

No. 63 *—"The Theory of Centrifugal Pumps, as supported by Experiment."¹ By the Hon. R. CLERE PARSONS, B.A., B.C.E. Dublin, Stud. Inst. C.E.

CENTRIFUGAL pumps are by no means a modern invention; the crude idea of which probably dates as far back as the middle of the last century, when the mathematician Euler brought out a primitive form of centrifugal pump, an account of which he published in the Proceedings of the Academy of Berlin for 1754. This pump is referred to by M. Combes,² the eminent French engineer, and also its theory, as deduced by Euler; but, as it has never come into practical use, it has probably failed, like most of the mechanical inventions of that great mathematician. From that period many rotatory and centrifugal pumps were invented, principally by French engineers, but none of them seems to have yielded even a reasonably good efficiency. The first mention³ of a centrifugal pump at all to be compared with those of the present day is in the year 1830, when one was erected by Mr. McCarty in the Naval Yard at New York, and some improvements upon this machine were patented by him in the following year. No experiments upon this pump seem to have been published, consequently it is uncertain whether it yielded a good efficiency. The next epoch in the history⁴ of the centrifugal pump is the Exhibition of the year 1851, when the late Mr. Appold achieved the great success with his pump of trebling the efficiency obtained by any other exhibitor. Mr. Appold, by making numerous experiments, at length determined that the efficiency mainly depended upon the form of the blades in the fan, and, further, that the best form was a curved blade pointing in the opposite direction to that in which the fan revolved (Plate 6, Fig. 1). Two other forms of fan which he tried were, one with straight radial blades, and another with straight blades inclined at an angle of 45° to the circumference (Plate 6,

¹ This communication was read and discussed at a meeting of the Students on the 11th of February, 1876, and was awarded the Miller Scholarship in the Session 1875-76. Mr. Parsons was subsequently elected an Associate of the Institution.

² "Recherches théorétiques et expérimentales sur les roues à réaction ou à tuyaux."

³ "Report of Juries of International Exhibition of 1851."

⁴ *Ibid.*

Figs. 2, 3). Both these fans yielded efficiencies far inferior to the one with curved blades.

Another detail which affects the efficiency, but not to the same extent as the form of the blades of the fan, is the form of the casing surrounding the fan. The old theory which Morin, Appold, and many others held, and which is still advocated in some recent books, viz., that as long as the casing outside the fan is large enough it is immaterial of what shape it is, can be proved to be false both by theory and experiment. In January 1875 Mr. Anderson, M. Inst. C.E., of the firm of Messrs. Eastons and Anderson, deputed the Author, in company with Mr. Hesketh, Stud. Inst. C.E., to make some experiments upon centrifugal pumps, both with a view to determine the laws which regulate their discharge, and at the same time, if possible, to improve their efficiency. The first experiments were made upon a pump whose suction and discharge pipes were 10 inches in diameter. The fan was 14 inches in diameter, and the casing surrounding the fan was circular, as is shown in Plate 6, Figs. 4, 5.

This pump was driven by a single-acting engine working directly on to the shaft of the pump. The water was raised by the pump from a large cast-iron tank, and discharged back into it through a measuring tank. In the bottom of the measuring tank was a hole in a thin sheet-iron plate, and by the head of water maintained over this hole the discharge was calculated by the formula

$$Q = 0.62 \times A \times \sqrt{2gh} \text{ cubic feet per second;}$$

where A = area of hole in square feet,

h = head of water in feet over hole.

The revolutions per minute of the fan were measured by a counter attached to the shaft, and the lift was determined by means of a staff fastened to a float in the lower tank. The staff was graduated in feet and decimals of a foot, and was applied to a water-gauge glass connected with the discharge-pipe. In order to take account of the residual velocity of the water in the discharge-pipe in estimating the lift; the tube with which the water-gauge was connected had a bend whose extremity met the water flowing in the discharge-pipe; consequently the water in the gauge-glass stood at the same height as the water issuing from the discharge-pipe. The engine was supplied with steam from a portable boiler, into which the feed-water was injected by a hand-pump. During an experiment the speed of the engine was regu-

lated by an observer, so as to maintain a constant head of water in the measuring tank, and consequently insuring a uniform discharge from the pump. An experiment was continued until 30 gallons of water were evaporated in the boiler, and the time required to accomplish this was accurately noted. Then the number of lbs. of water raised by the pump per minute, divided by the number of lbs. of water evaporated in the boiler per minute, is proportional to the efficiency of the pump. These experiments were repeated for different discharges, and tabulated in the following form :—

TABLE 1.

Gallons discharged per Minute.	Lift in Feet.	Foot-lbs. raised per Minute.	Foot-lbs. raised per lb. of Water evaporated.	Revolutions per Minute.	Remarks.
..	5·667	305	Appold's fan in circular casing.
745	6·000	44,886	7,779	363	
879	6·250	54,937	8,244	380	
989	6·666	65,936	11,883	393	
1,153	7·000	80,710	11,385	416	

The casing round the fan was next altered from the circular to the spiral form, as is indicated by the dotted lines in Plate 6, Fig. 4, by fitting wooden blocks inside; and a fresh series of experiments was now made under circumstances resembling the former set as much as possible. The following are the results, tabulated in a similar form :—

TABLE 2.

Gallons discharged per Minute.	Lift in Feet.	Foot-lbs. raised per Minute.	Foot-lbs. raised per lb. of Water evaporated.	Revolutions per Minute.	Remarks.
..	5·750	320	Appold's fan in spiral casing.
577	6·500	37,505	8,960	346	
746	6·925	51,600	10,809	363	
878	6·750	59,265	11,264	368	
999	7·085	70,029	12,088	387	
1,150	7·750	89,125	13,248	403	
1,288	8·333	107,329	15,996	423	

Thus, by comparing the first and second tables, it will be noticed how greatly the spiral casing has improved the efficiency, and also increased the discharge, the boiler pressure remaining the same. With the same casing, but another fan, designed on the principles

laid down by Rankine in his "Applied Mechanics," and also advocated by Glynn in his treatise on Water Power, some experiments were made similar to the preceding ones.

The following tables show the results of these experiments:—

TABLE 3.

Gallons discharged per Minute.	Lift in Feet.	Foot-lbs. raised per Minute.	Foot-lbs. raised per lb. of Water evaporated.	Revolutions per Minute.	Remarks.
..	5·500	300	Rankine's fan in circular casing.
578	5·925	34,200	7,203	316	
741	6·167	45,695	8,556	335	
880	6·333	55,733	8,377	348	
993	6·583	65,372	10,748	355	

TABLE 4.

Gallons discharged per Minute.	Lift in Feet.	Foot-lbs. raised per Minute.	Foot-lbs. raised per lb. of Water evaporated.	Revolutions per Minute.	Remarks.
..	5·416	300	Rankine's fan in spiral casing.
580	6·333	36,731	9,675	324	
743	6·667	49,528	10,857	334	
879	7·000	61,530	11,692	343	
996	7·333	73,036	12,954	353	

These results prove that Rankine's fan is far inferior to that of Appold. The blades of this fan were for half their length, from the centre outwards, similar to those of Appold; but for the remaining half of their length they curved forwards in the direction in which the fan revolves, and ended in radial tips (Plate 6, Fig. 6). This method of estimating the efficiency of a pump is by no means an accurate one; but in altering a pump it is a convenient way of determining whether the alteration has proved a success or the reverse.

A new casing was now designed, of a spiral form (Plate 6, Fig. 7), and a fresh series of experiments undertaken. These experiments were of a much more elaborate nature than those just described, and as it may make the deductions from them more easily understood, their general arrangement will be described. In order, as far as possible, to insure a constant lift, the pump was supported on the side of a large barge, and the engine for the purpose of driving it was placed in the bottom. The power transmitted to the pump was estimated by the dynamometer used by Messrs.

Eastons and Anderson at the Royal Agricultural Society's shows. The method of measuring the amount of water raised by the pump described in the former experiments was repeated in this instance; the water being raised from the Thames, in which the barge was floating, and discharged back into it through the measuring tank. The lift was measured by taking the difference of the levels of the water in the river and that in the discharge-pipe, as shown by two gauge-glasses. The revolutions of the pump were indicated by a counter attached to the pump-shaft. A number of experiments were made upon this pump, and the results tabulated as follows:—

TABLE 5.

No. of Experiments.	Gallons of Water discharged per Minute.	Lift in Feet.	Foot-lbs. raised per Minute.	Foot-lbs. indicated per Minute.	Revolutions per Minute.	Efficiency per Cent.	Corrected Efficiency per Cent.
1	1,012	14·67	148,461	298,438	392	49·74	58·57
2	1,108	14·70	162,875	317,158	394	51·35	59·99
3	1,197	14·65	175,364	332,136	395	52·80	61·08
4	1,280	14·70	188,160	343,754	398	54·74	62·99
5	1,350	14·75	199,128	357,194	399	55·75	63·78
6	1,431	14·75	211,073	374,954	400	56·20	63·95
7	1,501	14·70	220,650	388,897	402	56·69	64·16
8	1,568	14·75	231,280	404,737	403	57·01	64·29
9	1,630	14·80	241,240	409,612	404	58·90	66·19
10	1,695	14·75	251,987	419,790	405	60·17	67·18
11	1,753	14·80	259,450	435,630	406	59·42	66·39
12	1,012	17·40	176,088	370,458	424	47·53	54·06
13	1,108	17·20	190,576	388,316	425	48·97	55·56
14	1,197	17·20	205,884	404,156	427	51·09	57·33
15	1,280	17·30	221,440	417,214	428	53·08	59·51
16	1,350	17·30	233,550	433,054	429	53·93	60·19
17	1,431	17·40	248,994	447,552	431	53·63	61·86
18	1,501	17·40	261,174	460,512	432	56·71	62·47
19	1,568	17·40	272,832	471,552	433	57·86	63·97
20	1,630	17·60	286,880	479,810	434	59·79	65·98
21	1,695	17·60	298,310	486,050	435	61·37	67·64
22	1,753	17·60	308,528	494,210	436	62·43	68·63
23	1,012	11·81	119,517	238,603	..	50·09	..
24	1,197	11·80	141,246	268,970	..	52·51	..
25	1,350	11·83	159,681	297,829	..	53·61	..
26	1,501	11·83	177,568	321,918	..	55·16	..
27	1,630	11·92	194,196	341,127	..	56·93	..
28	1,753	12·00	210,360	357,007	..	58·92	..
29	2,029	12·33	250,175	416,957	..	60·00	..
30	2,301	12·12	278,881	463,268	..	60·13	..
31	2,544	12·17	309,604	503,759	..	61·46	..
32	2,765	12·17	336,500	553,346	..	60·80	..
33	2,933	12·75	373,867	583,870	..	64·04	..

Some experiments upon a vertical-spindle centrifugal pump will now be described.

The shaft or spindle in these pumps is vertical, as is shown in Plate 6, Fig. 8, and the fan revolves in a horizontal plane. Outside the fan is a circular casing attached to the bottom of a wrought-iron tube. The water enters the fan on one side only—in this case below—and is discharged into the casing; from thence it is carried in the wrought-iron tube to the height required, and is discharged through the orifice in the side. The wrought-iron tube both performs the office of a discharge-pipe, and serves as a support for the bearings of the vertical shaft. Above the fan, and attached to the casing, are six guide-blades, shown in section in Plate 6, Fig. 10 and dotted lines in Fig. 8, which receive the water escaping from the fan with a high tangential velocity, and conduct it up the vertical tube. In order to render the explanation of this pump more lucid the working parts of the fan are shown in black when they occur in section, whilst the stationary parts of the case are indicated in the ordinary way. In this form of pump the vertical pressure upon the fan arising from the column of water over it is entirely removed: *ff* are two holes, 1 inch in diameter, in the top of the fan, which admit the water into the space between the support of the centre bearing and the fan, and consequently the pressures on both sides of the fan are equalised. The Appold fan, used in the experiments upon this pump, is shown in Fig. 9, and the one made upon Rankine's principle was similar to that in Fig. 6. The pump was placed on the side of the barge in the same position as the pump last described, and the experiments were carried out in an exactly similar manner, and the results obtained are tabulated as follows:—

TABLE 6.

No. of Experiment.	Gallons discharged per Minute.	Lift in Feet.	Foot lbs. raised per Minute.	Foot-lbs. indicated per Minute.	Revolutions per Minute.	Efficiency per Cent.	Remarks.
1	3,079	8·1	249,399	559,600	206	44·3	Appold's fan.
2	3,343	„	270,783	576,460	208	46·9	
3	3,578	„	289,818	617,850	210	47·0	
4	3,821	„	309,501	674,600	214	49·9	
5	4,252	„	344,412	729,900	217	47·2	
6	4,437	„	359,397	766,830	218	46·6	
7	4,634	„	375,354	745,830	220	50·3	
8	4,810	„	383,610	795,700	220	48·9	
9	5,074	7·1	360,254	777,050	215	45·3	
10	3,079	6·2	190,898	503,000	163	37·9	Rankine's fan.
11	3,343	„	207,266	554,500	165	37·3	
12	3,578	„	221,836	578,000	165	38·3	
13	3,821	„	236,902	610,750	166	38·7	
14	4,042	„	250,604	656,750	167	36·6	
15	4,252	„	263,624	680,600	168	39·6	

These experiments also show the great superiority of Appold's fan over that proposed by Rankine.

The dynamical forces, produced in the centrifugal pump when it is in motion, will now be considered; and afterwards it will be shown how the theoretical investigations are supported by the experiments.

There are two totally different conditions in which a centrifugal pump may be situated while it is rotating. One in which it is revolving just fast enough to raise the water up to the discharge-pipe, and no further; and another in which it is revolving slightly faster, and discharging water out of this pipe. In the first case there is only centrifugal force, which is produced by the water in the fan rotating, that maintains the column of water in the discharge-pipe. In the second case this force is still produced, but in addition to it another, which may be called the force of impact, or in other words, the force with which the blades of the fan impinge against the water discharged by the pump.

The centrifugal force in the first case may be calculated as follows:—Assuming that the fan is a cylinder of water; every particle of this water as it rotates exerts a force outwards from the centre; consequently the force exerted at the circumference, or that which maintains the head in the discharge-pipe, is the sum of the forces of all the particles from the centre of the fan to the circumference. This force is given in lbs. per square inch by the formula

$$F = \int_0^R \frac{p x dx}{g} \omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Integrating this expression,

$$F = \frac{p R^2 \omega^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

p = weight in lbs. of a column of water 1 square inch in section and 1 foot long;

R = radius of fan in feet;

ω = angular velocity of fan;

g = dynamical force of gravity.

Now since $R \omega = v$, where v = velocity of circumference of fan, by replacing $R^2 \omega^2$ by v^2 in equation 3, it becomes

$$F = \frac{p v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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ing also that a particle of water moves in a curve similar to that of the blades—which is evidently the case. Then calling v the velocity of the water through the fan relatively to it, and ω its angular velocity, the linear velocity of a particle of water is given by the formula

$$V = r\omega - v \cos \theta. \quad (6)$$

θ being the angle indicated on the Fig., or $\cos \theta = \frac{\rho}{r}$.

Substituting for $\cos \theta$ its value, then

$$V = r\omega - v \frac{\rho}{r}. \quad (7)$$

Now the centrifugal force of a particle at the distance r from the centre, and moving with the velocity V , is

$$F = \frac{p}{g} \frac{V\omega}{dr} \quad (8)$$

Putting in for V its value in equation 6,

$$F = \frac{p}{g} \omega \left(r\omega - v \frac{\rho}{r} \right) dr \quad (9)$$

Now calling R and R' the external and internal radii of the fan, and integrating expression 9 between these limits,

$$F = \frac{p}{g} \int_{R'}^R \left(r\omega^2 - v\omega \frac{\rho}{r} \right) dr \quad (10)$$

but $\rho = \sqrt{r^2 - s^2}$;

substituting this in the last expression,

$$F = \frac{p}{g} \int_{R'}^R \left(r\omega^2 - v\omega \sqrt{1 - \frac{s^2}{r^2}} \right) dr \quad (11)$$

Expanding the second term,

$$F = \frac{p}{g} \int_{R'}^R \left\{ r\omega^2 - v\omega \left(1 - \frac{1}{2} \frac{s^2}{r^2} - \frac{1}{8} \frac{s^4}{r^4} \right) \right\} dr \quad (12)$$

¹ This expression is not strictly correct, but is a near approximation.

$$F = \frac{p}{g} \left\{ \frac{R^2 - R'^2}{2} \omega^2 - v \omega \left[R - R' + \frac{1}{2} s^2 \left(\frac{1}{R} - \frac{1}{R'} \right) + \frac{1}{24} s^4 \left(\frac{1}{R^3} - \frac{1}{R'^3} \right) \right] \right\} \dots \dots \dots (13)$$

which is the whole centrifugal force exerted at the circumference of the fan in lbs. per square inch. The second force exerted in this case is the force of impact. It is estimated by the maximum tangential velocity generated in the water passing through the fan, which takes place just as it is escaping at the circumference. The reason advanced for the assertion that this force can be estimated by the tangential velocity produced, is that no other force can produce this velocity. The centrifugal force just calculated can only produce a radial force or a radial velocity, but can in no case produce a tangential force or a tangential velocity. This latter force can only be made use of by gradually reducing the velocity of the water issuing from the fan, and this condition is effected by the spiral casing and conical discharge-pipe, which will be referred to presently, and can be easily calculated by multiplying v by cosine θ' , the angle made by the blade of the fan at its outer extremity, with the tangent to the fan; and subtracting this from V , the velocity of the circumference, the absolute tangential velocity of the water leaving the fan is obtained, viz. :—

$$v' = V - v \cos \theta' \dots \dots \dots (14)$$

The head then due to this velocity is given by the formula

$$H_2 = \frac{v'^2}{2g} \dots \dots \dots (15)$$

This, in other words, is the height that the water would rise, supposing that there was no friction to impede it. Now the centrifugal force has been estimated in lbs. per square inch; but by dividing it by 0.434 it is reduced to feet head of water. Then by adding these two heads together the theoretical height to which the pump lifts the water is obtained, *i.e.*,

$$H + H_2 = \frac{F}{.434} + \frac{v'^2}{2g} \dots \dots \dots (16)$$

This theoretical lift is always greater than that deduced by experiment, and it is only in a perfect pump that these two lifts would coincide. Consequently, if the practical lift be divided by the theoretical lift, and the result multiplied by 100, the percentage efficiency of the pump is obtained.

These calculations are worked out for the last series of experiments upon the spiral-cased Appold pump, and the results tabulated as follows:—

TABLE 7.

No. of Experiment.	Angular Velocity, ω .	ω^2 .	$0.218 \omega^2$.	Velocity through Fan, v .	$0.316 v \omega$.	$0.218 \omega^2 - 0.316 v \omega$.	Centrifugal Force in Feet Head, H_1 .	Velocity due to Impact, $R \omega - v \cos \theta$.	Head due to Impact, H_2 .	Head by Experiment, H_1 .	$H + H_2$.	Efficiency, $\frac{H_1}{H + H_2}$.
1	41.04	1,684	368.1	2.699	35.05	333.05	10.32	28.573	13.50	14.67	23.82	61.12
2	41.24	1,701	370.8	2.955	38.56	332.24	10.30	28.469	12.66	14.70	22.96	64.02
3	41.35	1,710	372.8	3.192	41.76	331.04	10.26	28.316	12.53	14.65	22.79	64.12
4	41.67	1,736	378.4	3.413	45.00	333.40	10.33	28.339	12.55	14.70	22.88	64.24
5	41.77	1,745	380.4	3.600	47.58	332.82	10.32	28.229	12.45	14.75	22.77	64.78
6	41.88	1,754	382.4	3.816	51.01	331.39	10.27	28.196	12.42	14.75	22.69	65.00
7	42.08	1,771	386.1	4.000	53.27	332.83	10.32	28.065	12.30	14.70	22.62	65.25
8	42.19	1,780	388.0	4.182	55.82	332.18	10.30	27.882	12.14	14.75	22.44	65.73
9	42.30	1,789	390.0	4.349	58.21	331.79	10.29	27.882	12.14	14.80	22.43	65.91
10	42.40	1,798	392.0	4.520	60.38	331.62	10.28	27.789	12.07	14.75	22.35	66.22
11	42.50	1,806	393.7	4.673	62.86	330.84	10.26	27.712	11.99	14.80	22.25	66.51
12	44.39	1,970	429.5	2.699	37.92	391.58	12.33	31.126	15.14	17.40	27.47	63.30
13	44.49	1,979	431.4	2.955	41.61	389.79	12.28	31.046	15.06	17.20	27.34	63.36
14	44.70	1,998	435.6	3.192	45.16	389.44	12.27	30.869	14.89	17.20	27.16	63.33
15	44.81	2,008	437.7	3.413	48.40	389.30	12.26	30.732	14.75	17.30	27.01	63.68
16	44.91	2,017	439.7	3.600	51.16	388.54	12.24	30.621	14.65	17.30	26.89	64.34
17	45.12	2,036	443.8	3.816	54.49	389.31	12.26	30.465	14.49	17.40	26.75	65.04
18	45.23	2,046	446.0	4.000	57.26	388.74	12.24	30.465	14.49	17.40	26.73	65.09
19	45.33	2,055	448.0	4.182	59.99	388.01	12.22	30.359	14.40	17.40	26.62	65.32
20	45.44	2,065	450.2	4.349	62.54	387.66	12.21	30.276	14.32	17.60	26.53	66.33
21	45.54	2,074	452.1	4.520	65.06	387.04	12.19	30.181	14.18	17.60	26.37	66.74
22	45.64	2,083	454.1	4.673	67.50	386.60	12.18	30.104	14.15	17.60	26.33	66.84

By comparing this with Table 5, it will be noticed that the theoretical efficiencies are considerably higher than those deduced by experiment. This is owing to the friction of the bearings of the pump, and of the strap by which it is driven; a small amount is also due to the outer sides and edges of the fan revolving in the water, not having been subtracted from the power indicated by the dynamometer. Some experiments were afterwards made in order to determine this friction, and it was found to amount to about 45,000 foot-lbs. per minute. The second column of efficiencies in Table 5 is obtained by making the correction.

The slight discrepancies that are now found to exist between the theoretical and experimental methods of determining the efficiencies are within the limits of observation, as the experiments to determine the loss by friction were not made at the same time, or with the same pump as those tabulated above.

Some of the practical advantages of being able to calculate the efficiency of a centrifugal pump theoretically may here be mentioned. Since the efficiency is the lift as measured by experiment divided by that deduced by calculation, the more the latter can be reduced relatively to the former the greater is the efficiency of the pump.

The first principle to be attended to in effecting this condition is to avoid giving sudden shocks to the water, a principle which is ably discussed by M. Combes.¹ This is done in the fan by designing the blades so as to enter the water in a direction tangential to their surface at their inner edges; and at the same time having their outer edges so as to leave the water moving in a direction as nearly as possible tangential to the circumference of the fan.

The internal angles vary both with the lift and discharge with which the pump is intended to work, and their value can be found in the following way:—Assuming, as before, that their form is an involute of a circle, which is the best curve in consequence of its property of allowing a constant area of passage between the successive blades, and thus admitting of no change of velocity in the water whilst traversing the fan:

Let R = external radius of fan;

r = internal radius of fan;

ω = angular velocity of fan;

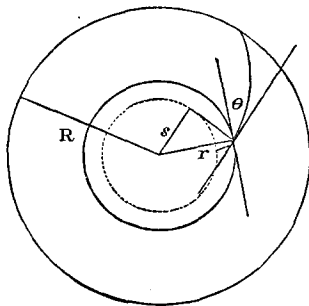
s = radius of generating circle of involute blades;

v = velocity of water through the passages of fan in feet per second;

v' = radial velocity of water entering the blades;

$V' = r\omega$ = tangential velocity of the internal circumference of fan in feet per second;

θ = angle made by the blades with the internal circumference of the fan.



¹ "Recherches théorétiques et expérimentales sur les roues à réaction ou à tuyaux."

Then referring to Fig. 2.

$$\sin \theta = \frac{8}{9} \quad (17)$$

and the condition that the blades should enter the water tangentially is evidently

$$\tan \theta = \frac{v'}{V'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Eliminating θ between equations 17 and 18, an equation is obtained for determining s , viz.,

$$g = \frac{v' r}{\sqrt{V'^2 + v'^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If then the blades of the fan be described with a generating circle, having a radius equal to that given by the preceding formula, they will cut the water in a tangential direction. This formula involves, first, the radial velocity of the water entering the fan, which is easily calculated from the required discharge. The best velocity for the water to flow through the passages of a fan is from 6 to 8 feet per second. Next, the internal radius, which is generally made half the external. And, lastly, the velocity of the internal periphery of the fan. This is given by the following formulæ—

$$\mathbf{V} = r \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad , \quad (20)$$

$$\omega = 9 \frac{\sqrt{h}}{R} \cdot g p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

A general equation which unites all the variables in a centrifugal pump will now be considered. Assuming at present that the pump has a spiral case, and that its mean theoretical efficiency be 64 per cent., as it was found to be by experiment; then the lift under which the pump is to work must be this percentage of the theoretical lift. Putting this condition into symbols, in the general equation

$$\text{Lift} = 0.64 \left[\frac{p}{g \times .434} \left\{ \frac{R^2 - r^2}{2} \omega^2 - v \omega \left(R - r + \frac{1}{2} g^2 \left(\frac{1}{R} - \frac{1}{r} \right) \right) \right\} + \frac{(V - v)^2}{2g} \right]. \quad (22)$$

But $V = R \omega$.

Substituting this in equation 22, an equation is found involving R , ω , v , and lift, any three of which being known, the fourth can be determined. This equation is of great practical value; for if, for instance, it is required to be known how fast a pump of given dimensions ought to be driven, so as to deliver a given quantity of water at a given lift, it is only necessary to put in the given dimensions, the discharge required, and the lift, and to solve the quadratic equation for ω , which gives the angular velocity at once. The general equation is applicable to all centrifugal pumps, but the theoretical efficiency of each kind must be determined before it is introduced into this equation, if great accuracy is required; but for all practical purposes the co-efficients 0.60 or 0.64 are sufficiently accurate.

The next thing to be borne in mind is to proportion the passages throughout the pump, so as to have a gradually increasing velocity in the water until it arrives at the circumference of the fan, and then to have a gradually decreasing velocity until it issues from the discharge-pipe. This condition is effected by having a conical end to the suction-pipe; and what is much more important is to have a spiral casing surrounding the fan. The importance of the last detail is shown most conclusively by the first series of experiments with the circular and spiral cases.

The form of the casing should be an Archimedian spiral, which has the property that the water flowing round the case moves with the same velocity as that issuing from the fan. The casing should then gradually open out into the discharge-pipe, and, if practicable, a conical adjutage should be added, similar to those described by Rittinger.¹

The same conditions are as nearly as possible fulfilled in the vertical-spindle pump previously described. Below the fan is a conical opening which produces a gradually increasing velocity in the water entering the fan. The water escaping from the fan is gradually turned into an upward direction by the guide-blades over the fan, and is at the same time checked in its velocity in consequence of the apertures between the guide-blades being smaller near the fan than above. The vertical-spindle pump was first tried without any guide-blades over the fan, and the maximum discharge obtained with an 8 HP. engine was 1,500 gallons per minute. Six guide-blades, similar to those in Plate 6, Figs. 8 and

¹ "Centrifugal-Ventilatoren und Centrifugalpumpen."

10, were then put in, and the maximum discharge, with the same engine and same boiler pressure, amounted to 5,000 gallons per minute. This fact, is sufficient to show how important it is to make use of the velocity of impact, which by Table 7 produces a force greater than that of the centrifugal force. Professor Rankine¹ in discussing centrifugal pumps states that guide-blades are unnecessary, and even useless; but it seems that the results arrived at by experiment, if not those deduced by theory, clearly prove how important guide-blades are for the attainment of high efficiencies with the vertical-spindle centrifugal pump.

By referring to the column of calculated centrifugal forces in Table 7, it will be observed that the faster the fan rotates—the lift remaining constant—the smaller is the centrifugal force. This seems to be a paradox at first sight, but the reason is evident. As the discharge increases, the velocity of the water in the casing more nearly approaches that of the water leaving the fan; consequently the efficiency of the pump improves, and the theoretical lift diminishes, and with it the centrifugal force.

A remarkable property of centrifugal pumps may be mentioned, which is illustrated very clearly by the preceding experiments, viz., that a small increase in the number of revolutions of the pump, when it has begun to discharge, produces a very large increase in the delivery. Thus in Table 5 the difference in discharge between experiments 1 and 11 is 741 gallons per minute, and a small increase of only fourteen revolutions; or in other words, while the discharge is nearly doubled the revolutions are only increased by $\frac{1}{2}g$. This property has been made use of by Professor James Thomson as a speed-indicator,² and has proved successful.

In conclusion, the whole aim of this Paper has been to show that the calculated efficiency is identically the same as that deduced by experiment. Most people have tried to calculate the friction in the pipes and passages in these pumps to effect this, but they have been unable to do so. The Author, however, has proceeded entirely on hydrodynamical principles, and has merely taken the velocities generated in the water, by which means the friction through the passages of the pump and the pipes has been eliminated from the calculations, in consequence of the velocities through them being directly affected by the friction, and these

¹ Rankine's "Applied Mechanics;" article on Centrifugal Pumps.

² "Mechanics' Magazine," January 1851.

were determined by experiment. The only friction it was necessary to measure was that of the strap required to drive the pump, and the friction in the bearings, and of the outside of the fan revolving in the water. This latter friction was ascertained by driving the pump at a speed of about 410 revolutions per minute, which corresponded nearly with a mean of the revolutions run during the previous experiments, and, as has been before stated, amounted to about 8 per cent. This is only an approximation; but owing to the great difficulty in maintaining a constant speed in a pump when it is not discharging, a more accurate series of experiments was not made.

Fig: 1.

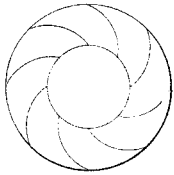


Fig: 2.

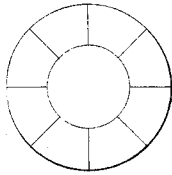


Fig: 3.

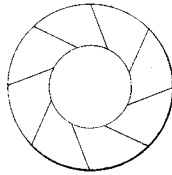


Fig: 8.

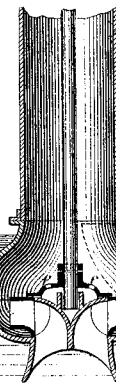
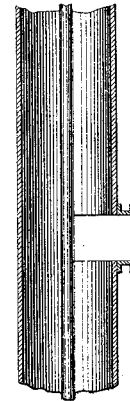
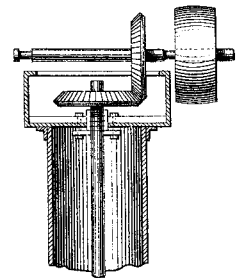
Scale $\frac{1}{48}^{th}$.

Fig: 4.

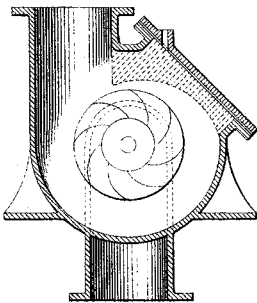


Fig: 5.

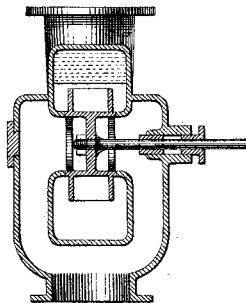


Fig: 7.

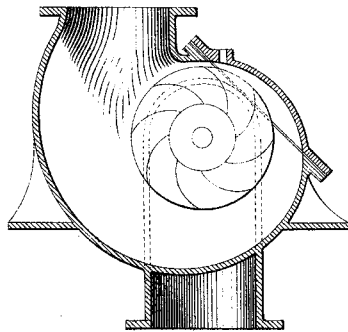
Scale $\frac{1}{24}^{th}$.

Fig: 6.

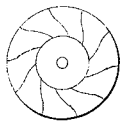


Fig: 9.

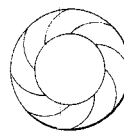


Fig: 10.

