



265. The Intersection of an in-Conic and Polar Circle

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and $ZN = ZN'$. Therefore angle $NZN' = \text{twice } DZN' = 2E'$. From the right-angled triangle $II'E'$, angle $IZI' = 2E'$. Thus angle $NZN' = IZI'$ and therefore angle $IZN = I'ZN'$. The triangles $IZN, I'ZN'$ are therefore congruent and $NI = N'I' = DN' - DI' = \frac{1}{2}R - r$.

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II. Let DN meet AE in L ; so that $DL = ED = DE'$ and $E'L$ is perpendicular to AE . ZIL is an isosceles triangle and DZ being parallel to IL touches at Z the circumcircle of ZIL . But $DZ^2 = DN \cdot DL$, therefore N lies on the same circumcircle, and angle $INL = IZL$. Let the circle on IE' as diameter cut DL in K . Then $DK \cdot DL = DI' \cdot DE'$; but $DL = DE'$ and therefore $DK = DI'$. Moreover, angle $IKL = IE'L = \frac{1}{2}IZL = \frac{1}{2}INL$, so that

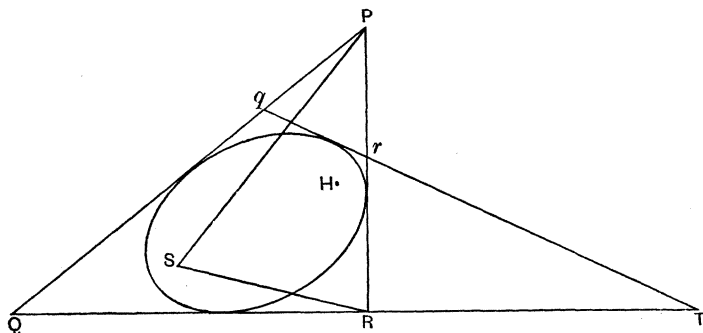
$$NI = NK = DN - DK = \frac{1}{2}R - r.$$

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265. [I¹. 17. e; K. 2. c.] *The intersection of an in-conic and polar circle.*

Let QRq be a quadrilateral circumscribed to an ellipse of foci S, H ; T, P the intersections of (QR, qr) and (Qq, Rr) .



Then if the tangents from Q, R, r, q subtend respectively angles $\alpha, \beta, \gamma, \delta$ at S

$$PSq = \frac{1}{2}(2\gamma + 2\delta) - \delta = \gamma,$$

$$TSR = \frac{1}{2}(2\beta + 2\gamma) - \beta = \gamma,$$

or $PSq = TSR$. When qr is parallel to QR , T goes to infinity and $PSq = SRQ = HRP$. The triangles PqS, PHR are therefore in this case similar, and $Pq \cdot PR (= Pr \cdot PQ) = SP \cdot HP$.

This theorem is due to Mr. E. P. Rouse; and is to be found in an article on "The Director Circle of an Inscribed Conic" in No. 2 of the *Mathematical Gazette*, July, 1894.

Mr. Rouse's application of this property to prove that the director circle of a conic inscribed in a triangle intersects orthogonally the polar circle of the triangle is here given for the benefit of a younger generation:

Draw RM (perpendicular to PQ) to meet PD (perpendicular to QR) in E the orthocentre of PQR .

From the well-known formula for the cosine of the angle between the tangents from an external point P (as given in Salmon and elsewhere)

$$\begin{aligned} 2SP \cdot HP \cos QPR &= SP^2 + HP^2 - 4CA^2 \\ &= 2(CP^2 + CS^2) - 4CA^2 \\ &= 2\{CP^2 - (CA^2 + CB^2)\}. \end{aligned}$$

