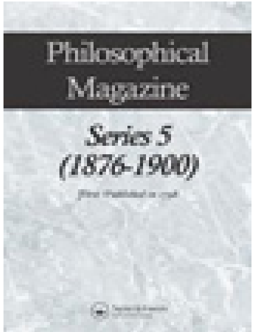

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VIII. An illustration of the crossing of rays

Walter Baily

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from which it appears that in every case the shifting is in the direction of propagation of waves of higher pitch, or towards the source of graver pitch.

According to Matthiessen, the shifting takes place with a velocity equal to half the difference of velocities of the component trains, *i. e.*

$$2V=\frac{n}{\kappa}-\frac{n'}{\kappa'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and in the direction of that component train which moves with greatest velocity. So far as regards the direction merely, the two rules come to the same thing for the range of pitch used by Lissajous and Matthiessen, since over this range the velocity increases with pitch. If, however, we have to deal with waves longer than the critical value (1·7 centim. for water), the two rules are at issue, since now the velocity increases as the pitch diminishes. The following are a few corresponding values, in C.G.S. measure, of wave-length, velocity, and frequency of vibration calculated by Thomson's formula (A).

Wave-length...	·5	1·0	1·7	2·5	3·0	5·0
Velocity	31·48	24·75	23·11	23·94	24·92	29·54
Frequency ...	62·97	24·75	13·60	9·579	8·306	5·908

I have examined the matter experimentally with the aid of vibrators making from 12 to 7 complete vibrations per second, and therefore well below the critical point, with the result that the transference is towards the source of graver pitch, although this is the direction of propagation of the component which travels with the smaller velocity. I reserve for the present a more detailed description of the apparatus, as I propose to apply it to the general verification of Thomson's law of velocities.

VIII. *An Illustration of the Crossing of Rays.* *By* Walter Baily*.

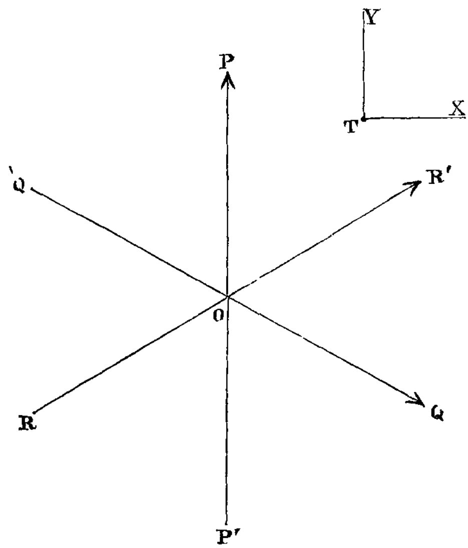
[Plate I.]

When rays of light are passing through a point, the resultant motion of the æther is in general far too complicated to be conceived; but if the light is homogeneous, it can readily be shown that the motion at each point is simply harmonic motion in an ellipse; so that in that case the

* Communicated by the Physical Society; read May 26, 1833.

complication consists only of the change in this ellipse in passing from one point to another. Hence a model might be constructed to represent the crossing of homogeneous rays by placing a number of ellipses to represent the motion at a number of separate points, through which the light might be supposed to be passing. If we further simplify the case by considering only rays parallel to one plane, and suppose them to be plane-polarized so that the vibrations are parallel to the same plane, the whole motion will be parallel to that plane, and might be represented by means of diagrams.

The case worked out in this paper is that of three rays of equal intensity parallel to one plane, plane-polarized so that the vibrations are parallel to that plane, and meeting one another at equal angles.



Take any point O, and let P′ O P, Q′ O Q, R′ O R be the rays through O. Take any other point T in the same plane; draw T X, T Y perpendicular and parallel respectively to P′ O P. Let p, q, r be the distances from O of the feet of the perpendiculars drawn from T on P′ O P, Q′ O Q, R′ O R respectively; these distances being considered positive if drawn towards P, Q, R, and negative if drawn towards P′, Q′, R′. Then it may be shown that

$$p+q+r=0. \qquad (1)$$

The position of T may be defined by any two of these quantities. The equations $p=$ const., $q=$ const., $r=$ const., are equations to straight lines perpendicular to P′ O P, Q′ O Q, R′ O R respectively; and the equations $q-r=$const, $r-p=$const., $p-q=$const. are equations to lines parallel to P′ O P, Q′ O Q, R′ O R respectively. When the constant is zero, the lines pass through O.

If we take any point in Q′Q and move perpendicularly to Q′ Q from this point, we can, without altering the phase of the vibration of the ray Q, reach a point at which the phase of the vibration of the ray R is the same. If we now move from this latter point in a direction parallel to P′ P, we shall keep the phases of Q, R equal to one another, and we can reach a point at which the phase of the ray P is equal to either of them. Take this point as the origin, and let the phases be zero at the initial time. Then at a time t the displacements due to the three rays at the point T will be $\sin 2\pi(t-p)$, $\sin 2\pi(t-q)$, $\sin 2\pi(t-r)$, the wave-length being taken as the unit of length, and the period as the unit of time.

Let x be the amount of displacement along TX and y that along TY, at the time t. Then

$$x= \sin\frac{\pi}{2}\sin 2\pi(t-p)+\sin\left(\frac{\pi}{2}+\frac{2\pi}{3}\right)\sin 2\pi(t-q)$$

$$+\sin\left(\frac{\pi}{2}-\frac{2\pi}{3}\right)\sin 2\pi(t-r),$$

$$y= \cos\frac{\pi}{2}\sin 2\pi(t-p)+\cos\left(\frac{\pi}{2}+\frac{2\pi}{3}\right)\sin 2\pi(t-q)$$

$$+\cos\left(\frac{\pi}{2}-\frac{2\pi}{3}\right)\sin 2\pi(t-r).$$

By means of (1) these equations may be written

$$x=\sin 2\pi(t-p)-\cos \pi(q-r)\sin 2\pi\left(t+\frac{p}{2}\right), \quad . \quad (2)$$

$$y=\sqrt{3}\sin \pi(q-r)\cos 2\pi\left(t+\frac{p}{2}\right). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In general the calculation of the phase and the ellipse would be laborious; but it may be readily effected along lines parallel to P′OP, Q′OQ, R′OR at distances $\frac{1}{\sqrt{3}}$ from one another as follows:—We have as equation to such lines parallel to P′OP, $q-r=n$, where n is an integer. Hence

$$y=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$x=\sin 2\pi(t-p)-\sin 2\pi\left(t+\frac{p}{2}\right)\cos n\pi.$$

If n is even,

$$x = -\sqrt{2-2\cos 3\pi p}\,.\cos 2\pi\left(t-\frac{p}{4}\right). \quad . \quad . \quad (5)$$

If n is odd,

$$x = \sqrt{2+2\cos 3\pi p}\,.\sin 2\pi\left(t-\frac{p}{4}\right). \quad . \quad . \quad . \quad (6)$$

Equation (4) shows that along these lines the vibrations are rectilinear, and perpendicular to direction of the ray.

Putting $p=\frac{m}{3}$, m being an integer, we see from (5) and (6) that there are points of no motion when m and n are both even or both odd. These conditions will be satisfied if p, q, and r are multiples of $\frac{1}{3}$. In order to satisfy (1), one of the quantities must be an even multiple, and the other two must be both even or both odd.

We may obtain similar equations in relation to Q′OQ and R′OR; and the points of no motion will be the same as those already obtained. If we draw the three sets of lines above considered, we shall form a series of triangles whose sides are parallel to the rays, each side being equal to $\frac{2}{3}$. These triangles will have the properties, that their angles will be nodes, and that the vibrations along their sides will be perpendicular to the sides, the displacement being given by equations (5) and (6) and the corresponding equations for the rays Q and R. The form of these triangles under displacement, when $t=0$, is shown in Pl. I. fig. 1.

The motion may be also readily obtained along lines perpendicular to the direction of the rays, at distances $\frac{1}{3}$ from each other, one of each set passing through the origin. p must be a multiple of $\frac{1}{3}$; and there are six different forms of equations (2) and (3) for six consecutive values of p, which are given in the following Table (n being an integer):—

p.	x.	y.
$2n+1$	$(1+A)\sin 2\pi t$	$-B\cos 2\pi t$
$2n+\frac{2}{3}$	$(1-A)\sin 2\pi(t+\frac{1}{3})$	$+B\cos 2\pi(t+\frac{1}{3})$
$2n+\frac{1}{3}$	$(1+A)\sin 2\pi(t-\frac{1}{3})$	$-B\cos 2\pi(t-\frac{1}{3})$
$2n$	$(1-A)\sin 2\pi t$	$+B\cos 2\pi t$
$2n-\frac{1}{3}$	$(1+A)\sin 2\pi(t+\frac{1}{3})$	$-B\cos 2\pi(t+\frac{1}{3})$
$2n-\frac{2}{3}$	$(1-A)\sin 2\pi(t-\frac{1}{3})$	$+B\cos 2\pi(t-\frac{1}{3})$

where $\quad A=\cos\pi(q-r), \quad B=\sqrt{3}\sin\pi(q-r).$

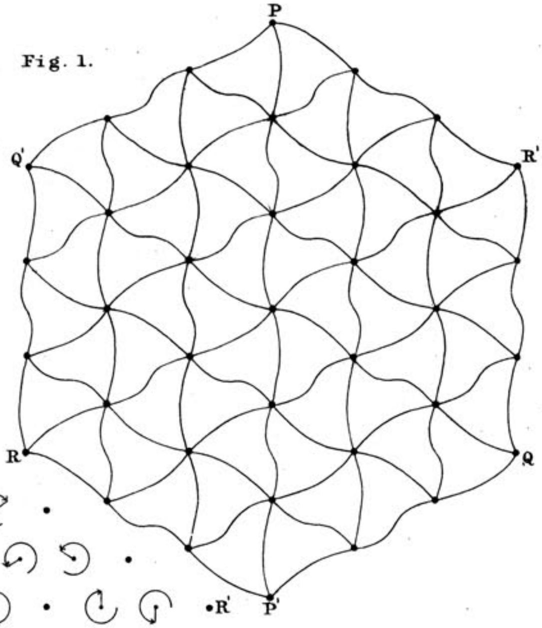


Fig. 1.

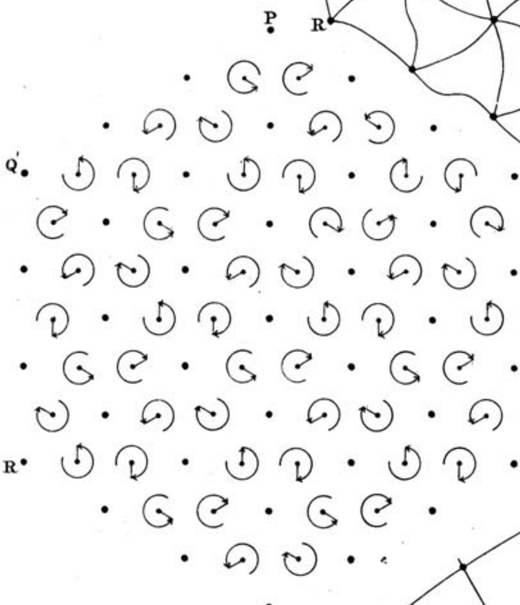


Fig. 2.

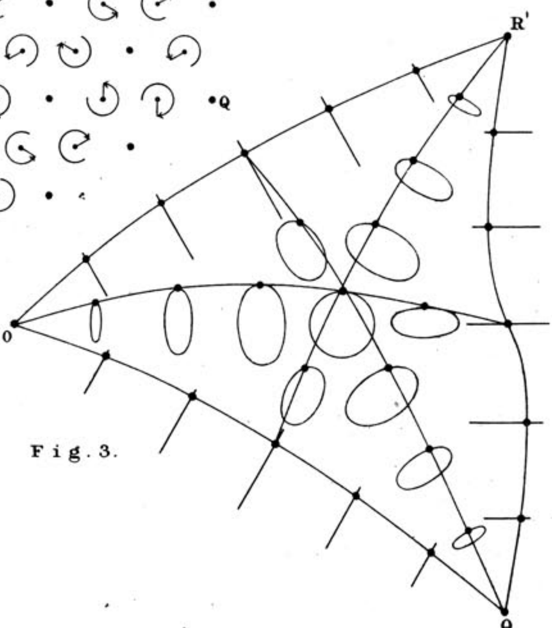


Fig. 3.

Mintern Bros lith.

These lines intersect the triangles (fig. 1) at their angles, and also at the bisection of their sides. At these points the motion has been already determined. The motion is circular if p is an even multiple of $\frac{1}{3}$, at the points for which $1-A=\pm B$ —that is, where $q-r=2m\pm\frac{2}{3}$ (m being an integer); and if p is an odd multiple of $\frac{1}{3}$, at the points for which $1+A=\pm B$ —that is, where $q-r=2m\pm\frac{1}{3}$.

These conditions are satisfied at the middle points of the triangles. In fig. 2 are shown the nodes and the circular points, the arrows indicating the phase when $t=0$. It will be noticed that at adjacent circular points the motion is in opposite directions.

It would be possible to construct a piece of apparatus to exhibit the motion approximately. A piece of elastic membrane, sufficiently stretched in all directions, should be fastened at a set of points corresponding to the points of rest, and the middle points of the triangles should then be displaced according to the phase (see fig. 2), and carried round their original positions in circles of equal size and period, the adjacent motions being in opposite directions—an arrangement which might easily be effected by a series of cogged wheels. We should then have a number of points fixed, and the correct motion given at other points where the motion is greatest. The motion of the rest of the membrane except near the edges would then be approximately correct.

In fig. 3 is given an enlarged view of one of the triangles, showing some of the points where the motion is elliptic, and the displacement of the lines through the nodes parallel and perpendicular to the rays.

IX. *On the Conservation of Solar Energy. Reply by* Sir William Siemens *to* Mr. E. H. Cook*.

ARTICLE LX. in the June Number of the 'Philosophical Magazine,' by E. H. Cook, B.Sc., calls for a reply to some of the objections raised against my Solar hypothesis, which I am the more readily disposed to give, inasmuch as they differ from those already raised by others, and involve moreover questions of general interest. Mr. Cook proves that CO_2 is distributed uniformly throughout our atmosphere; and concludes that the power of gaseous diffusion is such that, admitting (as he does) a universal plenum, the same gaseous proportion must prevail throughout space—that, in short, there must be as large a proportion of CO_2 and N in space

* Communicated by the Author.