile's distance from the eye, but does not persist up to the southern limit of the bay, which is bounded by high ground. I cannot remember whether it is visible when the observer is facing north.

Spencer Pickering.

Mirage effects similar to those referred to in Nature of January 29 (p. 565) have been noticed by me several times in Birmingham on tarred macadam or wood-block roads. The effect on a hot, sunny day is of a layer of water from 2 in. to 4 in. deep on the surface of the roadway, immersed in which are the feet of pedestrians and the wheels of vehicles about a hundred yards from the observer. The effect is best seen when the line of sight nearly coincides with the surface of the roadway, as, for instance, just before one breasts the summit of a slight rise, when the eye is practically level with the ground beyond the top of the rise. Stooping would produce a similar effect.

L. N. Norris-Rogers.

I first saw a mirage on a road in Colombo, and wondered how I was going to cross the apparent sheet of water in front of me. Since then I have seen it repeatedly in England, and instinctively look for it when the conditions are right. For the best effects these conditions are three: (*a*) Tarred roads (the reason is obvious); (*b*) bright sun; and (*c*) a slight gradient rising from the observer.

In very hot weather (*c*) may not be so necessary. At other times the mirage appears where the gradient reaches towards the level of the eyes. It is very clear, and reflections are as sharp as in water, especially of objects crossing near the further edge.

Harry Hillman.

117 Colmore Row, Birmingham.