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LXI. *On the Thermal Conditions and on the Stratification of the Antarctic Ice.* *By* O. FISHER, *Clk., M.A., F.G.S.**

SIR WYVILLE THOMSON delivered a lecture at Glasgow, November 23, 1876, "On the Condition of the Antarctic"†. In it he gave a most interesting and graphic account of observations made during the visit of the 'Challenger' to high southern latitudes. Towards the conclusion of the lecture, the author advanced some speculations about the probable thickness and physical conditions of the antarctic ice-cap. There appeared also in the 'Quarterly Journal of Science' for January last an article from the pen of Dr. Croll, which contained certain criticisms upon the conclusions arrived at by Sir Wyville Thomson, and giving reasons why the ice-cap may be considerably thicker than that philosopher supposes.

I venture to think that there are a few points left in a somewhat uncertain state by both these gentlemen, which a little mathematical treatment, of by no means an abstruse nature, may assist in clearing up.

The first of these relates to the limit of thickness which the effects of temperature might impose on the antarctic ice.

Dr. Croll remarks that there are but three sources from which the ice-cap can receive an appreciable amount of heat, viz. (1) the air above, (2) the earth beneath, (3) the work of compression and friction. The last of these he dismisses as inconsiderable, and without doubt rightly so.

* Communicated by the Author.

† 'Nature,' vol. xv. pp. 102, 120.

Sir Wyville Thomson remarks that "it is not easy to see why the temperature of the earth's crust, under a widely extended and practically permanent ice-sheet of great thickness, should ever fall below the freezing-point." Our first inquiry therefore shall be :—

(1) *If a level sheet of ice rest upon the earth, its upper surface being maintained at a uniform mean temperature lower than the freezing-point, to find the thickness beyond which melting at the lower surface must take place.*

It is assumed that the form of the ice-cap is permanent; so that we may consider that the masses, whether of rock or ice, have arrived at that state in which the temperature is also permanent. Probably any horizontal movement in the ice, so long as the thickness at any place remains unaltered, will only very slightly affect this supposition—because, if there be sliding of the parts one over another, ice abstracted by horizontal movement will be replaced by ice at the same temperature.

We know that there is a continual flow of heat out of the earth. This flow is such that it is found to raise the temperature of the strata of the earth by about $\frac{1}{60}$ of a degree Fahr. per foot on descending. This average increase prevails equally wherever observations have been taken. Even at Yakoutzk, where the ground is perpetually frozen beyond the depth pierced, the same rate obtains. This may therefore be admitted as an empirical fact. But the following reasoning may perhaps be accepted as explanatory of this uniformity of rate.

If the temperature of the surface of the earth were suddenly lowered at any place, the flow of heat to the surface would by that means be temporarily increased, and therefore also the rate of increase near the surface. But since the general internal temperature is no greater below that locality than elsewhere, this increased flow near the surface must take place at the expense of the superficial strata. Hence the isogeotherms near the surface, though temporarily brought closer together, will shortly begin to separate again, until the rate of increase falls to its normal value. At this juncture the flow of heat at the surface at the place in question will become equalized to that which obtains elsewhere. And since the supply of heat from the interior is the same here as elsewhere, the rate can fall no lower, and will afterwards continue equal to that which obtains in other regions. It follows that the permanent rate of increase of temperature near the surface of the earth will be everywhere the same, and the flow of heat also, whatever the temperature of the surface may be. This is what observation shows to be the case.

It is evident that this result will be independent of the cause which maintains the temperature of the surface (say) below the average, whether it be a cold climate or an ice-sheet. We may therefore assume that the rate of increase in the earth beneath the ice-sheet is $\frac{1}{60}$ of a degree Fahr. per foot of descent.

Let us now suppose that our sheet of ice is throughout below the melting-temperature, so that the level of that crucial temperature is situated within the rock. In this case the flow of heat from the earth will pass into the ice unaltered in amount, because none of it will be arrested and consumed in melting ice at the junction. The ice will then be under conditions which will render it sufficiently amenable to the following statement of Fourier:—

"The thermometric state of a solid enclosed between two parallel infinite planes, whose perpendicular distance is e, and which are maintained at fixed temperatures, a and b, is represented by the two equations"

$$v = a - \frac{a-b}{e}(e-z), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\left.\begin{aligned} F &= k\,\frac{a-b}{e} \\ F &= k\,\frac{dv}{dz}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where, for convenience in the present instance, we have taken the origin of z (the depth) at the surface.

v is the temperature at the depth z,
a " " of the lower surface,
b " " of the upper surface,
k the conductivity,

and F is the flow of heat upwards through the solid.

The last of these equations will be applicable to the rock if the proper value be assigned to k.

Let K be the conductivity of rock,
k " " ice.

Now $\frac{dv}{dz}$ represents the rate at which the temperature increases in descending. We know that for rock this is $\frac{1}{60}$, the units being the foot and degree Fahr. Suppose β to be the value of $\frac{dv}{dz}$ for the ice; F will be, under the circumstances supposed, the same both for the ice and rock. Hence equation (2) becomes,

$$\text{for the rock,}\quad F=K\frac{1}{60},$$

$$\text{for the ice,}\quad F=k\beta,$$

whence

$$\beta=\frac{K}{k}\frac{1}{60}.$$

Now the *ratio* of the conductivities will be the same whatever system of units we employ.

Referring to Professor Everett's 'Illustrations of the C.G.S. System of Units,' we find the mean of K for three kinds of rock *in situ*, as determined by Sir Wm. Thomson, to be ·00581, and the mean of k for ice ·00218; whence

$$\beta=\frac{581}{218}\frac{1}{60}$$

$$=\cdot 04441.$$

Hence, for the ice,

$$\frac{a-b}{e}=\cdot 04441,$$

in which expression a is the temperature of the surface in contact with the rock, and b is the temperature of the surface exposed to the air, e being the thickness of the ice.

If, therefore, we wish to find the thickness of ice which will just be sufficient not to induce melting at the bottom, we must put 32 for a; and if, with Dr. Croll, we suppose the mean temperature of the surface exposed to the air to be 0° F., we must put 0 for b. Whence

$$\frac{32}{e}=\cdot 4441;$$

$$\therefore\ e=743 \text{ feet.}$$

If the thickness of the ice be less than this, no melting will take place; but if greater, there will be melting at the junction of the ice and rock.

Here no account has been taken of the lowering of the melting-point by pressure. But that is easily allowed for. For, comparing the height of a column of ice whose pressure is equivalent to a column of mercury of 30 inches (or one atmosphere), it is 37 feet.

Since, then, the melting-temperature is lowered by 0·0137° F.

for each additional atmosphere of pressure, it will be lowered

$$\frac{e}{37} \times \cdot 0137^\circ \text{ for the thickness } e \text{ of ice.}$$

Hence we shall have, to determine the limiting thickness of the ice at which its lower surface will begin to liquefy,

$$\frac{32 - \frac{e}{37} \times \cdot 0137}{e} \times \cdot 04441;$$

$$\therefore e = 714 \text{ feet.}$$

(2) *No certain limit can be imposed upon the thickness to which the ice might accumulate, provided the snowfall be more than sufficient to counterbalance the melting at the bottom.*

We have found the critical value of the thickness of the ice so that melting should just not take place. Suppose the ice thicker than this; then the bottom of it will begin to melt, and consequently must be at the melting-temperature corresponding to the pressure. The flow of heat out of the earth will melt off a layer of it annually. But the whole of this flow of heat will not be so employed, because, the ice being maintained in a condition in which its upper and lower surfaces are at different temperatures, there must ensue a flow of heat through it. This flow of heat will be expressed by the equation

$$f = k\frac{a-b}{e}.$$

Or if, as before, we assume the temperature of the upper surface of the ice to be zero, and allow for the lowering of the melting-point by pressure,

$$f = k\frac{32 - e \times \cdot 00037}{e}.$$

Now it is obvious that there is no source from which this flow can be derived, except the flow out of the earth. Hence F, which is the flow out of the earth, must be split up into two portions, $F-f$ and f; of which $F-f$ goes to melt the ice at the bottom, while f is conducted away through the ice into space beyond. Now

$$f = k\frac{32 - e \times \cdot 00037}{e}$$

$$= k\left(\frac{32}{e} - \cdot 00037\right).$$

We see, then, that as the thickness of the ice is increased, f is diminished, and the less heat escapes through the ice; and when the thickness exceeds sixteen miles none will escape. But if none escaped, so that the whole of F was employed in melting the ice, Sir William Thomson asserts that it could only melt one fifth of an inch annually at the ordinary temperature and pressure. If, therefore, the snow-fall considerably exceeds this small amount, there seems to be no reason why the ice might not accumulate to a much greater thickness than the above, as far as melting at the bottom is concerned.

Note.—Professor Everett says, p. 45, that Sir Wm. Thomson's results are given in terms of the foot and second, and that consequently they have been multiplied by 929 to reduce them to the C.G.S. system. Hence to bring back K and k, as given by Professor Everett, to the system of units used here, we must put

$$K = \frac{\cdot 00581}{929}$$

and

$$k = \frac{\cdot 00218}{929}.$$

The thermometric scale used does not affect the value of the conductivity.

The next question which we will attempt to answer regards

(3) *The mode of origin of the stratification of the great tabular icebergs of the south.*

Supposing that each separate stratum, distinguished by alternations of more or less clear ice, is the product of the snowfall of a single year, Sir Wyville Thomson suggests the following as the mode by which the lower strata may have lost some of their original thickness:—"It is probable that, under the pressure to which the body of ice is subjected, a constant system of melting and regelation may be taking place, the water passing down by gravitation from layer to layer until it reaches the floor of the ice-sheet, and, finally, working out channels for itself between the ice and the land, whether the latter be subaerial or submerged."

Is this process of melting and regelation possible? If we consider the ice-sheet uniform in structure and at rest, it is obvious that the pressure upon any given area of section will be greatest at the bottom, where also the temperature will be highest. Consequently, under these circumstances, the bottom is the only place where the pressure would induce melting. But it may be replied that the ice is not at rest, but is moving outwards towards its free edge. And pressures in the horizontal direction may be connected with this movement greater than the vertical pressures due to the mere depth. It must, however, be recollected that the outward movement of the ice is not caused entirely by a horizontal pressure arising from a *vis a tergo*, but that at every place there is a tendency to flow outwards towards the unsupported free edge. In ordinary glaciers the strain thus produced occasionally gives rise to crevasses. It seems improbable, therefore, that the horizontal pressure should ever exceed the vertical; so that if the latter cannot induce melting, neither can the former do so.

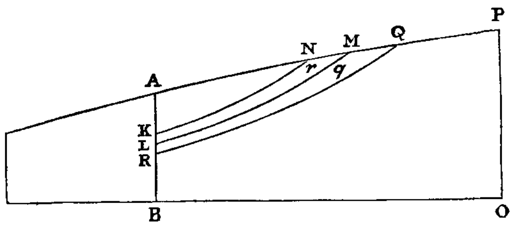
It is, however, more likely that what Sir Wyville Thomson contemplated was that the pressure of one layer upon the next is not evenly distributed, owing to the layers being in contact in some places but not in others. In this case the pressures upon the areas in contact might increase to a great amount, provided the areas of contact were sufficiently small. And it is quite likely to be partly in this way that *névé* is converted into solid ice; for the contact along any horizontal plane is confined to those areas in the *névé* which are occupied by ice and not by air, so that the pressure on these surfaces is much greater than if the ice were uniformly solid. In fact, the mean pressure upon the portion of any area which is occupied by ice is to the pressure due to the depth alone as the whole of that area is to the area occupied by ice. When the conversion of *névé* into ice has taken place, it is not easy to understand how the process of melting and regelation can go further.

But Dr. Croll has pointed out that the diminution of the thickness of the annual strata of ice from the top downwards may be accounted for by the fact that the ice radiates from a centre of dispersion. Although I am fully aware of the very slight practical value of any addition I can make to what Dr. Croll has said upon this part of the subject (for we are too ignorant of the physical properties of ice to arrive at any certain results), I propose nevertheless to offer a few further remarks upon the mode of origination of the stratified structure, and the consequences which follow as affecting the thickness of the strata.

The snowfall of each year is deposited upon that of the previous year; and there is no internal cause to alter this order of "conformable" arrangement of the strata. It is conceivable that the movement of the ice over a rough rocky surface might dislocate the strata, or that the liquefaction of the upper surface over partial areas might cause the snowfall of certain periods to be removed before the deposition of some later strata took place, and so render the strata unconformable. But these disturbing causes are supposed not to be present. It follows that, in any vertical column, the snowfall of every year intermediate between the earliest represented in it (which will be at the bottom) and the most recent (which will be at the top) must have its corresponding layer.

Joining the corresponding layers in contiguous columns, it appears that there must be a regular stratification, as was observed to be the case, and that the strata will not deviate far from horizontality. The question then arises, Where were these strata deposited? and what regulates their thickness?

Fig. 1.



Let a cylindrical surface A B be described in the ice-cap around the polar axis P O, with radius r approximately equal to P A. Also describe a ring on the surface of the ice, whose width is w_n; and let ρ_n be the distance of the edge of it further from the pole. N M is a section of this ring, and $\mathrm{PN}=\rho_n$; $\mathrm{NM}=w_n$.

Now, if the form of the ice-cap is permanent, the snow which falls upon the ring, and upon any part of it, will always follow the same route to meet the cylindrical surface at A B. Suppose N K to be the route taken by the particles which travel from N, and M L by those from M. It is evident, then, that the ice at A B will consist of layers, each of which is continually fed by the snowfall upon its own corresponding ring fixed in position upon the surface, and that the rings nearer the pole will supply the lower layers. This conclusion is independent of any assumption as to the forms of the paths pursued by the ice. It only supposes that these paths do not cross one another.

Suppose that $\mathrm{AB}=h$, $\mathrm{KL}=\delta h_n$. Then, if s be the depth of the annual snowfall upon N M, we must have the quantity of ice which passes outwards each year through the cylindrical surface at K L equal to the annual snowfall on the ring at N M.

Now the area of the ring at N M is

$$\pi\left(\rho_n^2-(\rho_n-w_n)^2\right)=\pi(2\rho_n w_n-w_n^2).$$

Taking v to represent the mean annual velocity through K L per annum, we must have, from the above consideration,

$$2\pi r v \delta h_n = \pi s (2\rho_n w_n - w_n^2).$$

$$\delta h_n = \frac{s}{vr}\left(\rho_n w_n - \frac{w_n^2}{2}\right).$$

This relation expresses the thickness of the layer of ice at A B which is derived from the snowfall on the ring whose section is N M.

If, then, δh_n be the thickness of a layer derived from one year's snowfall, or an annual stratum of ice at A B, this implies that it takes one year more for a particle to travel from M to L than from N to K.

(4) *To account for the downward diminution in thickness of the annual strata of ice.*

For the width of the ring next nearer to the pole we shall have to substitute for ρ_n the value $\rho_n - w_n$; and if the width of that ring be $w_n + \alpha$, and δh_{n-1} the thickness of the corresponding stratum at A B, taking s and v as constant for adjoining strata, we shall have

$$\delta h_{n-1} = \frac{s}{vr}\left((\rho_n - w_n)(w_n + \alpha) - \frac{(w_n + \alpha)^2}{2}\right),$$

whence it appears that

$$\delta h_n - \delta h_{n-1} = \frac{s}{vr}\left(w_n^2 + \frac{\alpha^2}{2} - \alpha(\rho_n - 2w_n)\right). \quad . \quad . \quad \text{(A)}$$

Now we do not know whether the rings which contribute the annual strata diminish or increase in width as they approach the pole. But we may gain a knowledge of the effect which a diminution or increase in the width of the rings would have upon the relative thickness of the successive strata at A B by putting $\delta h_n - \delta h_{n-1} = \beta$, and considering it as the ordinate of a curve whose abscissa is α. If we suppose A B to be near the free edge of the ice-cap, r will be large; and we may without much risk of error consider v constant for all depths above the water-level in that position.

Substituting β and suppressing the suffixes, and writing m for $\frac{vr}{s}$, which we now take as constant, and observe that it is large, because although v is small, yet r is large and s is small, we obtain

$$\left(\alpha - (\rho - 2w)\right)^2 = 2m\left(\beta + \frac{(\rho - 2w)^2 - 2w^2}{2m}\right).$$

This represents a parabola whose axis is vertical, and which has for the coordinates of its vertex,

$$\rho-2w \text{ and } -\frac{(\rho-2w)^2-2w^2}{2m}.$$

The *latus rectum* $\frac{2vr}{s}$ is independent of ρ and w, and may be taken as constant. Supposing that we draw the curve with assigned values of ρ and w, then the ordinate β to abscissa α will give the difference in thickness between the strata at A B which are derived from two contiguous rings whose widths are w and $w+\alpha$.

If we put $\alpha=0$, then $\beta=\frac{w^2}{m}$; so that the height at which the curve cuts the axis of β is independent of ρ, except so far as w depends upon ρ. The points at which the curve cuts the axis of α are given by the relation

$$\alpha=\rho-2w\pm\sqrt{(\rho-2w)^2-2w^2}.$$

This must be always positive.

Taking the smaller value, and observing that, except near the pole, $\rho-2w$ is much greater than w, we have, expanding,

$$\alpha=\frac{w^2}{\rho-2w} \text{ nearly,}$$

which, except near the pole, is much smaller than w. As soon as α exceeds this value, β will become negative.

The greatest negative value of β will be attained at the vertex, where

$$\alpha=\rho-2w.$$

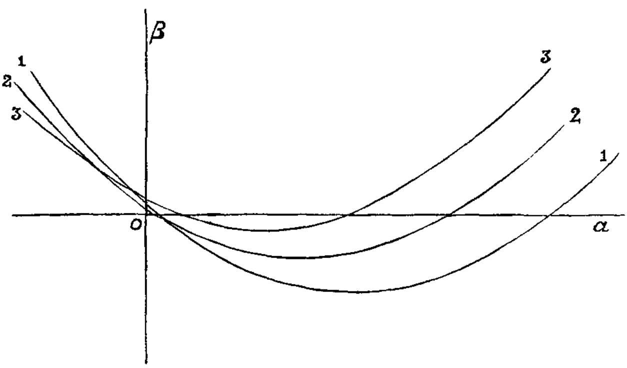
In this case it will be found, by reference to the distances measured from the pole, that this value of α would carry the ring whose width is $w+\alpha$ up to the pole itself.

We can now perceive in a general way how a decrease or an increase in the width of the rings as they approach the pole would affect the difference in the thickness of successive strata at A B.

The *latus rectum* of the parabola being constant, the curve always maintains the same size. We have therefore only to draw it with its axis vertical, and to place the vertex in the position corresponding to the assumed values of ρ and w. It will then necessarily cut the axis of β at the height $\frac{w^2}{m}$ above the origin; for this is the value of the ordinate corresponding to $\alpha=0$. This shows that, if the rings were of uniform width, the difference in thickness of the annual strata at A B would be constant; or, in other words, their thicknesses would form a decreasing arithmetical progression, whose common difference would be $\frac{w^2}{m}$.

Let curve 1 in fig. 2 be drawn with any assumed values

Fig. 2.



of ρ and α. And, first, suppose the rings to decrease in width as they approach the pole. Then the difference in thickness between the ring under consideration and the next nearer to the pole is given by the ordinate corresponding to the assumed value of α on the left of the origin. We observe these ordinates to be all positive. Hence the strata decrease in thickness.

Now take other values of ρ and w nearer to the pole. Then both ρ and w will be diminished. On account of the diminution of ρ, which we suppose much greater than w, the vertex of the parabola will be raised; and because $\frac{w^2}{m}$ is diminished, the point at which it cuts the axis of β will be lowered (compare curves 1 and 2), and the ordinate corresponding to the former negative value of α will be less than it was before, and less still for the probably smaller value of α which corresponds to the diminished value of ρ. Hence the difference in the thickness of the strata will become less and less for the lower ones.

Next suppose the width of the rings which supply the annual strata at A B to increase as they approach the pole. Then the difference in thickness between the stratum derived from the ring under consideration and the next to it nearer to the pole is given by the ordinate corresponding to the assumed value of α on the right of the origin, and, as before, is positive unless α is greater than $\frac{w^2}{\rho-2w}$.

Now take other values of ρ and w nearer to the pole. Then ρ is diminished, while w is increased. On account of the diminution of ρ and increase of w, the vertex of the parabola will be brought nearer to the origin, and raised (but more so than in the previous case, when the rings diminished). And because $\frac{w^2}{m}$ is increased, the point at which the curve cuts the axis of β will be raised (compare curves 1 and 3), and the ordinate corresponding to the former positive value of α will be greater than it was before; and for a slightly greater value of α, such as we may presume belongs to the next pair of annual rings towards the pole, it will be still greater; and so on for the next position of the curve. Consequently in this case the difference between the thicknesses of the annual strata will for large values of ρ be at first small, and will become larger in descending. But should the value of α increase rapidly, as it may possibly do on approaching the pole, then the difference would begin to decrease.

But the strata which are visible above the water-level must certainly be derived from rings distant from the pole, for which the values of ρ are large. Hence, on the whole, we may conclude that in the visible ice-cliff the differences of thickness in the strata in descending (1) would be constant if the rings from which they are derived were of uniform width, (2) the difference would diminish if the rings diminished in width, and (3) it would increase if they increased in width.

It does not appear that any sufficiently close observations have been, or perhaps could be, made to determine the rate of decrease in the thickness of the successive strata. In the address referred to, Sir Wyville Thomson tells us that the ice-cliff of a berg was on an average about 200 feet high, that at about 80 feet below the top the strata were about a foot thick, and near the water-line about three inches. These data are not sufficient to warrant any conclusion beyond the mere fact of the diminution in thickness.

Let the points r and q be so taken, in fig. 1, that the time which a particle of ice takes to travel from N to K is equal to that which it takes a particle to travel from r to L; and similarly from M to L and from q to R; then, if N M and M Q supply annual strata at A B, it will take one year for a particle to travel from M to r, and the same period for one to travel from Q to q. Hence the mean velocity through M r : mean velocity through Q q : : M r : Q q.

But it may be fairly assumed that M r is greater or less than Q q, according as N M is greater or less than M Q. Consequently, if the rings supplying the annual strata increase in width, the velocity of the ice near the surface also increases, as the pole is approached, and *vice versâ*.

The permanence of height in the ice-cap must depend upon the fact, that the resolved part of the annual velocity near the surface in the vertical direction is equal to the depth of the annual snowfall. Consequently, for a given path, a widening of the rings corresponding to a greater velocity would be consonant with a thinner ice-cap, and a narrowing of them with a thicker one.

LXII. *Action of Light upon the Soluble Iodides, with the Outlines of a New Method in Actinometry.* *By* ALBERT R. LEEDS, *Ph.D.**

THE question as to whether potassium iodide, in dilute solution, is decomposed by free sulphuric acid, has frequently been made a matter of controversy. Schönbein contended that it was not, and, in an acrimonious reply to Prof. Fischer (*Journ. für prakt. Chem.* 1845, xxxiv. p. 492), impugned the purity of the latter's chemicals. The same ground was taken by Houzeau, in a discussion with M. L. Sauvage (*Comptes Rendus*, 1868, lxvii. pp. 633, 714, 1138), the former going so far as to state that, when the solutions were a thousand times dilute, no decomposition took place even on protracted boiling. These discrepancies appear to have originated from an oversight of the essential part played by air or oxygen in the reaction. This is represented by the general equation $MI + HA + O = MA + H_2O + I$, where M indicates the basic and A the acid radical, coefficients being omitted. This holds true not only of the ordinary mineral acids, but has been verified in the three of the organic acids experimented upon—oxalic, tartaric, and acetic acids. *In the dark* the decomposition diminishes with the increase of dilution, being indeterminable, when the dilution has reached the one-thousandth, at the end of twelve hours, but plainly recognizable when the dilution has attained the one four-thousandth at the expiration of five days. These figures apply more especially to the potassium-iodide and sulphuric-acid solutions, upon which very numerous quantitative determinations of the liberated iodine were made. *In sunlight* the amount of iodine set free increases in the same ratio as the increase in surface of exposure to the

* Communicated by the Author.