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LVII. *On the Dimensions of a Magnetic Pole in the Electrostatic System of Units. (Second Article.)* By Prof. J. D. EVERETT\*.

MY appeal for an explicit statement of Maxwell's definition of the unit pole in the electrostatic system has brought me several communications from correspondents; and the diversity between them is a sufficient proof of the necessity for such an appeal. My correspondents do not refer me to any explicit definition by Maxwell himself, but give investigations which they regard as substantially his and which lead to his result. Two of these investigations seem quite satisfactory, and show that Maxwell's result can be obtained with as much simplicity as that of Clausius. They are given under the heads I., II., below.

Before entering on the points in dispute with respect to the electrostatic system, I may premise that the definitions of the unit quantity of electricity, the unit current, the unit electromotive force (or difference of potential), and the unit resistance (all of which may be called purely electrostatic definitions), are not in question, but are accepted by all parties. The divergence begins when we attempt to express magnetic quantities in an electrostatic system; and different results may be obtained according to the particular relation between magnetism and electricity which we select as the guiding principle in our definitions. In a strict sense there is no such thing as an electrostatic unit of any magnetic quantity; since magnetism and its relations to electricity lie outside the domain of electrostatics.

There are three laws of nature any one of which may be used to connect electrical with magnetic units.

I. The *galvanometer law*, as I may for brevity call it, because it is the law which determines the force which a current passing through the coil of a galvanometer exerts upon either pole of the needle. This law, stated without any assumption as to units, is that the force varies directly as the length of the wire, the strength of the current, and the strength of the pole, and inversely as the square of the distance of the wire from the pole. We must therefore, in every system, have

Force =  $k_1 \times \text{Current} \times \text{Pole} \times \text{Length} \div (\text{Distance})^2$ , (1)  
 $k_1$  being a factor which depends on the system employed. Maxwell's unit pole may be defined by making  $k_1 = 1$ . This gives, for determining the dimensions of a pole,

$$\text{MLT}^{-2} = \text{Current} \times \text{Pole} \times \text{L}^{-1}.$$

\* Communicated by the Author.

But

$$\text{Current} = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2};$$

therefore

$$\text{Pole} = M^{\frac{1}{2}} L^{\frac{1}{2}},$$

which is Maxwell's result. This proof was supplied to me by Professor Larmor. Clausius alludes to such a proof as having been given by Maxwell, and objects to it on the ground that the force which a current exerts upon a pole is not an electrostatical but an electrodynamical force. But, inasmuch as this objection applies equally to all definitions of "the unit pole in the electrostatic system," not excepting that offered by Clausius himself, it cannot be admitted as valid when we are discussing the merits of one as against another.

Again, we may employ

II. The *magneto-electric law*, which determines the electromotive force produced by moving a conductor in a magnetic field. This law, when stated without any assumption as to units, is that the electromotive force is directly as the length of the conductor, the velocity of its motion resolved in a certain direction, and the intensity of the field. Hence, bearing in mind that the intensity of the field due to a single pole is directly as the strength of the pole and inversely as the square of the distance, we must have, in every system,

$$\begin{aligned} \left. \begin{array}{l} \text{Electromotive} \\ \text{force} \dots \end{array} \right\} &= k_2 \times \text{Length} \times \text{Velocity} \times \text{Pole} \div (\text{Distance})^2 \quad (2) \\ &= k_2 \times L \times LT^{-1} \times \text{Pole} \times L^{-2} \\ &= \text{Pole} \times k_2 T^{-1}. \end{aligned}$$

If we define our unit pole by the condition  $k_2=1$ , we have

$$\begin{aligned} \text{Pole} &= \text{Electromotive force} \times T \\ &= M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \times T \\ &= M^{\frac{1}{2}} L^{\frac{1}{2}}. \end{aligned}$$

This mode of obtaining Maxwell's result was in substance supplied to me by Professor Fitzgerald. So far we have no discrepancy.

On the other hand, we may employ with Clausius,

III. The *law of the magnetic shell*, which asserts the equivalence of a current to a magnet. Taking the simplest case—that of a current in a plane circuit—the law of nature is that the moment of the equivalent magnet is jointly proportional to the strength of the current and the area of the circuit. Hence, since the moment of the magnet is the product of the

strength of either pole by the distance between the poles, we must have, in every system,

$$\text{Pole} \times \text{Length} = k_3 \times \text{Current} \times \text{Area}; \quad . \quad . \quad (3)$$

that is,

$$\text{Pole} = k_3 \times \text{Current} \times L,$$

where  $k_3$  depends on the system employed. Clausius defines his unit pole by making  $k_3=1$ , and thus obtains

$$\begin{aligned} \text{Pole} &= \text{Current} \times L \\ &= M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \times L \\ &= M^{\frac{1}{2}} L^{\frac{5}{2}} T^{-2}. \end{aligned}$$

This result disagrees with that of the two preceding investigations; and hence the two equivalent assumptions  $k_1=1$ ,  $k_2=1$  are inconsistent with the assumption  $k_3=1$ . Maxwell has chosen the former alternative, Clausius the latter. On Maxwell's system we have

$$k_1=1, \quad k_2=1, \quad k_3=T^2 L^{-2}.$$

On Clausius's system we have

$$k_1=T^2 L^{-2}, \quad k_2=T^2 L^{-2}, \quad k_3=1.$$

In fact it can be shown that, if  $v$  be the ratio of the electromagnetic to the electrostatic unit of quantity of electricity,  $k_3$  would be  $\frac{1}{v^2}$  on Maxwell's system, and  $k_1$  and  $k_2$  would each be  $\frac{1}{v^2}$  on Clausius's system. When we bear in mind how frequently laws I. and II. are applied in practical calculation, and how extremely rare is any practical application of law III., it seems clear that, if we were driven to employ an electrostatic system in calculations relating to magnetism, the best choice we could make would be Maxwell's.

It is further clear that electrostatic systems are essentially inconvenient for calculations relating to electromagnetism. The electromagnetic system makes  $k_1$ ,  $k_2$ , and  $k_3$  each unity, and also gives the value unity to the factor  $k_4$  which occurs in the general expressions for the attractions and repulsions between currents. For example, the force with which two parallel currents, one of very great length and the other of length  $l$ , attract or repel each other, is given in any system by the formula

$$\text{Force} = k_4 \times \text{Product of currents} \times 2l \div \text{Distance}, \quad . \quad (4)$$

where  $k_4$  has the value unity in the electromagnetic system, and the value  $\frac{1}{v^2}$  in the electrostatic system, there being here no difference between Maxwell and Clausius.

The mutual force of two magnetic poles is

$$k_5 \times \text{Product of poles} \div (\text{Distance})^2, \quad \dots \quad (5)$$

where  $k_5$  is  $v^2$  in the electrostatic system of Maxwell, and  $\frac{1}{v^2}$  in that of Clausius, but is defined as unity in the electromagnetic system.

On the other hand, the mutual force of two charges of electricity is

$$k_6 \times \text{Product of charges} \div (\text{Distance})^2, \quad \dots \quad (6)$$

where  $k_6$  is defined as unity in the electrostatic system, but has the value  $v^2$  in the electromagnetic system.

A comparison of the foregoing six equations shows how cautious we ought to be in asserting that a particular dimensional relation "must hold in every system of units." The laws of nature which connect dissimilar quantities are laws of *proportion*, and it is only by convention that they can be stated as laws of *equality*. Clausius was in error in the assertion, which I adopted from him in my last communication, that in every system the product of the current by the area enclosed must be *equal* to the moment of the equivalent magnet. *Proportional* would be the correct word; and the factor  $k_3$ , which remains constant as the current and area vary, does not necessarily retain the same value when we pass from one set of units of length and time to another. Maxwell says (§ 482):—"It has been shown by numerous experiments . . . . that the magnetic action of a small plane circuit . . . . is the same as that of a magnet . . . whose magnetic moment is equal to the area of the circuit multiplied by the strength of the current;" but the passage occurs in a discussion in which the "Electromagnetic system of measurement" is employed and defined (see § 479). I do not think that the charge of mistake brought against Maxwell has been substantiated. The controversy, however, has done good in exposing the difficulties and dangers which lurk in applications of electrostatical units to magnetism.