for scholarships. The residue of the estate (some $20,000) is left to Clark University.

DR. C. A. STRONG, associate professor of psychology in the University of Chicago, has been elected lecturer on psychology in Columbia College.

PROF. L. S. LUTHER, of Trinity College, Hartford, has been elected president of Kenyon College, Gambier, Ohio. Professor Theodore Stirling, the professor of natural science, has been during the last four years acting president.

PROF. THEODORE VON DER GOLTZ has been appointed professor of agriculture in the University at Bonn in the place of Prof. Dunkelberg, who has retired.

DISCUSSION AND CORRESPONDENCE.

QUATERNIONS.

EDITOR OF SCIENCE: The circular letter of Dr. Molenbroek and Mr. Kimura published in the issue of your journal for October 18th appears to me to be a distinct improvement upon their preceding letter published in *Nature* for October 3d. In the former letter they assume that Hamilton's Quaternions is a much more perfect method than it really is, and they affirm that the newer forms of vector theory invented by physicists are founded on definitions which are established by Quaternions, and are systems of notation rather than logical developments of a mathematical idea. They also advise the "many who are prejudiced against the calculus of quaternions and maintain the opinion that it is hard to understand and that it contains a great deal which is useless in addition to things immediately applicable" to "approach the calculus with proper care and meekness in the assurance that they will ere long rejoice in having at their disposal an instrument of research mightier far than they had the slightest notion of so long as they were in the domain of cartesian mathematics."

In recent years I have published a series of papers on Space Analysis, the express object of which is to unify and harmonize the several vector methods with one another and with the ordinary analysis. I exclude neither the idea of a vector nor the idea of a quaternion, and I do not attempt to make Nature simpler than she really is by identifying ideas that are different though complementary to one another. I look upon vector-analysis not as an independent and rival plant, but as a development of the old tree of mathematical analysis.

The greatest impediments to the progress of the method of Quaternions are not prejudice and false opinion in those to whom it is presented, but rather imperfections, mistakes and errors in the method itself. Hamilton ought to be reverenced for what he did accomplish, but that ought not to blind us to what he did not accomplish. It is an error to identify, as Hamilton does, vectors with quadrantal quaternions. It is an error to confound, as Hamilton does, successive with simultaneous addition; for thereby he failed to discover the generalization for space of the Exponential Theorem and of Taylor's Theorem. It is a mistake to introduce, as Hamilton does, a new notation which has no relation to the established notation of trigonometry, or to adopt conventions which do not harmonize with the established conventions of analysis.

To the amended proposal for an 'International Association for promoting the study of Quaternions and allied systems of Mathematics' there is no room for objection; for it does not assume the perfection and finality of Hamilton's work, but rather invites to the development and study of vector-analysis in its broadest sense. It will, I hope, receive a favorable response from all who are interested in the development or the teaching of space analysis. It is inevitable that there should be diversity of notation and warm discussion of principles among the pioneers in this region, but inasmuch as all are zealous for the truth, the proposed association would accelerate the progress to definite decisions, and thereby smooth the way for the spread of this, the highest development of the art of algebra.

Messrs. Molenbroek and Kimura refer to the remarkable advance in Electrical theory. That advance has been due in large measure to the practical manner in which electricians have discussed the principles and definitions of their science, finally settling all definitions by an authorized Congress. Doubtless the proposed association would eventually accomplish an

equal good in its line. Electricians are alive to the importance of this work also, and the indications are that they will have much influence in its settlement.

But since at the present time there are writers on space analysis who see nothing but vectors, and other writers who identify vectors with quadrantal quaternions, and since the principles commonly accepted by Quaternionists are not free from fundamental errors, it is evident that much time is still required for the discussion of principles before definite decisions about notation can be arrived at. The notation which is adopted must be built on an adequate analysis if it is to be lasting. And here the π muddle in the system of electric and magnetic units ought to act as a warning to make haste slowly.

The logical harmony and unification of the whole of mathematical analysis ought to be kept in view. The algebra of space ought to include the algebra of the plane as a special case, just as the algebra of the plane includes the algebra of the line. And as the algebra of space includes the spherical and higher forms of trigonometry, it ought to be made to harmonize as much as possible with the existing notations and conventions of trigonometrical analysis. When vector analysis is developed and presented so as not to contradict, but, on the contrary, to include the ordinary branches of analysis, we may expect to see many zealous cultivators, many fruitful applications, and, finally, its universal diffusion. Then there will be no need of arguments to prove its utility. May the movement initiated by Messrs. Molenbroek and Kimura hasten the realization of this happy result.

Alexander Macfarlane.

Lehigh University.

SCIENTIFIC LITERATURE.

De Saint Louis a Tripoli par le lac Tchad. Par le Lieutenant-Colonel P. L. Monteil. Paris, Alcan. 1895. Pp. x. and 463. Fifteen itinerary charts and one general map. Profusely illustrated by Riou.

This book may be considered as the fruit of the treaty between England and France which was entered into on August 5, 1890. The reason for the treaty was the necessity of fixing a boundary between the regions subject to their respective influences along an imaginary line drawn from Say on the Niger to Lake Tchad.

Monteil proposed to the French government to traverse this region, starting from St. Louis, in the French possessions on the west coast of Africa. His object was to obtain treaties with as many of the native potentates along the route as possible, and thereby fix the boundary as far as France was concerned.

He left St. Louis on October 9, 1890, with one white companion, Adjt. Badaire, and twelve natives, four of the latter deserting him quite promptly. For twenty-seven months from this time his experiences are given with considerable minuteness. He had the regulation 'ups and downs' which are the lot of the explorer everywhere, particularly in Africa. As far as Wagodogho he followed the itineraries of Binger and Crozat. Beyond this point everything was relatively unknown, except where light had been thrown upon various points along the line when his path crossed the track of his predecessors, Denham and Clapperton, Barth, Nachtigal and others.

His occupations were numerous, as he was at various times soldier, engineer, physician, botanist, astronomer, cartographer, pharmacist, trader, diplomat and magician. Photography did not prosper with him. His early attempts were crushed in Paris; where his plates going to one office and his letter of instructions to another, they were both opened separately with the consequent disastrous result to the negatives. A final blow was struck at this portion of his work when a native stole his camera, plates and all. One can imagine the 'joy and perplexity' of the average native while examining this piece of apparatus, as well as the feelings of the rightful owner under the circumstances.

The loss, however, is made good by the superb set of illustrations by Riou, which are one of the charms of the volume. The artist has so thoroughly caught the spirit of the author that, much as we regret the absence of the true copies of nature, we feel satisfied by the insight which the skillful sketches give us on the subject.

Another feature of the book which cannot be too highly praised is the series of itinerary