

heritability of species. Such an inference accords with the requirement of the mutation theory, but not with the theory which requires perpetual change in all species.

Certain groups of animals and plants have continued their existence without material change under very adverse conditions of physical environment which were seemingly more favorable for their existence than were those under which they originated. The theory of the origin of species by natural selection is at variance with such facts because it recognizes environing conditions as a constantly dominant factor in producing phylogenetic changes, on the one hand, and in preserving the integrity of faunal and floral types, on the other. The mutation theory recognizes the initiative action of an at present unknown determinate natural cause for phylogenetic changes. Environing physical conditions have doubtless been a potent cause in producing variations; and it has doubtless often had a controlling influence upon the destiny of faunas and floras; but such conditions have not been a direct factor in the origination of species.

Whatever criticisms may be made of the de Vriesian theory of mutation, the truth will remain that it accords with numerous important facts with which the theory of the origin of species by natural selection is quite inconsistent. Some theory which provides for the sudden, or rapid, origination of species is made necessary by a large array of geological facts, and the mutation theory more nearly meets that demand than does any other yet proposed.

CHARLES A. WHITE.

SMITHSONIAN INSTITUTION,  
February 20, 1905.

SCIENTIFIC BOOKS.

*Introduction à la Géométrie Générale.* Par GEORGE LECHALAS, Ingénieur en chef des ponts et chaussées. Paris, Gauthier-Villars. 1904. Pp. ix + 58.

*The Elements of Plane and Solid Analytical Geometry.* By ALBERT L. CANDY, Ph.D., assistant professor of mathematics in the University of Nebraska. Boston, D. C. Heath & Co. 1904. Pp. x + 248.

*The Elements of Analytic Geometry.* By PERCEY F. SMITH, Ph.D., professor of mathematics in the Sheffield Scientific School, Yale University, and ARTHUR S. GALE, Ph.D., instructor in mathematics in Yale College. New York, Ginn & Co. 1905. Pp. xii + 424.

*An Introduction to Projective Geometry and Its Applications.* By ARNOLD EMCH, Ph.D., professor of graphics and mathematics in the University of Colorado. New York, John Wiley & Sons. 1905. Pp. vii + 267.

*Descriptive Geometry for Students of Engineering.* Second edition. By JAMES A. MOYER, S.B., A.M., instructor in descriptive geometry in Harvard University. New York, John Wiley & Sons. 1905. Pp. iv + 198.

*The Elements of the Differential and Integral Calculus.* By DONALD F. CAMPBELL, Ph.D., professor of mathematics, Armour Institute of Technology. New York, The Macmillan Company. 1904. Pp. x + 364.

*Elements of the Differential and Integral Calculus.* By WILLIAM A. GRANVILLE, Ph.D., instructor in the Sheffield Scientific School, Yale University. New York, Ginn & Co. 1904. Pp. xiv + 463.

*The Boston Colloquium: Lectures in Mathematics.* By EDWARD B. VAN VLECK, HENRY S. WHITE and FREDERICK S. WOODS. New York, The Macmillan Company. (For the American Mathematical Society.) 1905. Pp. x + 187.

Except the last these works are all of them addressed to beginners in their respective subjects, and, except the first and last, they are designed for use as text-books in undergraduate study.

The aim of Lechalas's splendid essay is orientation among the fundamental divisions of modern geometry conceived in full generality. Especially intended for such as have not formed a systematic conception of the Euclidean, Lobatchevskian and Riemannian

theories, the motivity and principles of classification are presented in fresh and simple fashion, and the three grand types are made to appear vividly in their true light as together constituting a single exhaustive geometric unit or whole. To this end it is shown, in an introductory chapter, by easy considerations connected with symmetry, reversibility (*re-tournabilité*) and curvature, that the concept of super-dimensional spaces is logically indispensable. The second chapter accordingly deals with the concept and certain illuminating properties of Euclidean space of four dimensions. The essay closes with a third chapter devoted to analogous features of the geometry of spaces of negative curvature. From the simplest and most familiar considerations to the most remote and recondite, the reader is conducted swiftly and along geodesic paths, but the scenery along the way is rich and stimulating. We take pleasure in recording our judgment, independently formed and concordant with the author's, that the geometry of hyperspace is, at least for certain temperaments, genuinely geometric and not merely analytic as some have claimed. The notion of hyperspace, indeed, rests upon spatial intuition, and the higher spaces that reason has created, the imagination may very well yet learn to picture and illuminate. To those who may aspire to a reasonably competent acquaintance with the elements of *general* geometry, we commend this little book along with Jouffret's 'Traité élémentaire de Géométrie à quatre dimensions,' from which the former draws to some extent, Schubert's essay on fourfold space in his 'Mathematical Essays and Recreations,' and 'Mehrdimensionale Geometrie' by Paul Schoute.

The late Judge Cooley was accustomed to advise his students never to buy a law book not provided with an index. It would be harsh to apply that maxim to Professor Candy's book, for, though it lacks the feature mentioned, it contains others which make it one of the very best among the works of its class. We refer particularly to its emphasis upon the general analytic method as distinguished from limited methods especially available for the conics, and to its fusion and

simultaneous treatment of algebra, geometry, equation theory, analytics and calculus, as contrasted with the usual practice of presenting these subjects separately in succession and of so giving the injurious impression that they are as so many separate instruments, or insulated departments, of thought. In the respects indicated we believe the book will prove to be something of a pioneer.

Doctors Smith and Gale 'have endeavored to write a drill book for beginners which presents the elements in a manner conforming with modern ideas.' But these words are obviously not to be taken quite literally. For, on the one hand, a book that is relieved by such live and life-giving topics as invariants, parametric equations, conic systems, homothetic, similitude and symmetry transformations, inversion, systems of orthogonality, poles, polars and polar reciprocation, can hardly be adequately described as a mere drill book; and, on the other hand, its modernity, for it is modern in many respects, would have been, in our judgment, not a little improved by a bold introduction and use of the infinite elements and by allowing imaginaries to play conspicuously their familiar enlightening rôle. It is true that imaginary elements are not properly intuitable; they lack the property of definitely localizable exteriority, and on that account they tend to confuse at first, but only at first, and afterward they give a light which, if it be invisible to sensuous vision, reason at all events demands. We do not believe in the subordination of the intelligence to sight and imagination. It is to the *rational* spirit, which beholds many things inaccessible to the ordinary imagination, that analytic geometry, strictly and properly conceived, addresses itself. And in this subject, the languages of analysis and geometry should be coextensive. We are glad to note that in this work, as in Candy's, the accent falls upon the general analytic method rather than on specific curves, as those of second order. The book abounds in concepts, and these after all are the constituents of intelligence. These being well formed, ratiocination is easy, while without the former the latter is empty and vain.

Professor Emch's book is avowedly utilitari-

an in its aim. Just on that account, it meets a need long distinctly felt in its field. It does not seek to rival in their own way the logicians and 'arithmetizers,' or the purists like von Staudt, or the Italian geometer-ontologists like Veronese, or the 'visualizers' like Enriques. It is more in the spirit of such as Poncelet, Steiner and Chasle, who were less concerned with foundations, which to them were obvious enough, than with superstructure. But it is in no sense a slavish imitator of any type or school. It has a way of its own, and it is refreshing in these peering microscopic days to find your cruder intelligence enlisted without ado, to be ushered at once into the midst of things and in course of the first score of pages to find the atmosphere charged with such cardinal notions as anharmonic ratio, involution, projective transformation, and projective and involutonic pencils and ranges. A second chapter deals with 'Collineation,' a third with 'Theory of Conics,' a fourth with 'Pencils and Ranges of Conics and the Steinerian Transformation in Connection with Cubics,' and a final chapter of forty-five pages with 'Applications to Mechanics.' The method, consistently with the emancipated spirit of modern geometry, is now analytic, now geometric and now a combination of them. For the pure mathematician as such, Professor Emch's book can not be regarded as a substitute for the Cremona and Reye classics, but it will serve for much more than an admirable introduction to them. It differs from them in spirit, content and method. The exposition of the interesting connection between collineations and the surprisingly beautiful doctrine of linkages deserves special mention, as do also the clearness, directness and swiftness of style in which the book is written.

Mr. Moyer has not aimed to write for the geometrician, but for the student of practical engineering. Hence his book does not say that, if such and such propositions be granted, such and such others will follow as logical consequences. It says, if you desire certain specified results, you should proceed thus and so. From some points of view, it seems a pity that such important practice should be so

divorced from the theory which constitutes its ground and rational justification. Such detachment, however, is not fatal. The Romans were but meager mathematicians, but they were excellent engineers in their day. Nevertheless we believe that the interests of both theory and practise would be better served if the instruction offered by Mr. Moyer were combined with such a course as that afforded by Professor Emch's book. The time element has of course to be reckoned with. Mr. Moyer's experiment of adopting the notation of mechanical drawing and of introducing many concrete graduated exercises has been eminently successful and these features have been retained in the second edition.

The above listed books of the calculus are evidently, both of them, products of conscientious workmanship. Both of them were composed in the composite light of teaching experience and of modern knowledge of the subject. Guided by the needs of his own classes and therefore excluding all topics not directly bearing upon engineering, Professor Campbell has been enabled, without making his book large or cumbersome, to give a brief introduction to mechanics and differential equations and at the same time to dwell upon fundamental notions and processes. In this last regard, his painstaking and repeated explanation of the way in which summations and integrals are constituted deserves especial mention. Though written primarily for technological students, the book is far from ill-adapted to the uses of so-called more liberal institutions. Professor Granville's work is of larger range, embracing not only the subjects specially adapted to the needs of the student consciously destined for work in applied science, but also those that particularly appeal to the student looking towards specialization in pure mathematics. As well in its scope as in its spirit, the work is distinctly more than the author modestly styles it, 'essentially a drill book.' We especially like its graphic treatment of limits and continuity and its constant appeal to intuition as an indispensable aid to analysis. The geometric and physical illustrations of the significance of the integration constant are happy, to mention a single

feature. Commendable also is the effort throughout to render clear the *meaning* and *limitations* of cardinal theorems. We doubt, however, whether the vexed question whether a variable may attain its limit is, in spirit, quite settled by the ingenious example given on page 20.

The Boston Colloquium consists of three sets of lectures: three by Professor White on 'Linear Systems of Curves on Algebraic Surfaces'; three by Professor Woods on 'Forms of Non-Euclidean Space'; and six by Professor E. B. Van Vleck on 'Selected Topics in the Theory of Divergent Series and Continued Fractions.' The lecturers being all of them former pupils of Professor John Monroe Van Vleck recently retired from the chair of mathematics and astronomy at Wesleyan University, where he had served for a period of fifty years, this volume of lectures is inscribed to him. Each set of the lectures affords a compendious account of the advanced thought in its field, together with indications of existing problems and of the directions that further immediate developments will probably follow. The lectures being of highly technical character and being addressed to specialists, any adequate account of them must be reserved for journals specifically devoted to mathematics.

CASSIUS J. KEYSER.

COLUMBIA UNIVERSITY.

*Neudrucke von Schriften und Karten über Meteorologie und Erdmagnetismus herausgegeben von Professor Dr. G. HELLMANN. No. 15 (Schlussheft). Denkmäler Mittelalterlicher Meteorologie.* Berlin, A. Asher & Co. 1904. 4to. 46 pp. introduction + 270 pp. reprints + 12 pp. addenda and errata to previous numbers.

The publication of this volume, which is the fifteenth and final one of the series, affords an opportunity to congratulate Dr. Hellmann on the completion of so admirable a bibliographical work, which offers to students the advantage of reading in their original form many epoch-making papers relating to meteorology and terrestrial magnetism. Readers of SCIENCE are familiar with the nature of these reprints from the reviews that have appeared

in Vol. I., p. 302; Vol. IX., p. 910; Vol. XIII., p. 821; and Vol. XVI., p. 352. Five of the earlier volumes demonstrated that meteorology was actively cultivated in the fourteenth, fifteenth and sixteenth centuries, but in the present issue it is seen that much earlier, and during the entire middle ages, meteorological questions were continually discussed. As illustrations there are given, in part or *in extenso*, 26 writings, dating from the seventh to the fourteenth centuries, inclusive, among them the following examples of famous authors: 'De Natura Rerum,' by Isidorus Hispalensis and Beda Venerabilis; two papers by Albertus Magnus; Roger Bacon's 'Opus Majus,' and 'De Proprietatibus Rerum,' by Bartholomæus Anglicus. Most of the treatises are taken from printed books, but a few are printed from manuscripts for the first time. Certain tracts that were written in little-known languages have been translated into German and so are made accessible to a wider circle of readers, but those in early French, Italian and Dutch appear in the original tongues. It is hardly necessary to say that the facsimile reproductions have been made with the same care that characterized Dr. Hellmann's previous reprints, for the accuracy of which the writer can vouch, having compared several with the originals in his own possession. Their value is much enhanced by the explanatory and bibliographical notes which accompany each.

One or two copies of this last volume have been placed on sale at the Blue Hill Observatory, Hyde Park, Mass., and will be sent on receipt of the publisher's price, viz., 28 Marks, or \$7. In conclusion it may be stated that at least four of the earlier volumes are already out of print and command high prices.

A. LAWRENCE ROTCH.

#### SCIENTIFIC JOURNALS AND ARTICLES.

THE last number of *The American Journal of Mathematics* contains the following articles:

G. W. HILL: 'Deduction of the Power Series Representing a Function from Special Values of the Lateral.'